Classification Lecture Notes (3) (cse352)

DECISION TREE CLASSIFICATION

(Supervised Learning)

Professor Anita Wasilewska

Classification Learning
ALGORITHMS
Different Classifiers

- DESCRIPTIVE:
- Decision Trees (ID3, C4.5)
- Rough Sets
- Genetic Algorithms

- STATISTICAL:
- Neural Networks
- Bayesian Networks

Classification Data

- **Data format:** a data table with key attribute removed. Special attribute- class attribute must be distinguished

age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31…40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31…40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31…40	medium	no	excellent	yes
31…40	high	yes	fair	yes
>40	medium	no	excellent	no

Classification (Training) Data with objects

rec	Age	Income	Student	Credit_rating	Buys_computer (CLASS)
r1	<=30	High	No	Fair	No
r2	<=30	High	No	Excellent	No
r3	31…40	High	No	Fair	Yes
r4	>40	Medium	No	Fair	Yes
r5	>40	Low	Yes	Fair	Yes
r6	>40	Low	Yes	Excellent	No
r7	31…40	Low	Yes	Excellent	Yes
r8	<=30	Medium	No	Fair	No
r9	<=30	Low	Yes	Fair	Yes
r10	>40	Medium	Yes	Fair	Yes
r11	<=30	Medium	Yes	Excellent	Yes
r12	31…40	Medium	No	Excellent	Yes
r13	31…40	High	Yes	Fair	Yes
r14	>40	Medium	No	Excellent	No

Classification
by Decision Tree Induction

- Decision tree is

  A flow-chart-like tree structure;

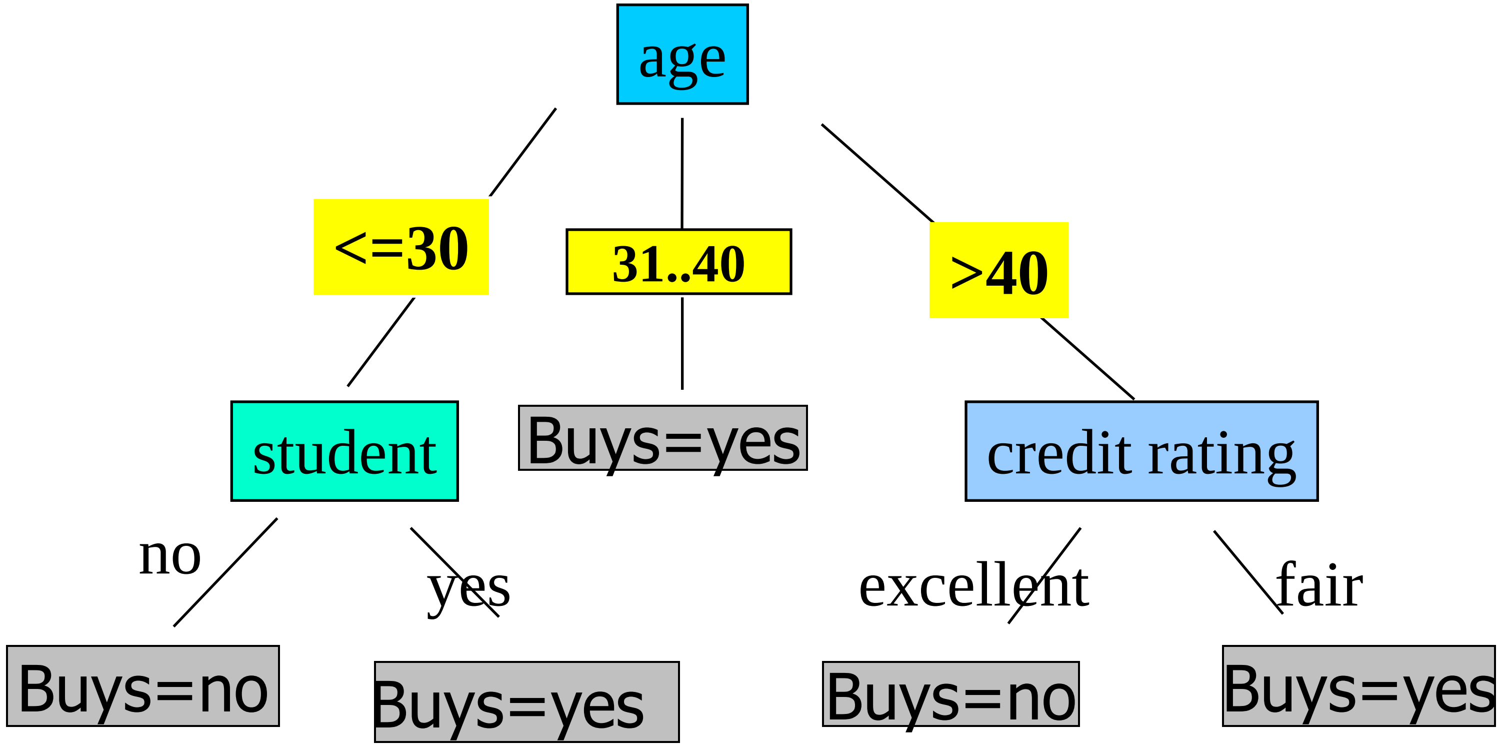
  Internal node denotes an attribute;

  Branch represents the values of the node attribute;

  Leaf nodes represent class labels or class distribution

DECISION TREE

An Example



Classification
by Decision Tree Induction

- **Decision tree generation** consists of two phases

Tree construction

- **We choose recursively internal nodes (attributes) with their proper values as branches.**
- **At start we choose one attribute as the root and put all its values as branches**
- **We Stop when all the samples (records) are of the same class, then the node becomes the leaf labeled with that class**
- **or there is no more samples (records) left. In this case we apply MAJORITY VOTING to classify the node.**
- **Majority Voting involves converting the node into a leaf and labeling it with the most common class in the training set.**

- **Some algorithms allow majority voting at any level of the tree, i.e. converting a given node into a leaf and labeling it with the most common class at the node.**
- **We call it General Majority Voting.**

Tree pruning

- **Identify and remove branches that reflect noise or outliers**

Classification by Decision Tree Induction

Crucial point

Good choice of the root attribute and internal nodes attributes is a crucial point.

Bad choice may result, in the worst case in a just another knowledge representation: relational table re-written as a tree with class attributes (decision attributes) as the leaves.

- **Decision Tree Induction Algorithms differ on methods of evaluating and choosing the root and internal nodes attributes.**

Decision Tree CONSTRUCTION:
Example 1.

Consider our TRAING Dataset (next slide)

We build the DECISION TREE choosing

the attribute AGE as the root of the tree.

Training Data with objects

rec	Age	Income	Student	Credit_rating	Buys_computer(CLASS)
r1	<=30	High	No	Fair	No
r2	<=30	High	No	Excellent	No
r3	31…40	High	No	Fair	Yes
r4	>40	Medium	No	Fair	Yes
r5	>40	Low	Yes	Fair	Yes
r6	>40	Low	Yes	Excellent	No
r7	31…40	Low	Yes	Excellent	Yes
r8	<=30	Medium	No	Fair	No
r9	<=30	Low	Yes	Fair	Yes
r10	>40	Medium	Yes	Fair	Yes
r11	<=30	Medium	Yes	Excellent	Yes
r12	31…40	Medium	No	Excellent	Yes
r13	31…40	High	Yes	Fair	Yes
r14	>40	Medium	No	Excellent	No

Building The Tree: we choose "age" as a root

age

<=30

income	student	credit	class
high	no	fair	no
high	no	excellent	no
medium	no	fair	no
low	yes	fair	yes
medium	yes	excellent	yes

>40

income	student	credit	class
medium	no	fair	yes
low	yes	fair	yes
low	yes	excellent	no
medium	yes	fair	yes
medium	no	excellent	no

31…40

income	student	credit	class
high	no	fair	yes
low	yes	excellent	yes
medium	no	excellent	yes
high	yes	fair	yes

Building The Tree: “age” as the root

age

<=30

income	student	credit	class
high	no	fair	no
high	no	excellent	no
medium	no	fair	no
low	yes	fair	yes
medium	yes	excellent	yes

>40

income	student	credit	class
medium	no	fair	yes
low	yes	fair	yes
low	yes	excellent	no
medium	yes	fair	yes
medium	no	excellent	no

31…40

class=yes

Building The Tree: we chose "student" on <=30 branch

age

<=30

>40

student

no

yes

in	cr	cl
h	f	n
h	e	n
m	f	n

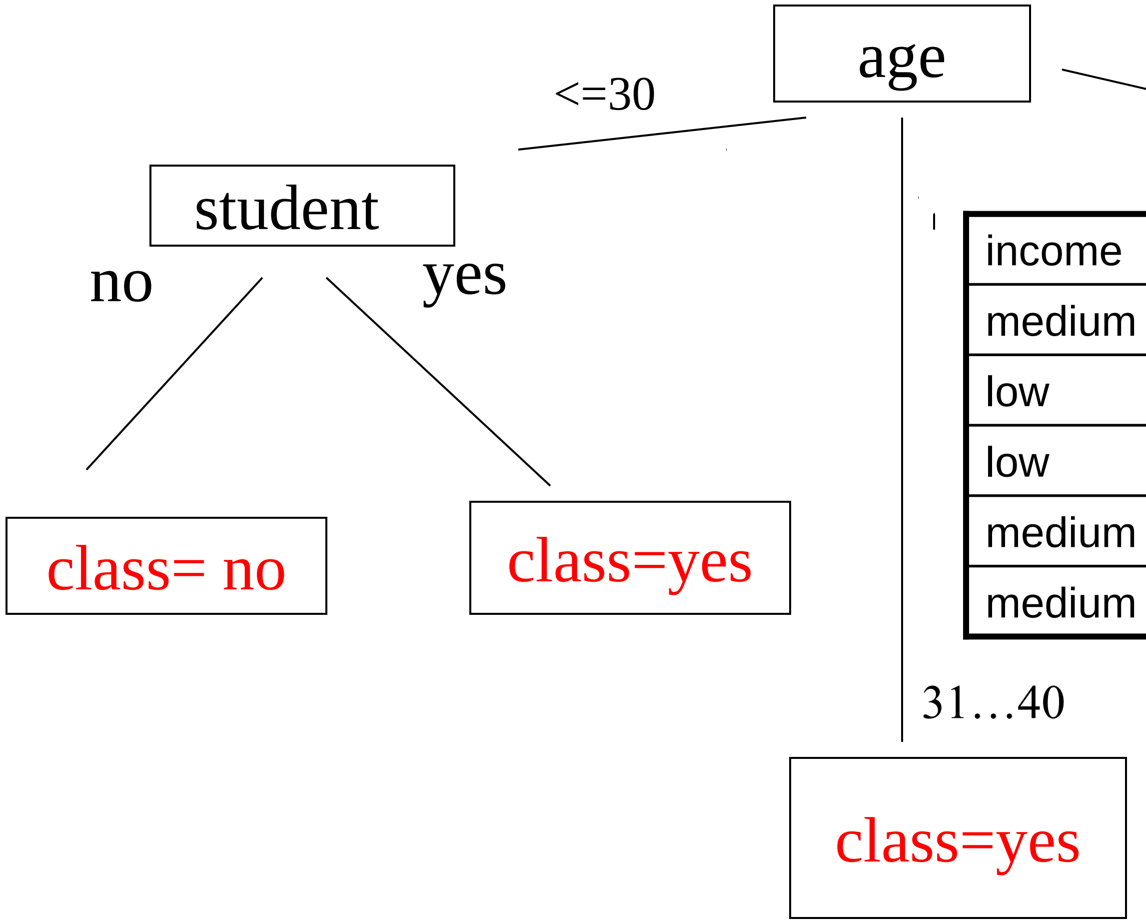
in	cr	cl
l	f	y
m	e	y

income	student	credit	class
medium	no	fair	yes
low	yes	fair	yes
low	yes	excellent	no
medium	yes	fair	yes
medium	no	excellent	no

