CSE 230
Intermediate Programming in C and C++
Recursion

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Stony Brook University

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What is recursion?

- Sometimes, the best way to solve a problem is by solving a smaller version of the exact same problem first
- Recursion is a technique that solves a problem by solving a smaller problem of the same type

Recursive Function

- A function is called recursive if it calls itself
- In C, all functions can be used recursively
- Example:

```
#include <stdio.h>

int main(void)
{
  printf("The universe is never ending\n");
  main();
  return 0;
}
```

– *This will act like an infinite loop*

Recursive Function: Example

- This code computes the sum of first n positive integers.
- For n = 4

```
int sum(int n)
{
    if(n <= 1)
        return n;
    else
        return (n+sum(n-1));
}
```

Function Call	Value returned
sum(1)	1
sum(2)	2+sum(1) or 2+1
sum(3)	3+sum(2) or 3+2+1
Sum(4)	4+sum(3) or 4+3+2+1

Recursive Function

- There is a **base case** (or cases) that is tested upon entry
- And a general **recursive case**
  - in which one of the variables, is passed as an argument in such a way as to ultimately lead to the base case.

```
int sum(int n)
{
    if(n <= 1)
        return n;
    else
        return (n+sum(n-1));
}
```

Problems Defined Recursively

- There are many problems whose solution can be defined recursively

Example: *factorial n*

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ (n-1)! * n & \text{if } n > 0 \end{cases} \qquad \text{(\textit{recursive} solution)}$$

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ 1*2*3*\ldots*(n-1)*n & \text{if } n > 0 \end{cases} \qquad \text{(\textit{closed form} solution)}$$

