Propositional Resolution Part 2

Short Review

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GOAL: Use Resolution to prove/ disapprove $\models A$

PROCEDURE

Step 1: Write $\neg A$ and transform $\neg A$ info set of clauses **$CL_{\{\neg A\}}$** using Transformation rules.

Step 2: Consider **$CL_{\{\neg A\}}$** and look at if you can get a deduction of {} from **$CL_{\{\neg A\}}$**

ANSWER

1. **$CL_{\{\neg A\}}$** $\vdash_R \{\}$ – Yes, $\models A$
2. **$CL_{\{\neg A\}}$** $\nvdash \{\}$ (i.e. you never get {}) – No, $\not\models A$

Rules of transformation

- Rules of transformation of a formula A into a logically equivalent set of clauses CL_A
- **Rule U: (AUB) ______ A,B + Information**

What "Information" mean?

Example: a, b, (a U ¬(a=> b)), ¬c

a, b, a ,¬(a=> b), ¬c

a,b, ¬c is Information

Rule (U) : I , (AUB), J

I, A, B, J

I,J --- Information

Implication Rule (=>)

- **I, (A=>B), J** **(A=>B)**

I, ¬A, B, J **¬A , B**

Example: a, (a U b), (a => ¬a), (a ∧ b), c

(=>)

a, (a U b), ¬ a, ¬ a, (a ∧ b), c

(U)

a, a, b, ¬ a, ¬ a, (a ∧ b), c

next step?

we need (∧) Rule!

Conjunction Rule (∧)

I, (A ∧B), J

(∧)

I , A, J **I , B, J**

(A∧B)

(∧)

A **B**

Example:

a, a, b, ¬a, ¬a, (a ∧ b), c

(∧)

a, a, b, ¬a, ¬a, a, c

a, a, b, ¬a, ¬a, b, c

STOP when get only literals and form clauses out of leaves

Conjunction Rule (∧)

- {a, a, b, ¬a, ¬a, a, c} = {a, b, ¬a, c}
- {a, a, b, ¬a, ¬a, b, c} = {a, b, ¬a, c}
- Observe that we en-up with only one set of clauses
- $\triangle$ = {{a, b, ¬a, c}}

Negation of Implication Rule (¬ =>)

I, ¬ (A =>B), I

(¬=>)

I , A, I   I , ¬B, I

¬ (A => B)

(¬=>)

A   ¬B

Example:

a, b, a, ¬ (a => b), ¬ c

(¬=>)

a, b, a, a, ¬c   a, b, a, ¬b, ¬c

Stop – when only literals :

Form clauses out of a, b, a, a, ¬c   and

a, b, a, ¬b, ¬c

Clauses

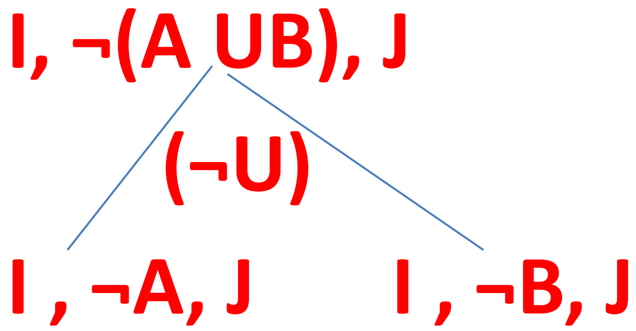
- Leaf: a, b, a, a, ¬c   makes clause {a, b, ¬c}
- Leaf: a, b, a, ¬b, ¬c makes clause {a, b, ¬b, c}

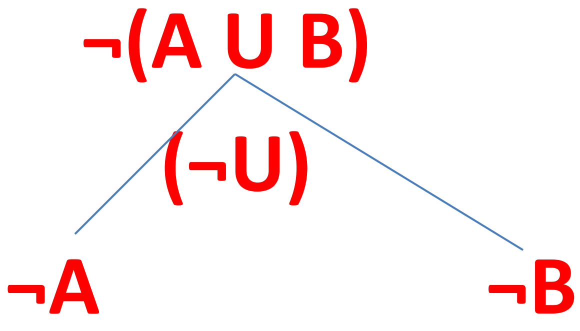
- **CL** = {{a, b, ¬c}, {a, b, ¬b, c}}

- **CL** is set of clauses corresponding to

  a, b, a, ¬ (a => b), ¬ c

Negation of Disjunction Rule (¬ U)