31…40

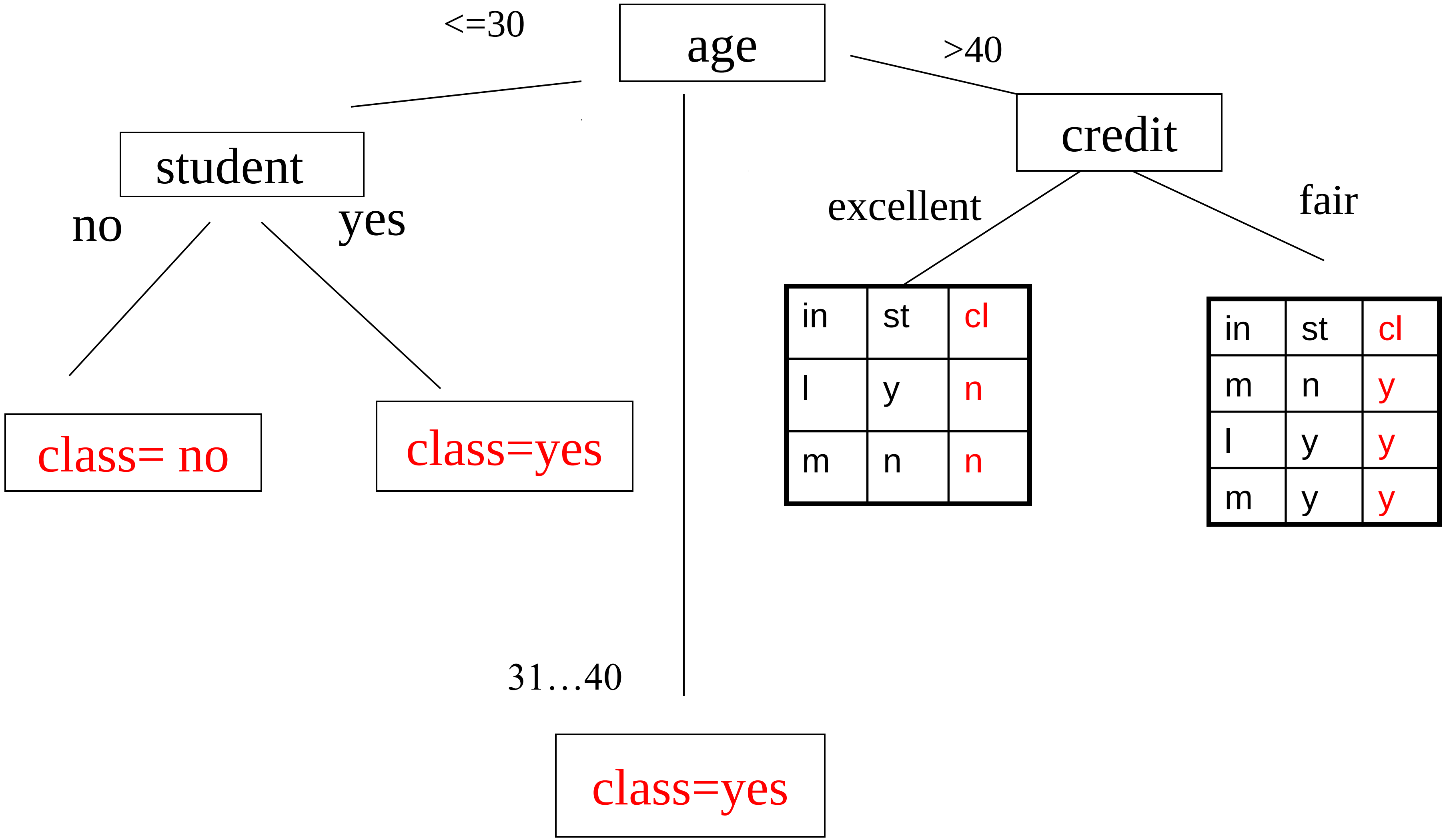
class=yes

Building The Tree: we chose “student” on <=30 branch



income	student	credit	class
medium	no	fair	yes
low	yes	fair	yes
low	yes	excellent	no
medium	yes	fair	yes
medium	no	excellent	no

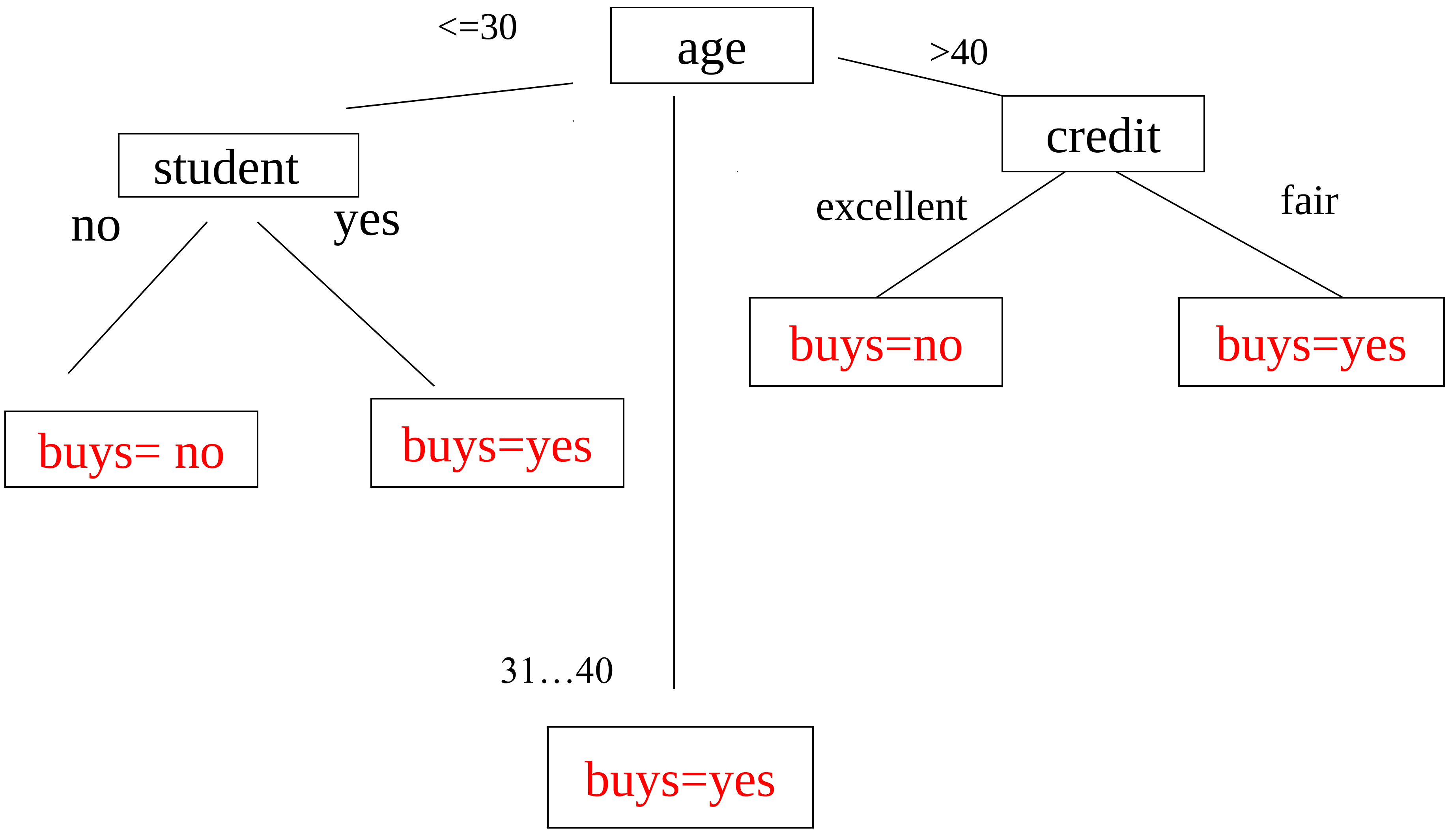
Building The Tree: we chose "credit" on >40 branch



in	st	cl
l	y	n
m	n	n

in	st	cl
m	n	y
l	y	y
m	y	y

Finished Tree for class="buys"



Extracting Classification Rules from Trees

- Goal: Represent the knowledge in the form of
- IF-THEN rules (determinant);
- One rule is created for each path from the root to a leaf;
- Each attribute-value pair along a path forms a conjunction;
- The leaf node holds the class prediction;
- Rules are easier for humans to understand

RULES (Discriminant) extracted from our TREE

- The rules are:

IF *age* = "<=30" AND *student* = "*no*" THEN *buys_computer* = "*no*"

IF *age* = "<=30" AND *student* = "*yes*" THEN *buys_computer* = "*yes*"

IF *age* = "31…40" THEN *buys_computer* = "*yes*"

IF *age* = ">40" AND *credit_rating* = "*excellent*" THEN *buys_computer* = "no"

IF *age* = ">40" AND *credit_rating* = "*fair*" THEN *buys_computer* = "yes"

Rules format for testing and applications

- In order to use rules for testing, and later when testing is done and predictive accuracy is acceptable we write rules in a predicate form:

IF *age*(x, <=30) AND *student(x, no)* THEN

buys_computer (x, *no*)

IF *age(x,* <=30) AND *student* (x, *yes*) THEN

buys_computer (x, *yes)*

- Attributes and their values of the new record x are matched with the IF part of the rule and the record is is classified accordingly to the THEN part of the rule.

Exercise: Predictive Accuracy

Calculate the predictive accuracy of our set of rules with respect of the TEST data given by the next slide.

R1: IF *age* = "<=30" AND *student* = "*no*" THEN *buys_computer* = "*no*"

R2: IF *age* = "<=30" AND *student* = "*yes*" THEN *buys_computer* = "*yes*"

R3: IF *age* = "31…40" THEN *buys_computer* = "*yes*"

R4: IF *age* = ">40" AND *credit_rating* = "*excellent*" THEN *buys_computer* = "no"

R5: IF *age* = ">40" AND *credit_rating* = "*fair*" THEN *buys_computer* = "yes"

TEST Data
for predictive accuracy evaluation

rec	Age	Income	Student	Credit_rating	Buys_computer(CLASS)
r1	<=30	Low	No	Fair	yes
r2	<=30	High	yes	Excellent	No
r3	<=30	High	No	Fair	Yes
r4	31…40	Medium	yes	Fair	Yes
r5	>40	Low	Yes	Fair	Yes
r6	>40	Low	Yes	Excellent	yes
r7	31…40	High	Yes	Excellent	Yes
r8	<=30	Medium	No	Fair	No
r9	31…40	Low	no	Excellent	Yes
r10	>40	Medium	Yes	Fair	Yes

Basic Idea of ID3/C4.5 Algorithm (1)

- The basic algorithm for decision tree induction is a greedy algorithm that constructs decision trees in a top-down recursive divide-and – conquer manner.
- The basic strategy is as follows.
- Tree STARTS as a single node representing all training dataset (data table with records called samples)
- IF the samples (records in the data table) are ALL in the same class, THEN the node becomes a LEAF and is labeled with that class

  (some algorithms apply general majority voting or other method to decide the class on the leaf at any level of the tree)
- OTHERWISE, the algorithm uses an entropy-based measure known as *information gain* as a heuristic for selecting the ATTRIBUTE that will best separate the samples (split the data table) into individual classes. This attribute becomes the node-name (test, or tree split decision attribute)

Basic Idea of ID3/C4.5 Algorithm (2)

- A branch is created for each value of the node-attribute (and is labeled by this value - this is syntax) and the samples (data table at the node) are partitioned accordingly
- The algorithm uses the same process recursively to form a decision tree at each partition.
- **Observe that once an attribute has occurred at a node, it need not be considered in any other of the node's descendants**
- The recursive partitioning **STOPS** only when any one of the following conditions is true.

Basic Idea of ID3/C4.5 Algorithm (3)

- TERMINATION CONDITIONS:
- All records (samples) for the given node belong to the same class or
- There are no remaining attributes on which the samples (records in the data table) may be further partitioned.
- In this case we convert the given node into a LEAF and label it with the class in majority among original training samples.
- This is called a ***majority voting***
- There is no records (samples) left – a leaf is created with majority vote for training sample

Heuristics: Attribute Selection Measures

- Construction of the tree depends on the order in which root attributes are selected.
- Different choices produce different trees; some better, some worse
- Shallower trees are better; they are the ones in which classification is reached in fewer levels.
- These trees are said to be more efficient as the classification, and hence termination is reached quickly

Attribute Selection Measures

- Given a training data set (set of training samples) there are many ways to choose the root and nodes attributes while constructing the decision tree
- **Some possible choices:**
- Random
- Attribute with smallest/largest number of values
- Following certain order of attributes
- We present here a special order: information gain as a measure of the goodness of the split
- The attribute with the highest information gain is always chosen as the split decision attribute for the current node while building the tree.