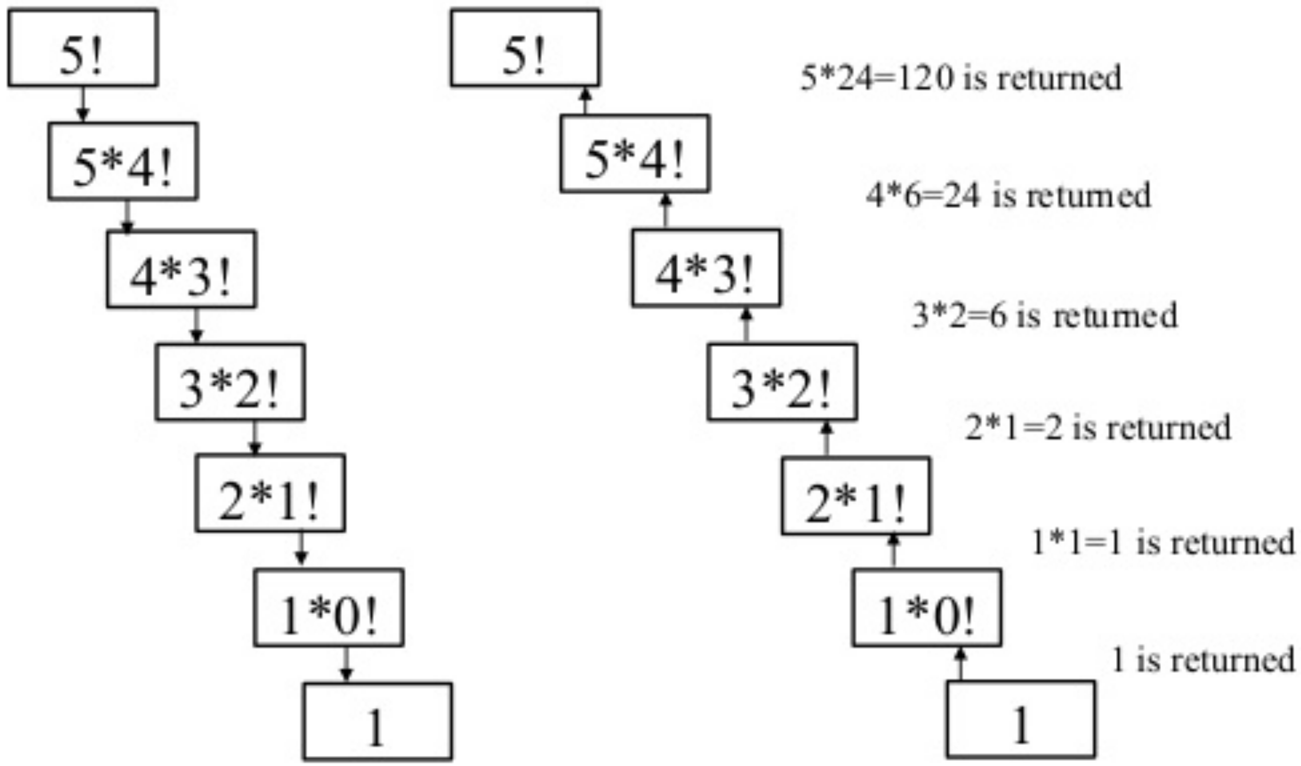
Coding the Factorial Function

- Recursive Implementation

```
int Factorial(int n)
{
 if (n==0)   // base case
    return 1;
 else
    return n * Factorial(n-1);
}
```

- For n > 12 this function will return incorrect value as the final result is too big to fit in an integer

Trace of Recursion: Factorial



Coding the Factorial Function (cont.)

- Iterative Implementation

```
int Factorial(int n)
{
 int fact = 1;

 for(int count = 2; count <= n; count++)
   fact = fact * count;

 return fact;
}
```

- Both recursive and iterative version returns same value

Another Example: *n choose k* (combinations)

- Given *n* things, how many different sets of size *k* can be chosen?

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}, \quad 1 < k < n$$ *(recursive solution)*

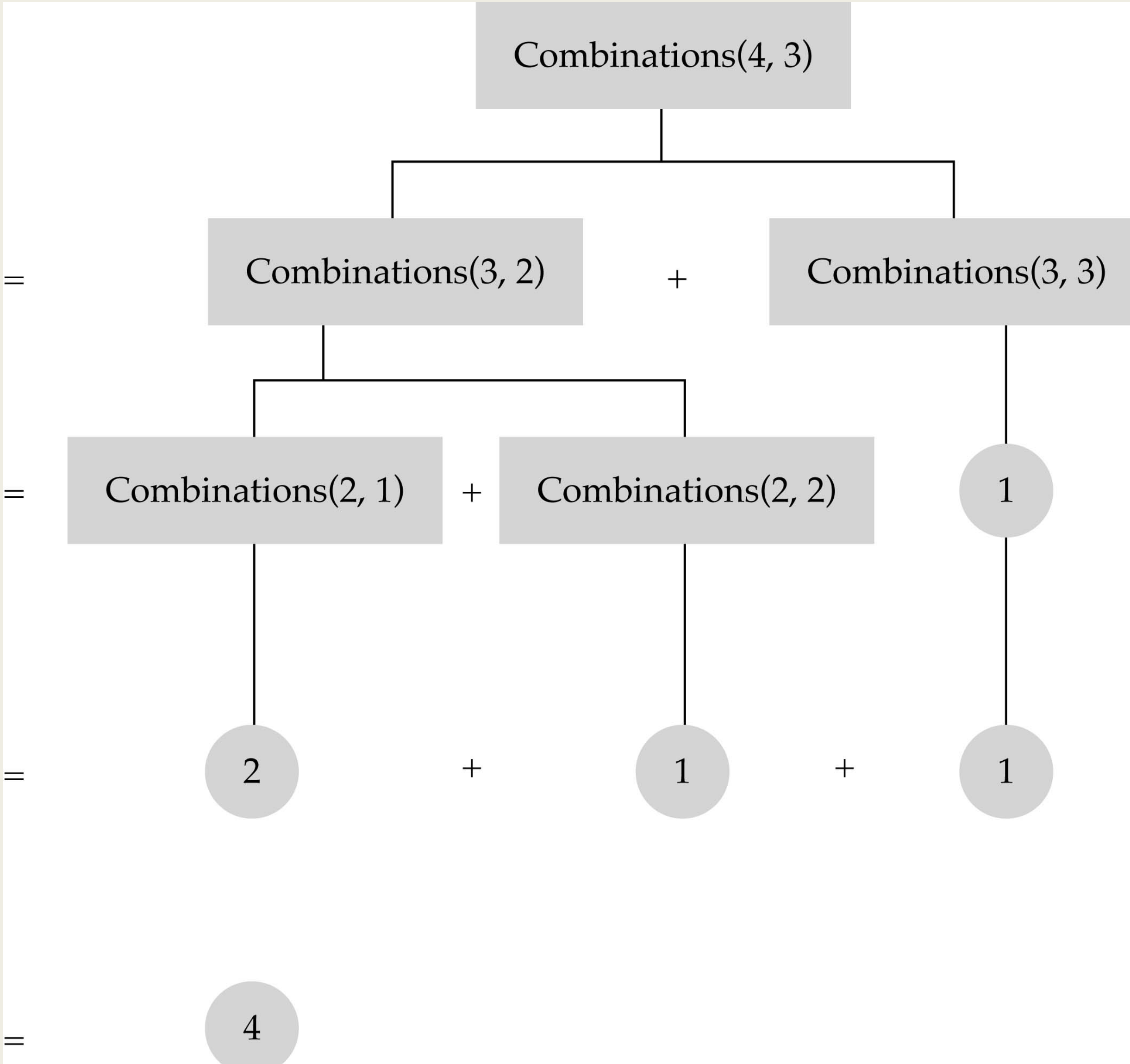
$$\binom{n}{k} = \frac{n!}{k!(n-k)!}, \quad 1 < k < n$$ *(closed-form solution)*