- Use DeMorgan Law:

  ¬(AUB) ≡ (¬A ∧ ¬B)

Negation of Conjunction Rule (¬∧)

I, ¬(A ∧B), J

(¬∧)

I , ¬A, ¬B, J

¬(A ∧ B)

(¬∧)

¬A, ¬B

- Use :

  ¬(A ∧ B) ≡ (¬A U ¬B)

Negation of Negation Rule (¬¬)

I, ¬¬ (A), J

(¬¬)

I , A, J

¬¬(A)

(¬¬)

A

- Use :

¬¬ (A) ≡ A

- Transformation Rules :
- (Λ), (U), (=>), (¬Λ), (¬U), (¬=>)

Transformation Rules Shorthand Form

(AUB) (U)

A, B

(A ∧B) (∧)

A B

(A=>B) (=>)

¬A, B

¬¬A (¬¬)

A

¬(A U B) (¬U)

¬A ¬B

¬(A ∧ B) (¬∧)

¬A, ¬B

¬(A => B) (¬=>)

A ¬B

+ Keep all Information

End when all leaves are literals

Example

- Let A be a Formula $(((a \Rightarrow \neg b) \, U \, c) \wedge (\neg a \, U \, \neg b))$
- Find $\mathbf{CL}_A$
- $(((a \Rightarrow \neg b) \, U \, c) \wedge (\neg a \, U \, \neg b))$

$((a \Rightarrow \neg b) \, U \, c)$ $\qquad$ $(\neg a \, U \, \neg b)$

$(a \Rightarrow \neg b), c$ $\qquad$ $\{\neg a, b\}$ **STOP**

$\{\neg a, \neg b, c\}$ **STOP**

$\mathbf{CL}_A = \{\{\neg a, \neg b, c\}, \{\neg a, b\}\}$

$A \equiv \mathbf{CL}_A$

ARGUMENTS (rules of inference)

- From (premises) $A_1, \ldots\ldots, A_n$ we conclude C

A_1 or $\dfrac{A_1, \ldots\ldots, A_n}{C}$

:

:

$\underline{A_n}$

C

Definition:

Argument $\dfrac{A_1, \ldots\ldots, A_n}{C}$ is VALID iff

$$\models ((A1 \wedge \ldots\ldots, \wedge An \Rightarrow C)$$

ARGUMENTS

- Otherwise

  Argument is NOT VALID

Valid Arguments ≡ Tautologically Valid

$A_1, \ldots\ldots, A_n, C$

Are formulas of Propositional or Predicate Language

Validity of Arguments

Remember: $\models A$ iff $=\mid \neg A$

Tautology (always true), Contradiction (always false)

This means that if we want to decide $\models A$ we decide $=\mid \neg A$ and use Resolution for that

STEPS: **Step 1:** Negate A; i.e. take ¬A and find set of clauses corresponding to ¬A i.e. $\mathbf{CL}_{\{\neg A\}}$

Step 2: Use Completeness of Resolution

$\models A$ iff $\mathbf{CL}_{\{\neg A\}} \vdash_R \{\}$ i.e.

1. Look for deduction of {}:
2. if YES – we have $\models A$
3. If there is no deduction of {} we have: $\not\models A$

Basic Theorems

1. $=|$ **CL** iff **CL** $\vdash_R \{\}$

   CL is inconsistent iff there is a resolution deduction of $\{\}$ from **CL**

2. For any formula A, there is a set of clauses **CL**$_A$ such that $A \equiv$ **CL**$_A$

3. $|= A$ iff $=| \neg A$,

By 2 we get that

$|= A$ iff $=|$ **CL**$_{\{\neg A\}}$

And by 1 and 3 we get

4. $|= A$ iff **CL**$_{\{\neg A\}} \vdash_R \{\}$

Exercise

- Prove By Propositional Resolution
- $\models (\neg(a \Rightarrow b) \Rightarrow (a \wedge \neg b))$
- Remember: $\models A$ iff $=\mid \neg A$ + use resolution

Steps

Step 1: Find set of clauses corresponding to ¬A
i.e. **$CL_{\{\neg A\}}$**

Step 2: Find deduction of {} from **$CL\{\neg A\}$**
i.e. show that **$CL_{\{\neg A\}} \vdash_R \{\}$**

DO IT!