Information Gain Computation (ID3/C4.5): Case of Two Classes

- **Assume there are two classes, *P* (positive) and *N* (negative)**

Let S be a training data set consisting of s examples (records):
$|S|=s$

And S contains p elements of class P and n elements of class N

The amount of information, needed to decide if an arbitrary example (record) in S belongs to P or N is defined as

$$I(p,n) = -\frac{p}{p+n}\log_2\frac{p}{p+n} - \frac{n}{p+n}\log_2\frac{n}{p+n}$$

Information Gain Measure

- Assume that using attribute **A** a set *S* will be partitioned into sets $\{S_1, S_2, \ldots, S_v\}$ (v is number of values of the attribute A)

If S_i contains p_i examples of P and n_i examples of N, the entropy E(**A**), or the expected information needed to classify objects in all sub-trees S_i is

$$E(A) = \sum_{i=1}^{v} \frac{p_i + n_i}{p + n} I(p_i, n_i)$$

- The encoding information that would be gained by branching on *A*

$$Gain(A) = I(p, n) - E(A)$$

Example: Attribute Selection by Information Gain Computation

- **Class P: buys_computer = "yes"**
- **Class N: buys_computer = "no"**
- **I(p, n) = I(9, 5) =0.940**
- **Compute the entropy for**

age	p_i	n_i	$I(p_i, n_i)$
<=30	2	3	0.971
31…40	4	0	0
>40	3	2	0.971

$$E(age) = \frac{5}{14} I(2,3) + \frac{4}{14} I(4,0) + \frac{5}{14} I(3,2) = 0.694$$

Hence

$$Gain(age) = I(p,n) - E(age)$$

Gain(age)=0.246

Similarly

$$Gain(income) = 0.029$$

$$Gain(student) = 0.151$$

$$Gain(credit\_rating) = 0.048$$

The attribute "age" becomes the root.

Decision Tree Induction, Predictive Accuracy and Information Gain Calculations

EXAMPLES

Example 1

TASK: Use Decision Tree Induction algorithm

Use different choices of the root attribute to FIND discriminant rules that determine whether a person buys a computer or not.

Compute Information gain for all nodes of the tree.

1. We choose attribute *buys_computer* as the CLASS attribute.
2. We perform DT algorithm "by hand" using different choices of the root attribute, and different "by hand" choices of the following nodes.
3. We build two trees with attributes: *Income* and *Credit Rating* respectively, as the root attribute to derive rules

Training Data

age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31…40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31…40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31…40	medium	no	excellent	yes
31…40	high	yes	fair	yes
>40	medium	no	excellent	no

Training Data with objects

rec	Age	Income	Student	Credit_rating	Buys_computer
r1	<=30	High	No	Fair	No
r2	<=30	High	No	Excellent	No
r3	31…40	High	No	Fair	Yes
r4	>40	Medium	No	Fair	Yes
r5	>40	Low	Yes	Fair	Yes
r6	>40	Low	Yes	Excellent	No
r7	31…40	Low	Yes	Excellent	Yes
r8	<=30	Medium	No	Fair	No
r9	<=30	Low	Yes	Fair	Yes
r10	>40	Medium	Yes	Fair	Yes
r11	<=30	Medium	Yes	Excellent	Yes
r12	31…40	Medium	No	Excellent	Yes
r13	31…40	High	Yes	Fair	Yes
r14	>40	Medium	No	Excellent	No

EXAMPLE 1 Solution 1

- **BOTH TREES of the following Example 1 Solutions ARE NOT CORRECT !!!**
- **FIND STEPS** where the construction didn’t follow the ALGORITHM and **CORRECT THEM!!**
- Write the CORRECT Solutions for the EXAMPLE 1
- Perform Exercises 1 and 2 for the corrected trees.

Index: 1

Income

Gain=0.027

Low

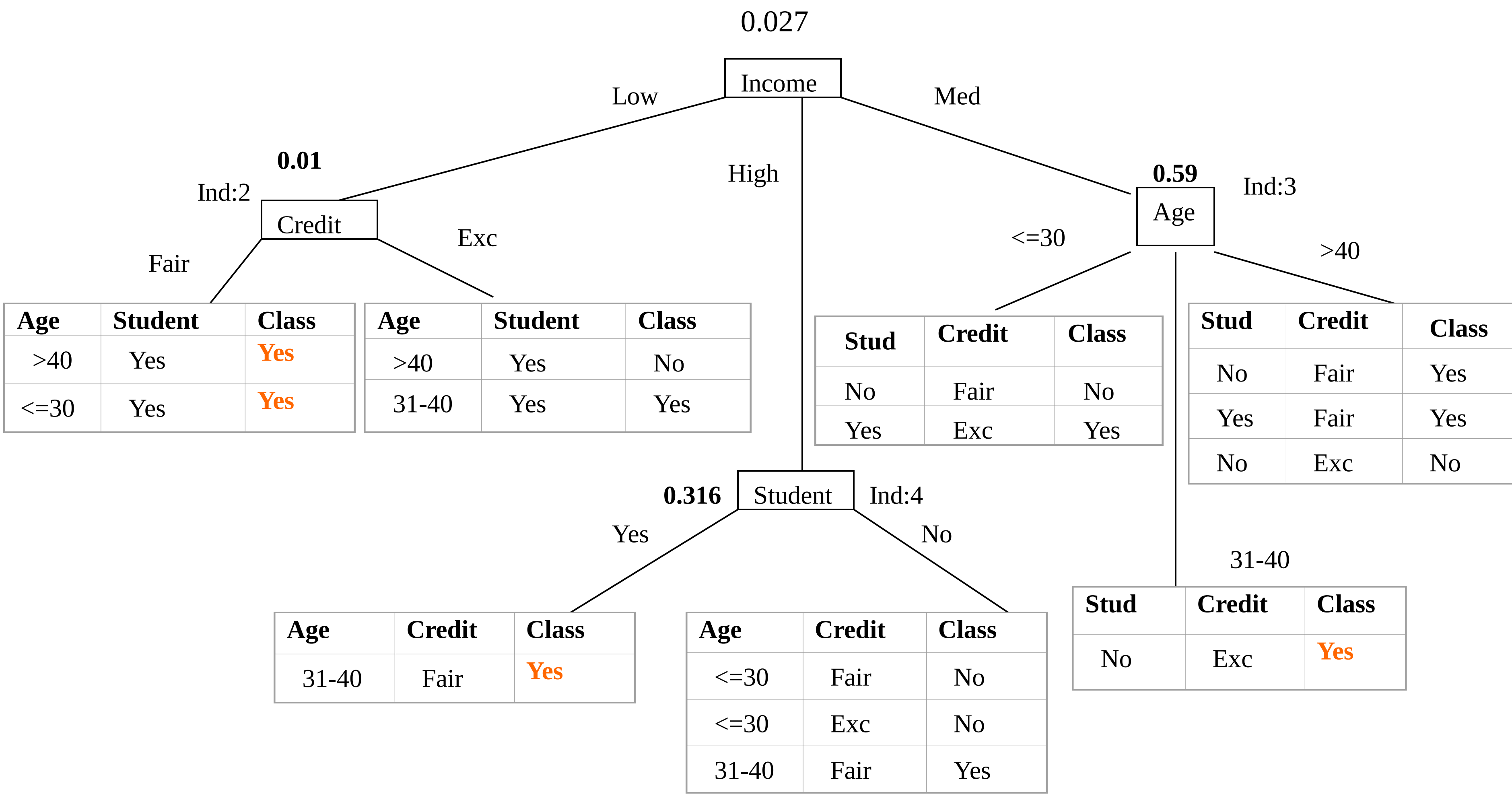
Age	Student	Credit	Class
>40	Yes	Fair	Yes
>40	Yes	Exc	No
31-40	Yes	Exc	Yes
<=30	Yes	Fair	Yes

High

Age	Student	Credit	Class
<=30	No	Fair	No
<=30	No	Exc	No
31-40	No	Fair	Yes
31-40	Yes	Fair	Yes

Med

Age	Student	Credit	Class
>40	No	Fair	Yes
<=30	No	Fair	No
>40	Yes	Fair	Yes
<=30	Yes	Exc	Yes
31-40	No	Exc	Yes
>40	No	Exc	No



0.027
Income
Low
High
Med
0.01
Ind:2
Credit
Fair
Exc

Age	Student	Class
>40	Yes	Yes
<=30	Yes	Yes

Age	Student	Class
>40	Yes	No
31-40	Yes	Yes

0.59
Age
Ind:3
<=30
>40
31-40

Stud	Credit	Class
No	Fair	No
Yes	Exc	Yes

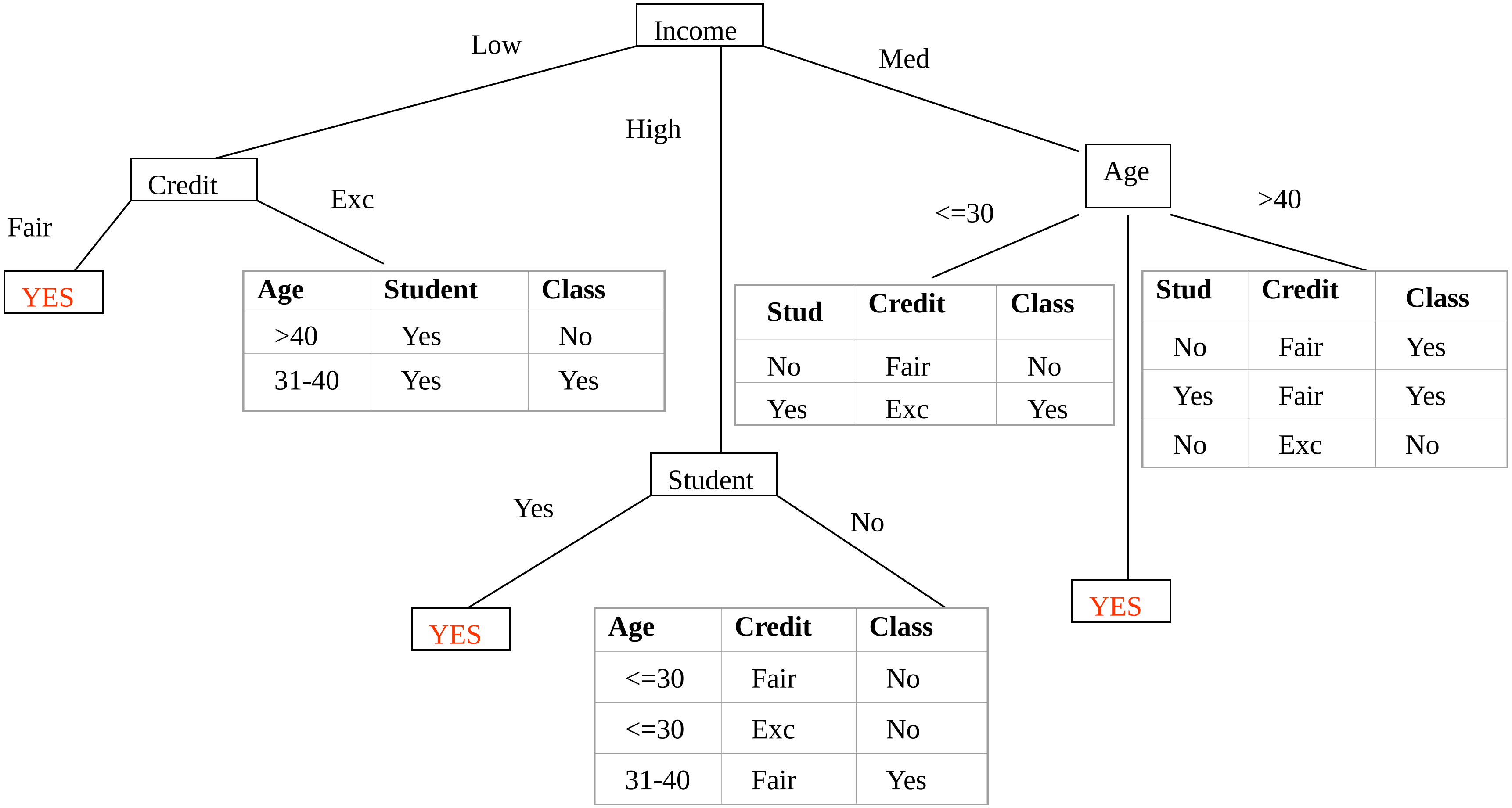
Stud	Credit	Class
No	Fair	Yes
Yes	Fair	Yes
No	Exc	No