with base cases:

$$\binom{n}{1} = n \ \ (k = 1), \quad \binom{n}{n} = 1 \ \ (k = n)$$

n choose k implementation

```
int Combinations(int n, int k)
{
 if(k == 1)  // base case 1
   return n;
 else if (n == k)  // base case 2
   return 1;
 else
   return(Combinations(n-1, k) +
  Combinations(n-1, k-1));
}
```


Combinations(4, 3)
=
Combinations(3, 2)
+
Combinations(3, 3)
=
Combinations(2, 1)
+
Combinations(2, 2)
1
=
2
+
1
+
1
=
4

Recursion vs Iteration

- Iteration can be used in place of recursion
  - *An iterative algorithm uses a looping construct*
  - *A recursive algorithm uses a branching structure*
- Recursive solutions are often less efficient, in terms of both *time* and *space*, than iterative solutions
- Recursion can simplify the solution of a problem, often resulting in shorter, more easily understood source code

How to write a recursive function?

- Determine the size factor
- Determine the base case(s)

  (the one for which you know the answer)
- Determine the general case(s)

  (the one where the problem is expressed as a smaller version of itself)
- Verify the algorithm

  (use the "Three-Question-Method")

Three Question Verification

1. The Base-Case Question
   - *Is there a non-recursive way out of the function, and does the routine work correctly for this "base" case?*
2. The Smaller-Caller Question
   - *Does each recursive call to the function involve a smaller case of the original problem, leading inescapably to the base case?*
3. The General-Case Question
   - *Assuming that the recursive call(s) work correctly, does the whole function work correctly?*