Exercise Solution

- **Step 1:** Negate A; ¬A and find the set of clauses for ¬A i.e. **CL**$_{\{\neg A\}}$
- ¬(¬(a=>b) => (a∧¬b))

¬(a=>b) ¬(a∧¬b)

a ¬b ¬a, ¬¬b

{a} {¬b} {¬a, b}

CL $_{\{\neg A\}}$ ={{a}, {¬b}, {¬a, b}}

{b}

{}

Step 2: Check if **CL**$_{\{\neg A\}} \vdash_R$ {} – **YES!**

Remark: $\not\models$A iff there is no deduction of {} from **CL**$_{\{\neg A\}}$

Back To Arguments

- **Use resolution to show that from $A_1, \ldots\ldots, A_n$ we can deduce B**
- "We can" deduce B from $A_1, \ldots\ldots, A_n$ means validity of argument

$$\frac{A_1, \ldots\ldots, A}{B}$$

| | iff By definition

$$\models (A_1 \wedge \ldots\ldots \wedge A_n \Rightarrow B)$$

We have to use Resolution to prove this Tautology

Arguments

- $\models (A_1 \wedge \ldots\ldots. \wedge A_n \Rightarrow B)$ iff
- $=\mid \neg (A_1 \wedge \ldots\ldots. \wedge A_n \Rightarrow B)$ iff
- $=\mid (A_1 \wedge \ldots\ldots. \wedge A_n \wedge \neg B)$
- **Step 1:** we transform $(A_1 \wedge .. \wedge A_n \wedge \neg B)$ to clauses
- Take A1,……., An and find **CL_{A1} U ….U CL_{An}**
- Also find **$CL_{\neg B}$.**

All Together form

CL_{A1} U …. U CL_{An} U $CL_{\neg B}$ = CL

Step 2: examine whether **$CL \vdash_R \{\}$**

Remember

- **Argument $\dfrac{A1, \ldots\ldots, An}{B}$ is valid iff**

$$CL_{A1} \cup \ldots \cup CL_{An} \cup CL_{\neg B} \vdash_R \{\}$$

Not Valid

iff never $CL_{A1} \cup \ldots \cup CL_{An} \cup CL_{\neg B} \vdash_R \{\}$

A lot of cases strategies (complete) of resolution cut down number of cases to consider

Example

Check if you can deduce B = (¬(a U ¬ b)=> (¬ a ∧ b))

from A1 = ((a=>¬b)=>a) and A2 = (a=>(b=>a))

Procedure:

1. **Find** $\mathbf{CL}_{\{A1\}}$, $\mathbf{CL}_{\{A2\}}$ and $CL_{\{\neg B\}}$
2. **Form** $\mathbf{CL} = \mathbf{CL}_{\{A1\}} \cup \mathbf{CL}_{\{A2\}} \cup \mathbf{CL}_{\{\neg B\}}$
3. Check if $\mathbf{CL} \vdash_R \{\}$ or never $\mathbf{CL} \vdash_R \{\}$

Yes, we can

No, we can’t

Example Solution

A1 = ((a => ¬b) => a)

¬(a=>¬b), a

a, a

¬¬b, a

b, a

CLA1 = {{a}, { b, a}}

A2 = ((a=>(b=>a))

¬a , (b=>a)

¬a, ¬b, a

CLA2 = {¬a, ¬b, a}

Example Solution

- $\neg B = \neg(\neg(a \cup \neg b) \Rightarrow (\neg a \wedge b))$

$\neg(a \cup \neg b)$ $\qquad\qquad$ $\neg(\neg a \wedge b)$

$\neg a$ $\qquad$ $\neg\neg b$ $\qquad\qquad$ $\neg\neg a, \neg b$

b $\qquad\qquad$ $a, \neg b$

CL = {{a}, {b, a}, {¬a, ¬b, a}, {¬a}, {b}, {a, ¬b}}

Remove Tautology Strategy (later)

CL' = {{a}, {b, a}, {¬a}, {b}, {a, ¬b}}

Example Solution

- **CL** = {{a}, {b, a}, {¬a, ¬b, c}, {¬a}, {b}, {a, ¬b}}

{a} (on b)

{ }

Yes Argument is Valid

Next : Strategies for Resolution