Stud	Credit	Class
No	Exc	Yes

0.316
Student
Ind:4
Yes
No

Age	Credit	Class
31-40	Fair	Yes

Age	Credit	Class
<=30	Fair	No
<=30	Exc	No
31-40	Fair	Yes

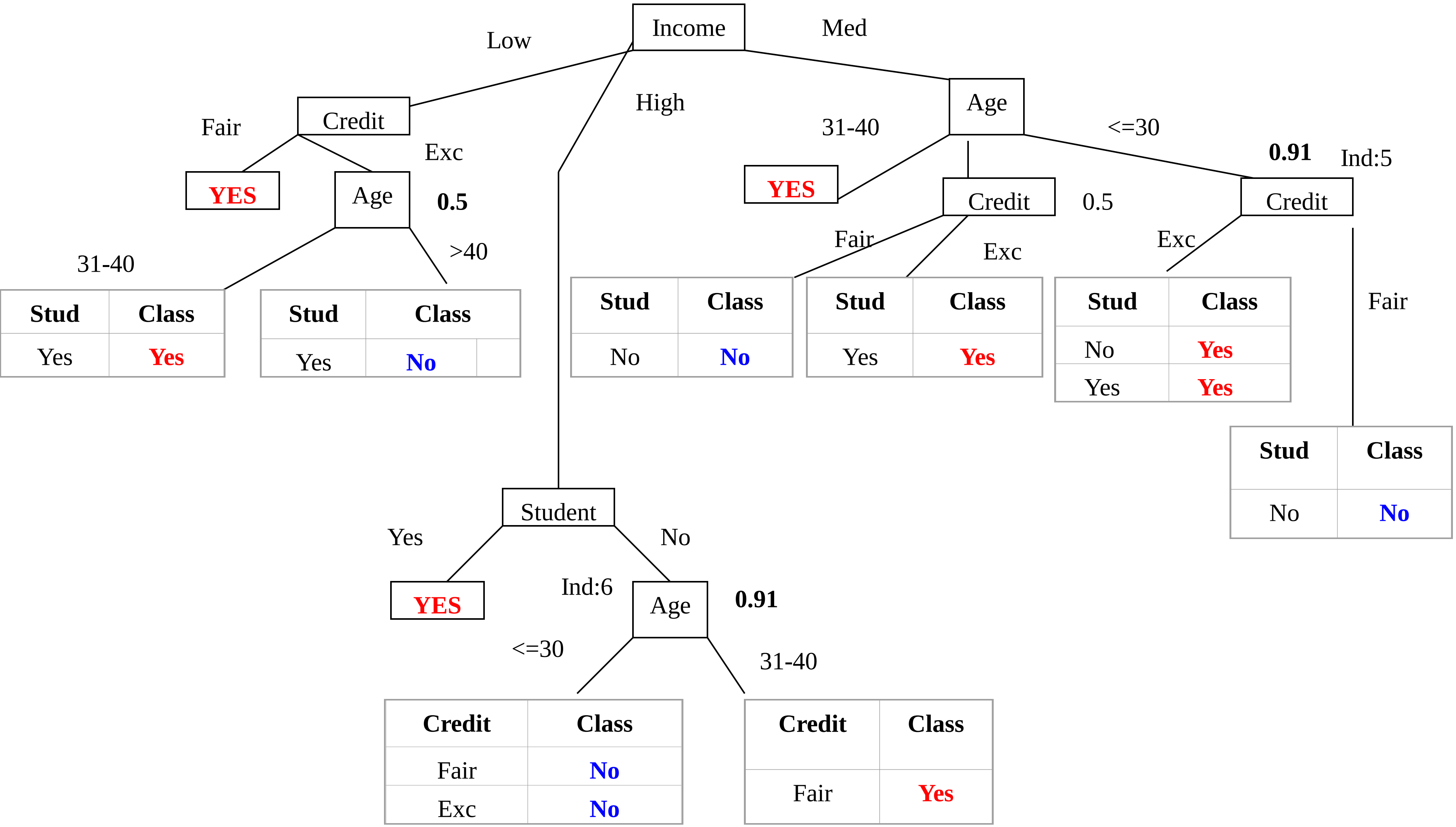


Age	Student	Class
>40	Yes	No
31-40	Yes	Yes

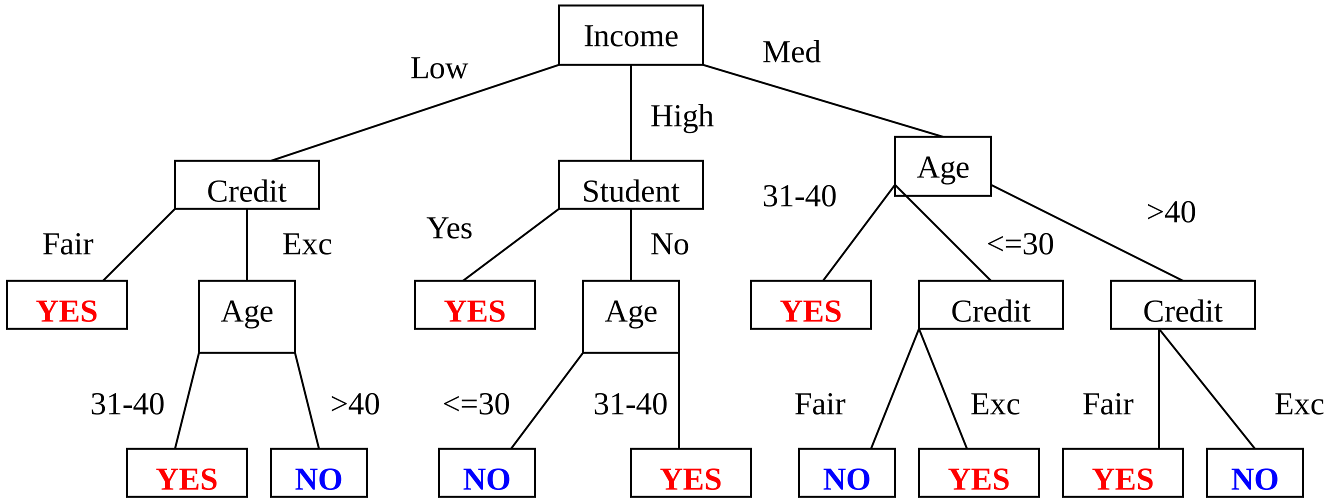
Stud	Credit	Class
No	Fair	No
Yes	Exc	Yes

Stud	Credit	Class
No	Fair	Yes
Yes	Fair	Yes
No	Exc	No

Age	Credit	Class
<=30	Fair	No
<=30	Exc	No
31-40	Fair	Yes


Income
Low
Med
High
Credit
Fair
Exc
YES
Age
0.5
31-40
>40
Stud | Class
Yes | Yes
Stud | Class
Yes | No
Age
31-40
<=30
YES
Credit
0.5
Fair
Exc
Stud | Class
No | No
Stud | Class
Yes | Yes
0.91
Ind:5
Credit
Exc
Fair
Stud | Class
No | Yes
Yes | Yes
Stud | Class
No | No
Student
Yes
No
YES
Ind:6
Age
0.91
<=30
31-40
Credit | Class
Fair | No
Exc | No
Credit | Class
Fair | Yes

Tree 1 with root attribute Income



Rules derived from tree 1 (predicate form for testing)

1. Income(x, Low) ^ Credit(x, Fair) -> buysComputer(x, Yes).
2. Income(x, Low) ^ Credit(x, Exc) ^ Age(x, 31-40) -> buysComputer(x, Yes).
3. Income(x, Low) ^ Credit(x, Exc) ^ Age(>40) -> buysComputer(x, No).
4. Incomex, (High) ^ Student(x, Yes) -> buysComputer(x, Yes).
5. Income(x, High) ^ Student(x, No) ^ Agex(x, <=30) -> buysComputer(x, No).
6. Income(x, High) ^ Student(x, No) ^ Age(x, 31-40) -> buysComputer(x, Yes).
7. Income(x, Medium) ^ Age(x, 31-40) -> buysComputer(x, Yes).
8. Income(x, Medium) ^ Age(x, <=30) ^ Credit(x, Fair) -> buysComputer(x, No).
9. Income(x, Medium) ^ Age(x, <=30) ^ Credit(x, Exc) -> buysComputer(x, Yes).
10. Income(x, Medium) ^ Age(x, >40) ^ Credit(x, Fair) -> buysComputer(x, Yes).
11. Income(x, Medium) ^ Age(x, >40) ^ Credit(x, Exc) -> buysComputer(x, No).

Tree 2 with root attribute Credit Rating

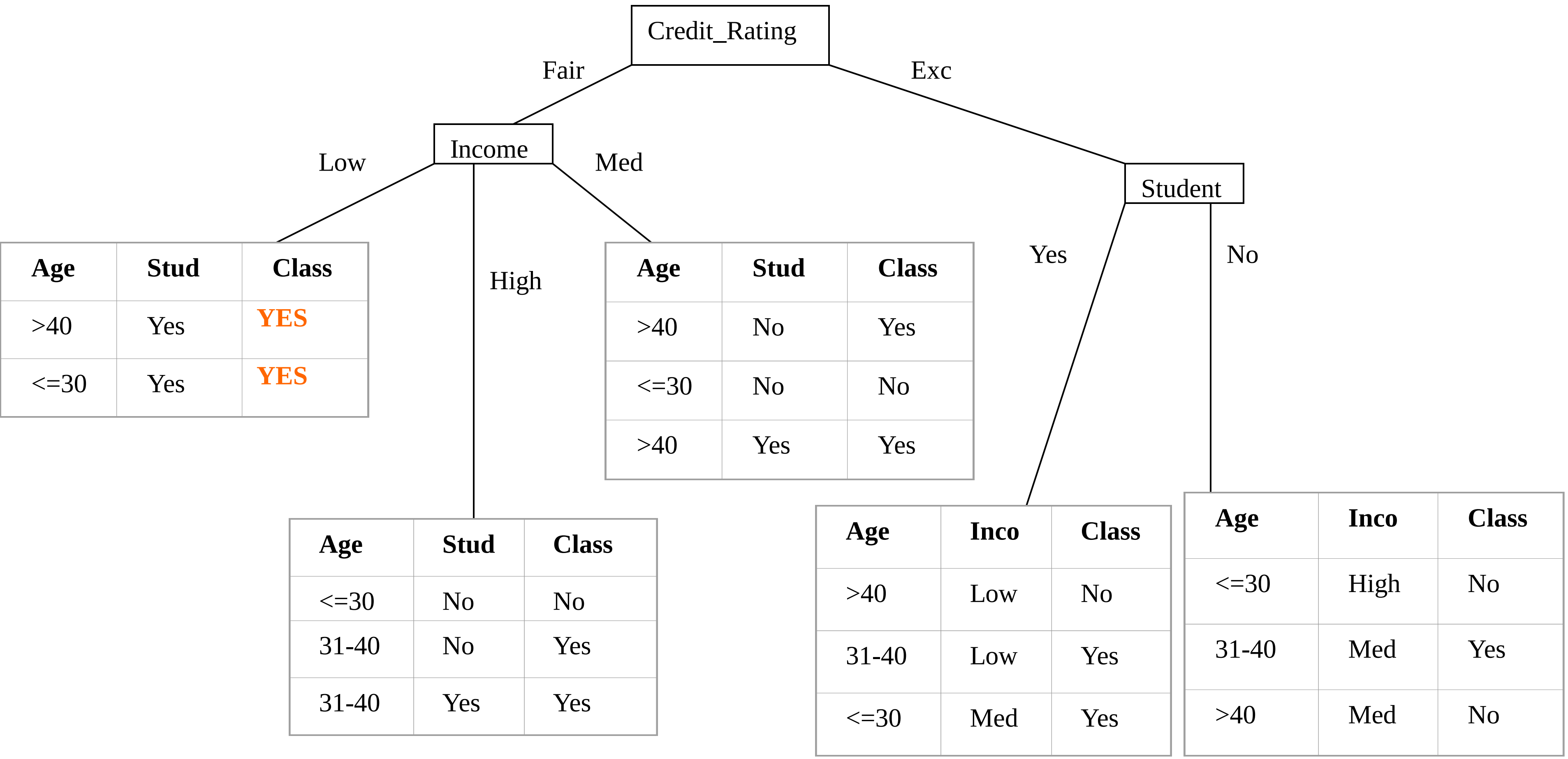
Credit_Rating

Fair

Age	Income	Student	Class
<=30	High	No	No
31-40	High	No	Yes
>40	Med	No	Yes
>40	Low	Yes	Yes
<=30	Med	No	No
<=30	Low	Yes	Yes
>40	Med	Yes	Yes
31-40	High	Yes	Yes