Recursion: Calculation of Fibonacci Sequence

- Recursive solution

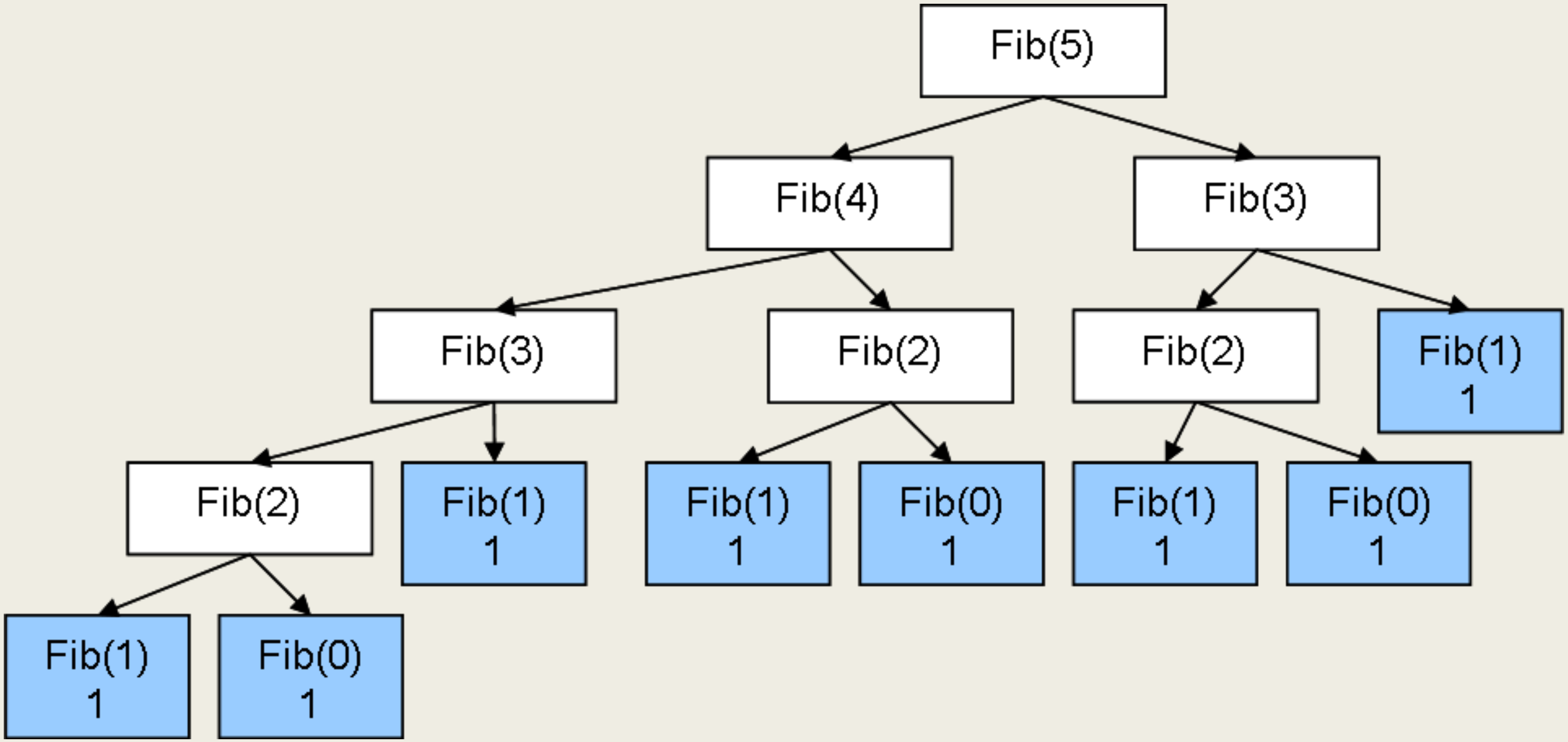
  $f_0 = 0, f_1 = 1, f_{i+1} = f_i + f_{i-1}$, for $i = 1, 2, \ldots$

  – *Except for* f_0 and f_1*, every element in the sequence is the sum of the previous two elements*

- The sequence begins 0, 1, 1, 2, 3, 5, 8, …

```
int Fibonacci(int n)
{
 if(n <= 1)  // base case
   return n;
else
   return(Fibonacci(n-1) + Fibonacci(n-2));
}
```

Recursion: Calculation of Fibonacci Sequence



Number of Function Calls for Recursive Fibonacci

Value of n	Value of Fibonacci(n)	#of function calls
0	0	1
1	1	1
2	1	3
…	…	…
23	28657	92735
24	46368	150049
…	…	…
42	267914296	866988873
43	433494437	1402817465

A large number of function call is required to compute the nth fibonacci for even moderate values of n

Pitfalls of Recursion

- Missing base case – failure to provide an escape case.
- No guarantee of convergence – failure to include within a recursive function a recursive call to solve a subproblem that is not smaller.
- Excessive space requirements - a function calls itself recursively an excessive number of times before returning; the space required for the task may be prohibitive.
- Excessive recomputation – illustrated in the recursive Fibonacci method which ignores that several sub-Fibonacci values have already been computed.