Exc

Age	Income	Student	Class
<=30	High	No	No
>40	Low	Yes	Yes
31-40	Low	Yes	Yes
<=30	Med	Yes	Yes
31-40	Med	No	No
>40	Med	No	No



Age	Stud	Class
>40	Yes	YES
<=30	Yes	YES

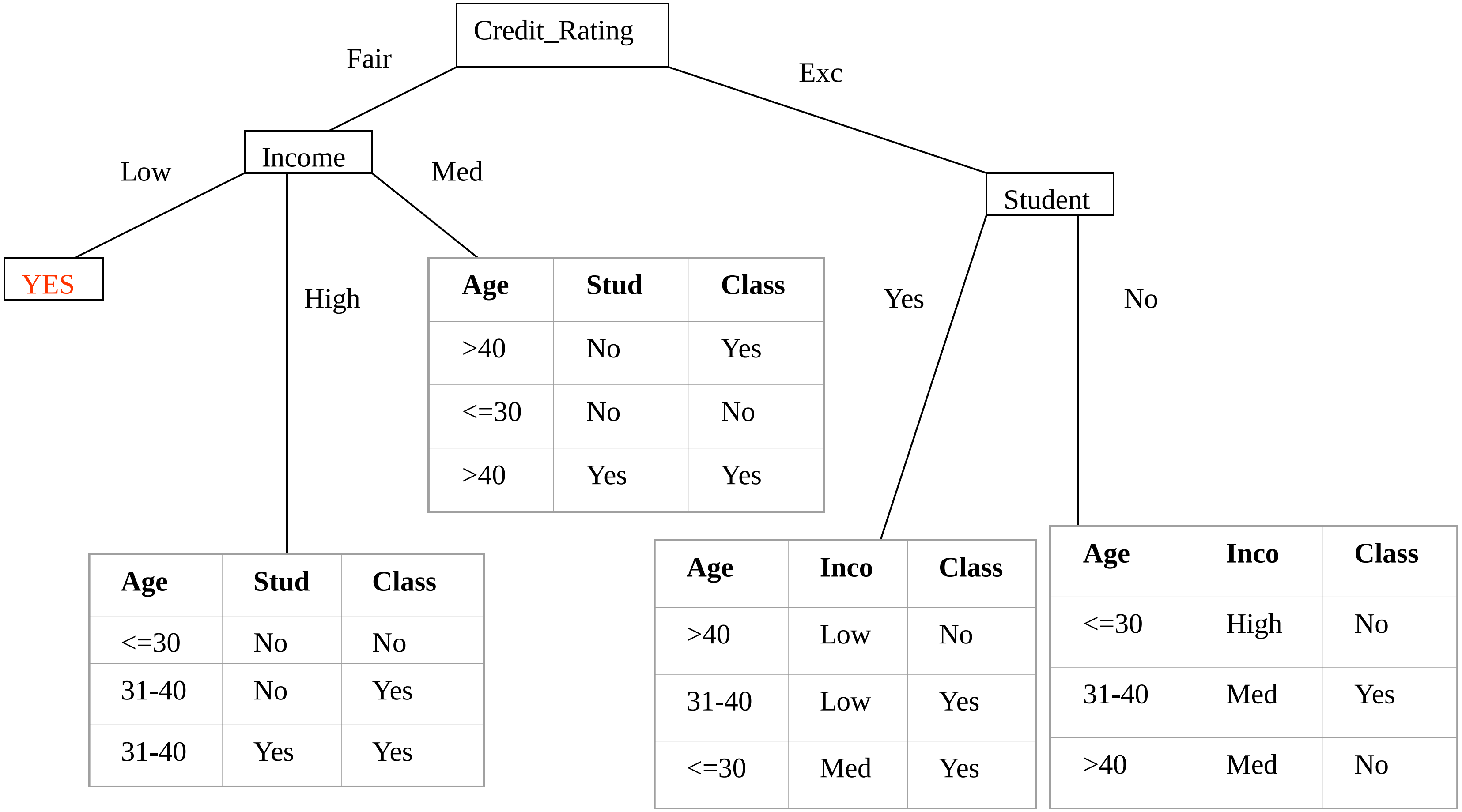
Age	Stud	Class
<=30	No	No
31-40	No	Yes
31-40	Yes	Yes

Age	Stud	Class
>40	No	Yes
<=30	No	No
>40	Yes	Yes

Age	Inco	Class
>40	Low	No
31-40	Low	Yes
<=30	Med	Yes

Age	Inco	Class
<=30	High	No
31-40	Med	Yes
>40	Med	No

Tree 2 with next level attributes Income and Student



Income = Med:

Age	Stud	Class
>40	No	Yes
<=30	No	No
>40	Yes	Yes

Income = High:

Age	Stud	Class
<=30	No	No
31-40	No	Yes
31-40	Yes	Yes

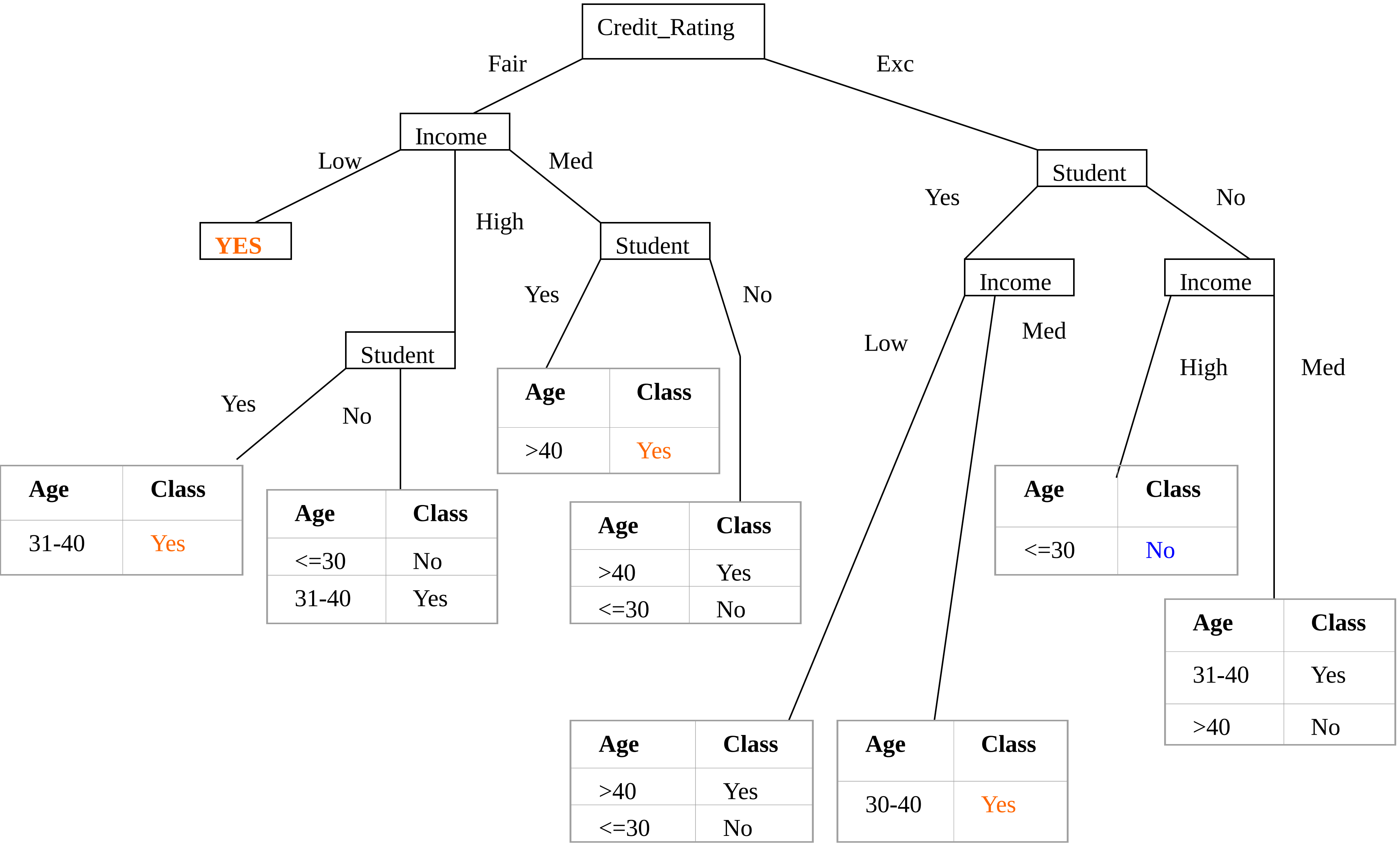
Student = Yes:

Age	Inco	Class
>40	Low	No
31-40	Low	Yes
<=30	Med	Yes

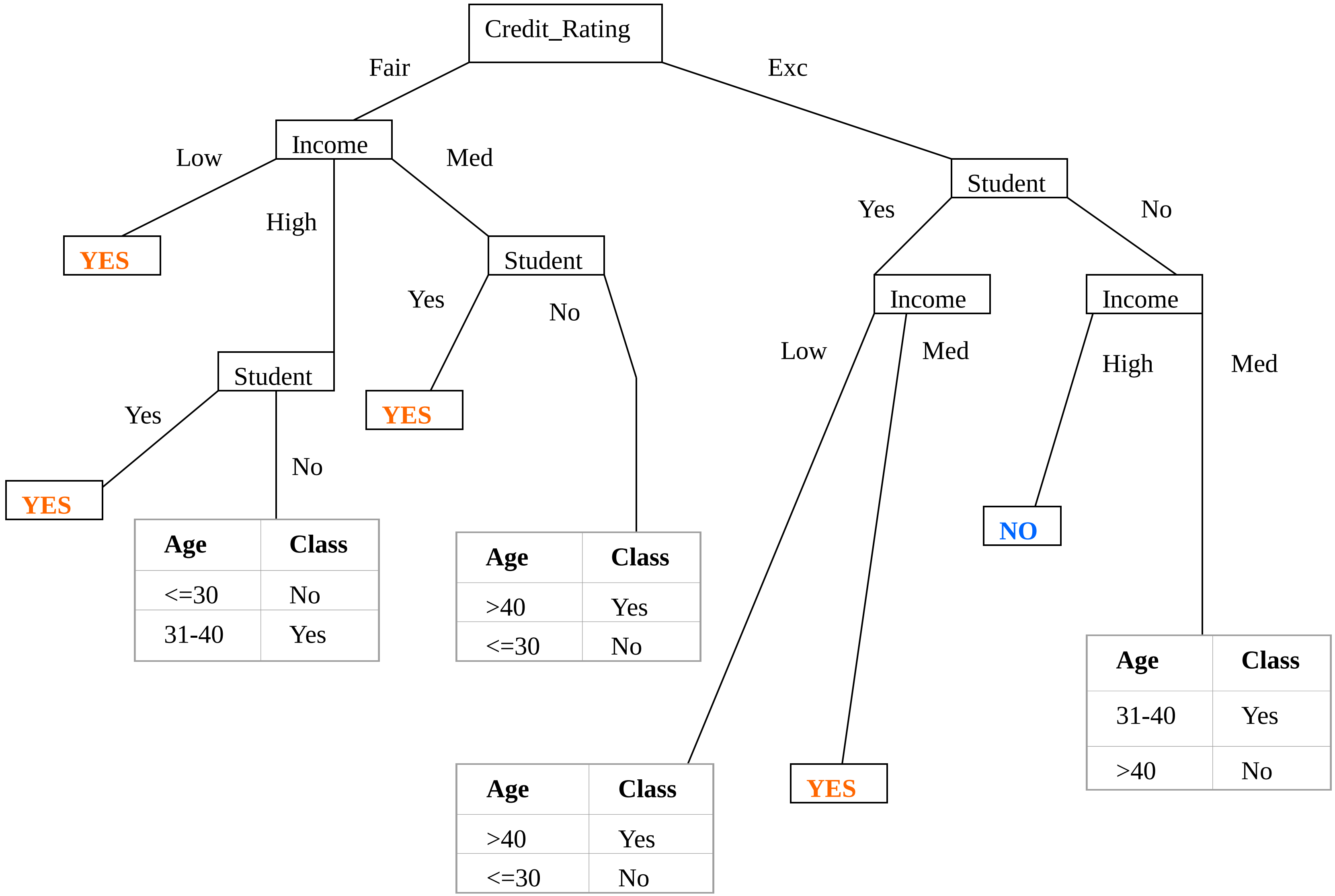
Student = No:

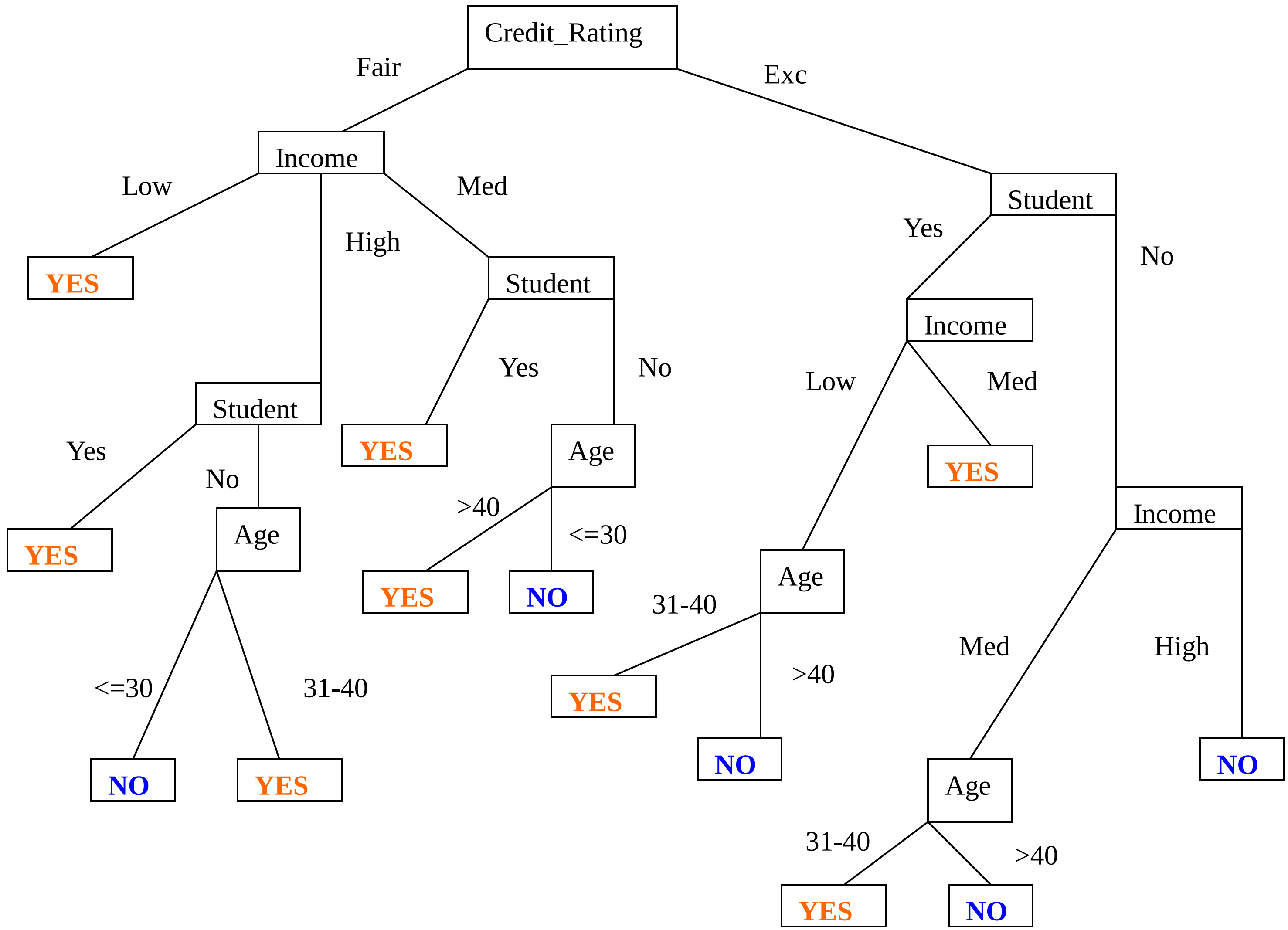
Age	Inco	Class
<=30	High	No
31-40	Med	Yes
>40	Med	No

Tree 2 with root attribute Credit Rating



Credit_Rating
Fair
Exc
Income
Low
Med
High
YES
Student
Yes
No
Student
Yes
No
Age
Class
31-40
Yes
Age
Class
<=30
No
31-40
Yes
Age
Class
>40
Yes
Age
Class
>40
Yes
<=30
No
Student
Yes
No
Income
Low
Med
Income
High
Med
Age
Class
<=30
No
Age
Class
31-40
Yes
>40
No
Age
Class
>40
Yes
<=30
No
Age
Class
30-40
Yes


Credit_Rating
Fair
Exc
Income
Low
High
Med
YES
Student
Yes
No
YES
Student
Yes
No
YES
Age Class
<=30 No
31-40 Yes
Age Class
>40 Yes
<=30 No
Student
Yes
No
Income
Low
Med
Age Class
>40 Yes
<=30 No
YES
Income
High
Med
NO
Age Class
31-40 Yes
>40 No



Final Tree 2 with root attribute Credit Rating

The Decision tree with root attribute *Credit_Rating* has produced 13 rules, two more than with root attribute *Income*

1. Credit(x, Fair) ^ Income(x,Low) -> buysComp(x,Yes).
2. Credit(x,Fair) ^ Income(x, High) ^ Student(x,Yes) -> buysComp(x, Yes).
3. Credit(x,Fair) ^ Income(x, High) ^ Student(x, No) ^ Age(<=30) -> buysComp(x, No).
4. Credit(x,Fair) ^ Income(x, High) ^ Student(x, No) ^ Age(31-40) -> buysComp(x, Yes).
5. Credit(x, Fair) ^ Income(x, Med) ^ Student(x, Yes) -> buysComp(x, Yes).
6. Credit(x, Fair) ^ Income(x, Med) ^ Student(x, No) ^ Age(>40) -> buysComp(x, Yes).
7. Credit(x, Fair) ^ Income(x, Med) ^ Student(x, No) ^ Age(<=30) -> buysComp(x, No).
8. Credit(x, Exc) ^ Student(x, Yes) ^ Income(x, Low) ^ Age(31-40) -> buysComp(x, Yes).
9. Credit(x, Exc) ^ Student(x, Yes) ^ Income(x, Low) ^ Age(>40) -> buysComp(x, No).
10. Credit(x, Exc) ^ Student(x, Yes) ^ Income(x, Med) -> buysComp(x, Yes).
11. Credit(x, Exc) ^ Student(x, No) ^ Income(x, Med) ^ Age(x, 31-40) -> buysComp(x, Yes).
12. Credit(x, Exc) ^ Student(x, No) ^ Income(x, Med) ^ Age(x, >40) -> buysComp(x, No).
13. Credit(x, Exc) ^ Student(x, No) ^ Income(x, High) -> buysComp(x, No).

EXERCISE 1

- We use some random records (tuples) to calculate the Predictive Accuracy of the set

  of rules from the Example 1.

Predictive Accuracy is the % of well classified records not from training set for which the class attribute is known.

Random Tuples to Check Predictive Accuracy based on three sets of rules

Obj	Age	Income	Student	Credit_R	Class
1	<=30	High	Yes	Fair	Yes
2	31-40	Low	No	Fair	Yes
3	31-40	High	Yes	Exc	No
4	>40	Low	Yes	Fair	Yes
5	>40	Low	Yes	Exc	No
6	<=30	Low	No	Fair	No

Predictive accuracy:

1. Against Lecture Notes: 4/6 = 66.66%
2. Against Tree 1 rules with root att. *Income*: 3/6 = 50%
3. Against Tree 2 rules with root att. *Credit*: 5/6 = 83.33%

EXERCISE 2

- **Predictive accuracy depends heavily on a choice of the test and training data.**

- **Find a small set of TEST records such that they would give a predictive accuracy 100%** for rules from Lecture Tree and trees 1 and 2 from Example 1

1. <u>TEST DATA applied against rules in Lecture Notes</u> that gives predictive accuracy 100%

No	Age	Income	Student	Credit_R	Class
1	<=30	Med	No	Exc	No
2	<=30	High	Yes	Fair	Yes
3	31-40	Low	No	Exc	Yes
4	>40	High	Yes	Exc	No
5	<=30	Low	No	Fair	Yes
6	31-40	High	Yes	Fair	Yes

2. TEST DATA that applied against the rules with root attribute *Income* give predictive accuracy 100%

No	Age	Income	Student	Credit_R	Class
1	31-40	Low	Yes	Fair	Yes
2	>40	Low	No	Exc	No
3	<=30	High	Yes	Fair	Yes
4	31-40	High	No	Exc	Yes
5	31-40	Med	No	Fair	Yes
6	>40	Med	Yes	Exc	No

3.TEST DATA that applied against the rules with root attribute *Credit Rating* gives predictive accuracy 100%

No	Age	Income	Student	Credit_R	Class
1	31-40	Low	No	Fair	Yes
2	<=30	High	Yes	Fair	Yes
3	<=30	Med	No	Fair	No
4	31-40	High	Yes	Exc	Yes
5	>40	Med	Yes	Exc	No
6	>40	Med	No	Exc	No

EXERCISE 1, Examples 1,2 CORRECTED

CORRECT Trees, Rules and Predictive Accuracy

Index: 1

Income

Gain=0.027

Low

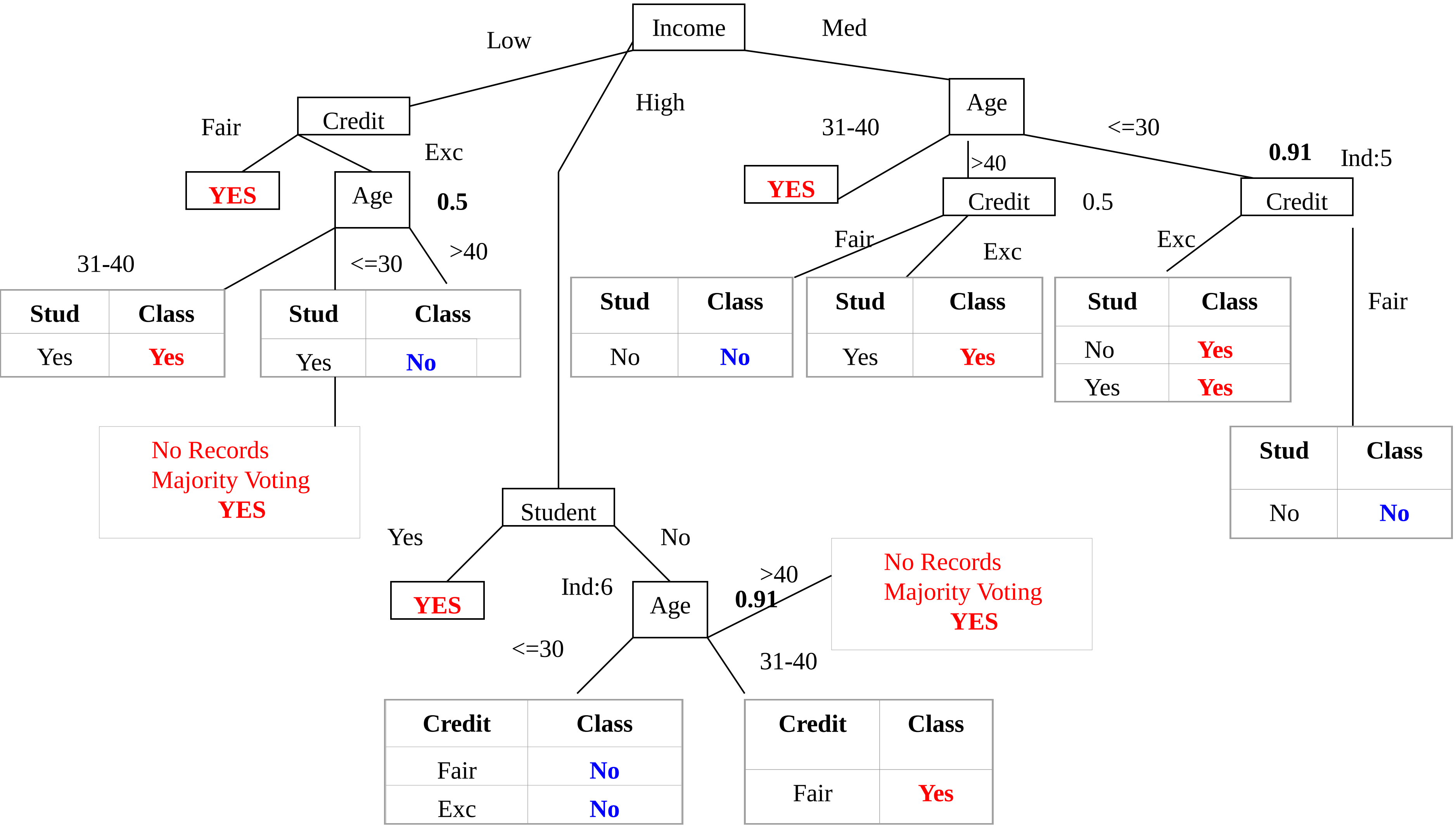
Age	**Student**	**Credit**	**Class**
>40	Yes	Fair	Yes
>40	Yes	Exc	No
31-40	Yes	Exc	Yes
<=30	Yes	Fair	Yes

High

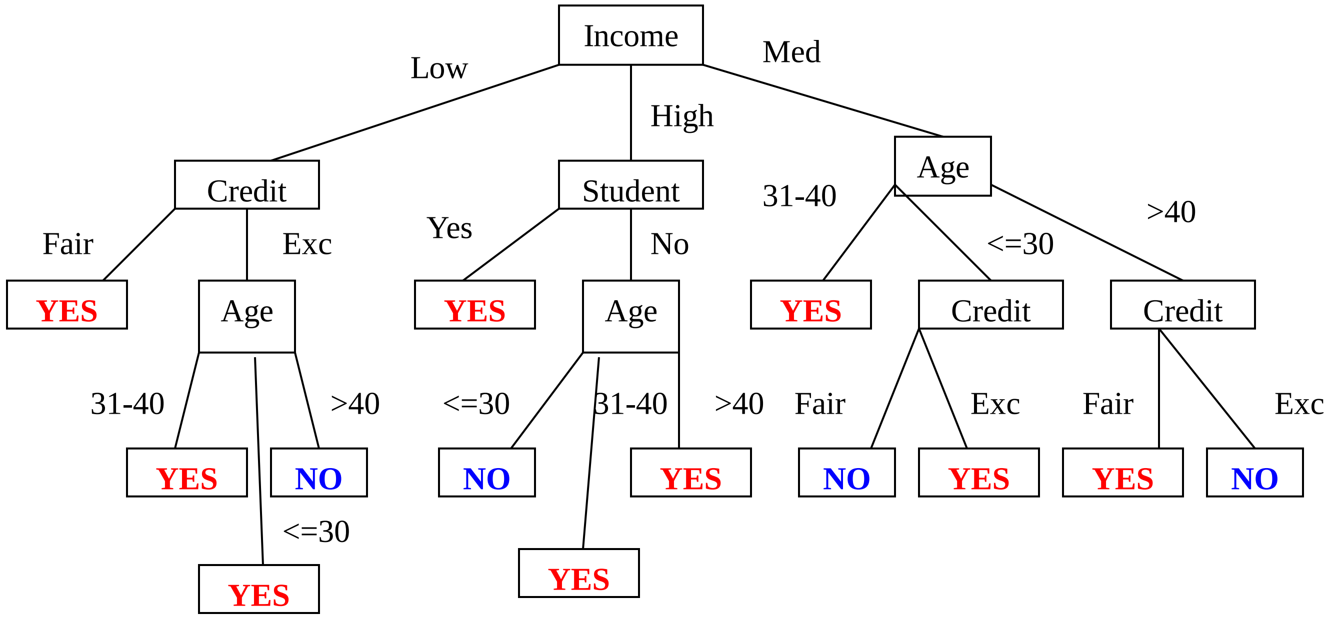
Age	**Student**	**Credit**	**Class**
<=30	No	Fair	No
<=30	No	Exc	No
31-40	No	Fair	Yes
31-40	Yes	Fair	Yes

Med

Age	**Student**	**Credit**	**Class**
>40	No	Fair	Yes
<=30	No	Fair	No
>40	Yes	Fair	Yes
<=30	Yes	Exc	Yes
31-40	No	Exc	Yes
>40	No	Exc	No


Income
Low
Med
High
Credit
Fair
Exc
YES
Age
0.5
31-40
<=30
>40
Stud Class
Yes Yes
Stud Class
Yes No
No Records
Majority Voting
YES
Age
31-40
<=30
>40
YES
Credit
0.5
Fair
Exc
Stud Class
No No
Stud Class
Yes Yes
Credit
0.91
Ind:5
Exc
Fair
Stud Class
No Yes
Yes Yes
Stud Class
No No
Student
Yes
No
YES
Ind:6
Age
>40
0.91
<=30
31-40
No Records
Majority Voting
YES
Credit Class
Fair No
Exc No
Credit Class
Fair Yes

Tree 1 with root attribute Income



Rules derived from tree 1 (predicate form for testing)

1. Income(x, Low) ^ Credit(x, Fair) -> buysComputer(x, Yes).
2. Income(x, Low) ^ Credit(x, Exc) ^ Age(x, 31-40) -> buysComputer(x, Yes).
3. Income(x, Low) ^ Credit(x, Exc) ^ Age(>40) -> buysComputer(x, No).
4. Incomex, (High) ^ Student(x, Yes) -> buysComputer(x, Yes).
5. Income(x, High) ^ Student(x, No) ^ Age(x, <=30) -> buysComputer(x, No).
6. Income(x, High) ^ Student(x, No) ^ Age(x, 31-40) -> buysComputer(x, Yes).
7. Income(x, Medium) ^ Age(x, 31-40) -> buysComputer(x, Yes).
8. Income(x, Medium) ^ Age(x, <=30) ^ Credit(x, Fair) -> buysComputer(x, No).
9. Income(x, Medium) ^ Age(x, <=30) ^ Credit(x, Exc) -> buysComputer(x, Yes).
10. Income(x, Medium) ^ Age(x, >40) ^ Credit(x, Fair) -> buysComputer(x, Yes).
11. Income(x, Medium) ^ Age(x, >40) ^ Credit(x, Exc) -> buysComputer(x, No).
12. Income(x, Low) ^ Age(x, <=30) ^ Credit(x, Exc) -> buysComputer(x, Yes). Majority Voting
13. Income(x, High) ^ Student(x, No) ^ Age(x>40) -> buysComputer(x, Yes). Majority Voting

Tree 2 with root attribute Credit Rating

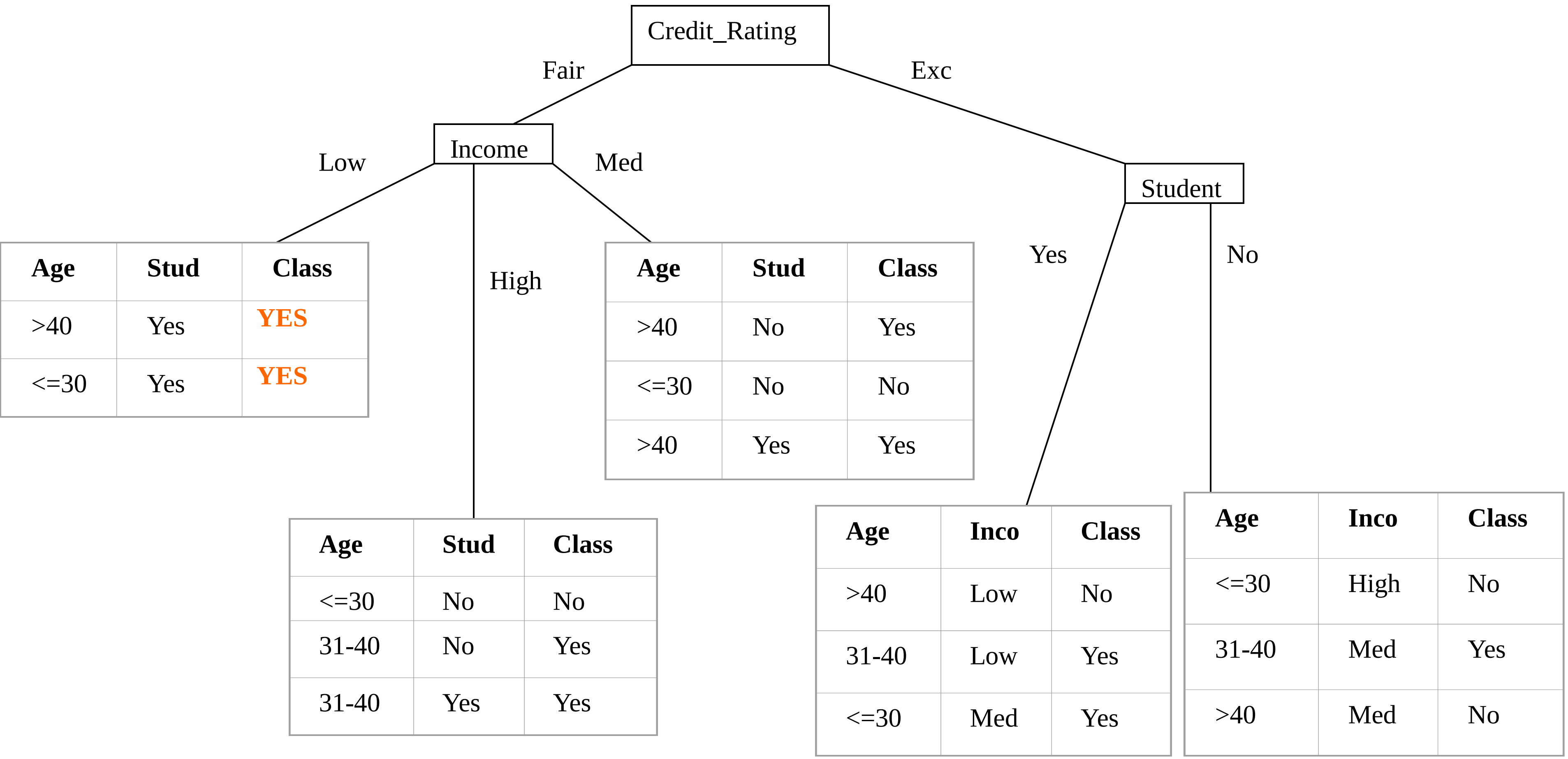
Credit_Rating

Fair

Age	Income	Student	Class
<=30	High	No	No
31-40	High	No	Yes
>40	Med	No	Yes
>40	Low	Yes	Yes
<=30	Med	No	No
<=30	Low	Yes	Yes
>40	Med	Yes	Yes
31-40	High	Yes	Yes

Exc

Age	Income	Student	Class
<=30	High	No	No
>40	Low	Yes	Yes
31-40	Low	Yes	Yes
<=30	Med	Yes	Yes
31-40	Med	No	No
>40	Med	No	No



Age	Stud	Class
>40	Yes	YES
<=30	Yes	YES

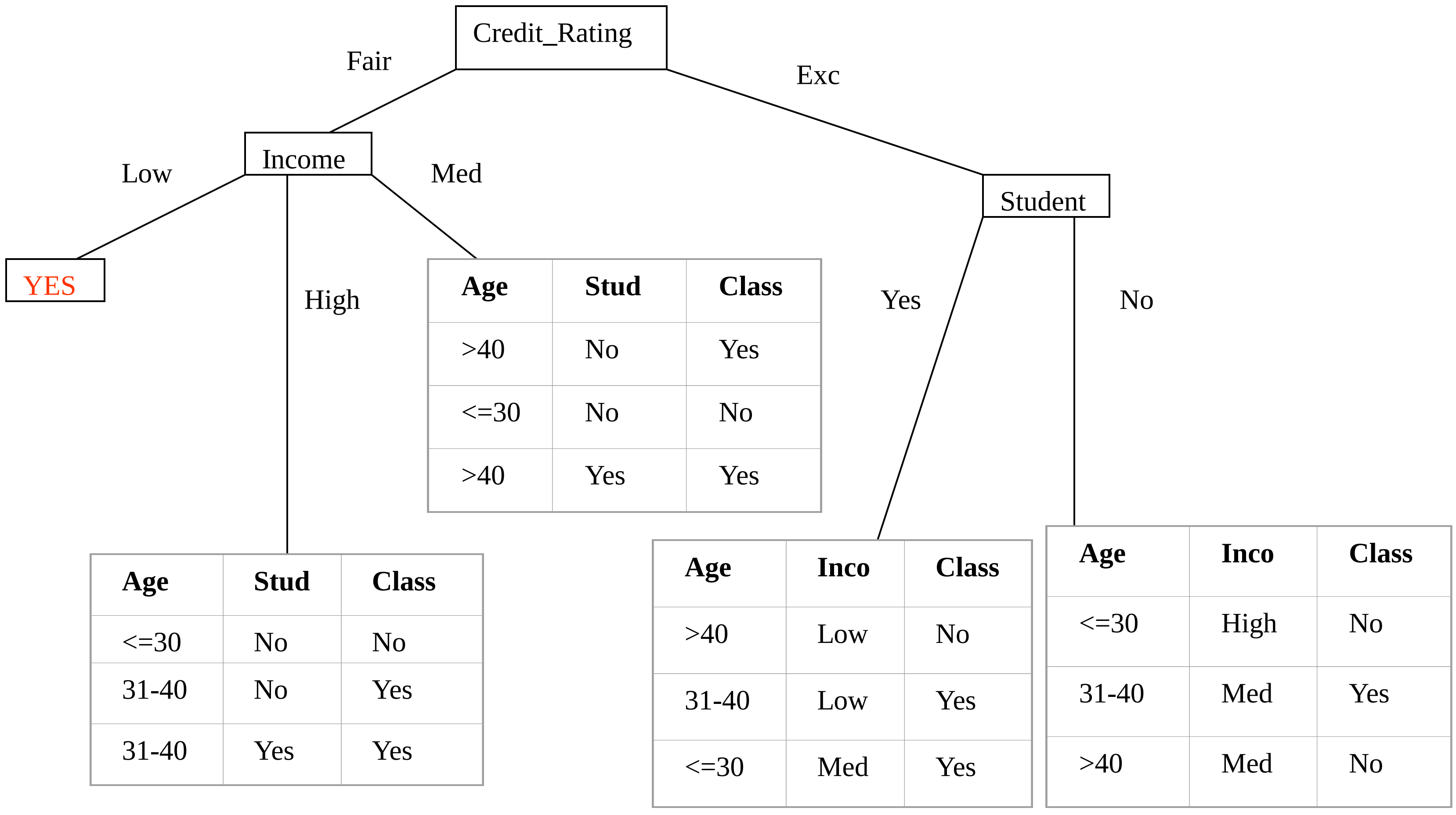
Age	Stud	Class
<=30	No	No
31-40	No	Yes
31-40	Yes	Yes

Age	Stud	Class
>40	No	Yes
<=30	No	No
>40	Yes	Yes

Age	Inco	Class
>40	Low	No
31-40	Low	Yes
<=30	Med	Yes

Age	Inco	Class
<=30	High	No
31-40	Med	Yes
>40	Med	No

Tree 2 with next level attributes Income and Student



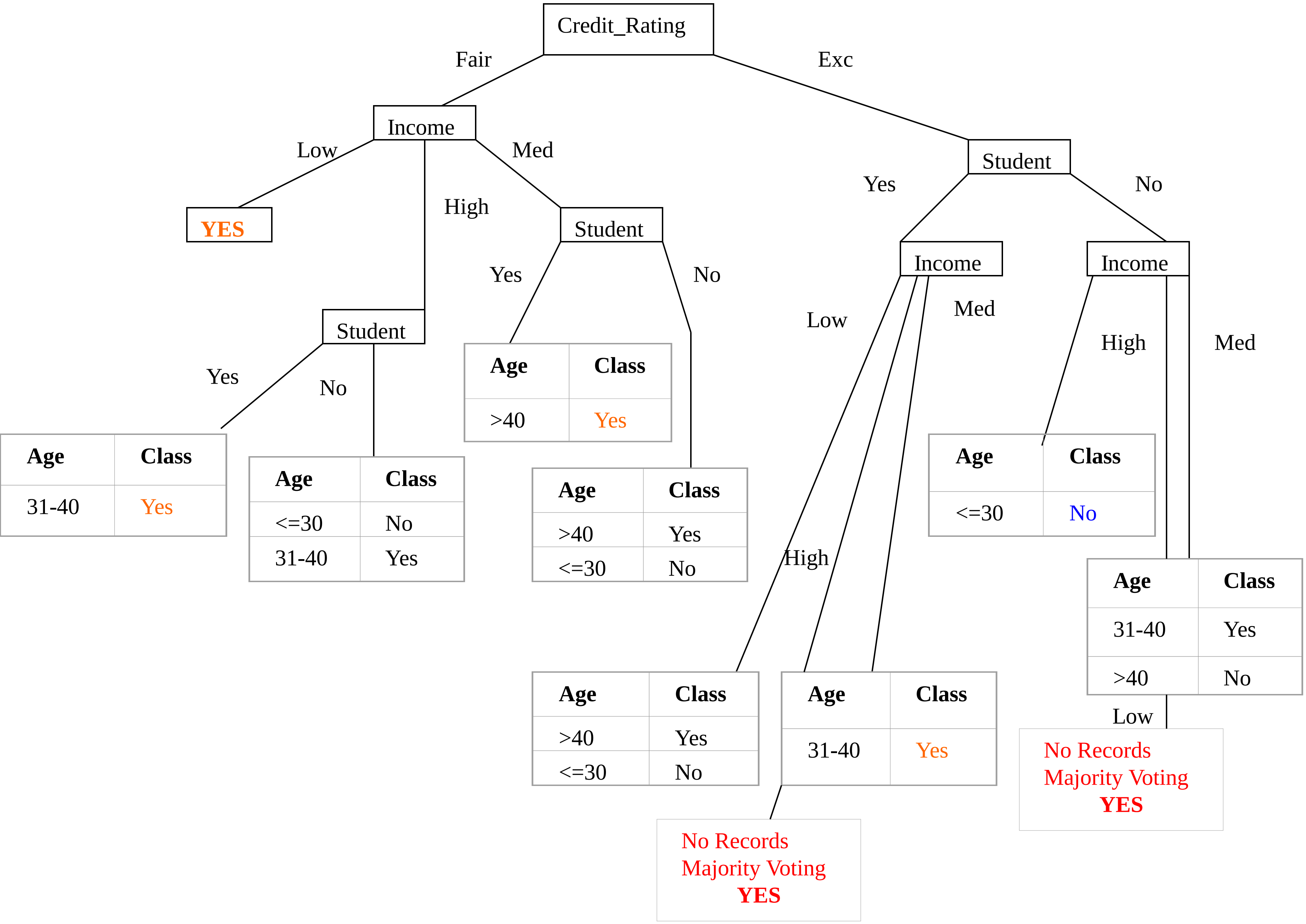
Age	Stud	Class
>40	No	Yes
<=30	No	No
>40	Yes	Yes

Age	Stud	Class
<=30	No	No
31-40	No	Yes
31-40	Yes	Yes

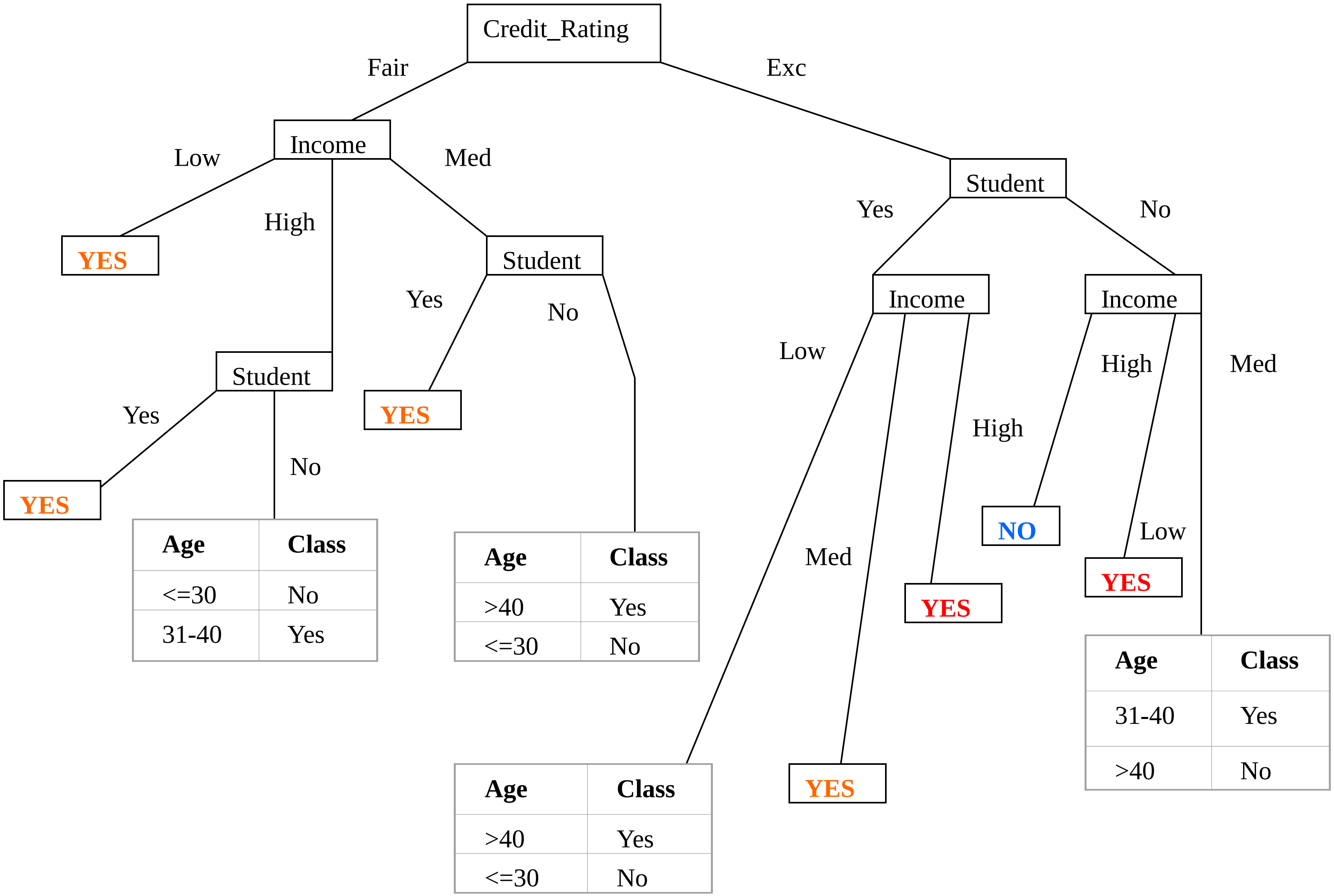
Age	Inco	Class
>40	Low	No
31-40	Low	Yes
<=30	Med	Yes

Age	Inco	Class
<=30	High	No
31-40	Med	Yes
>40	Med	No

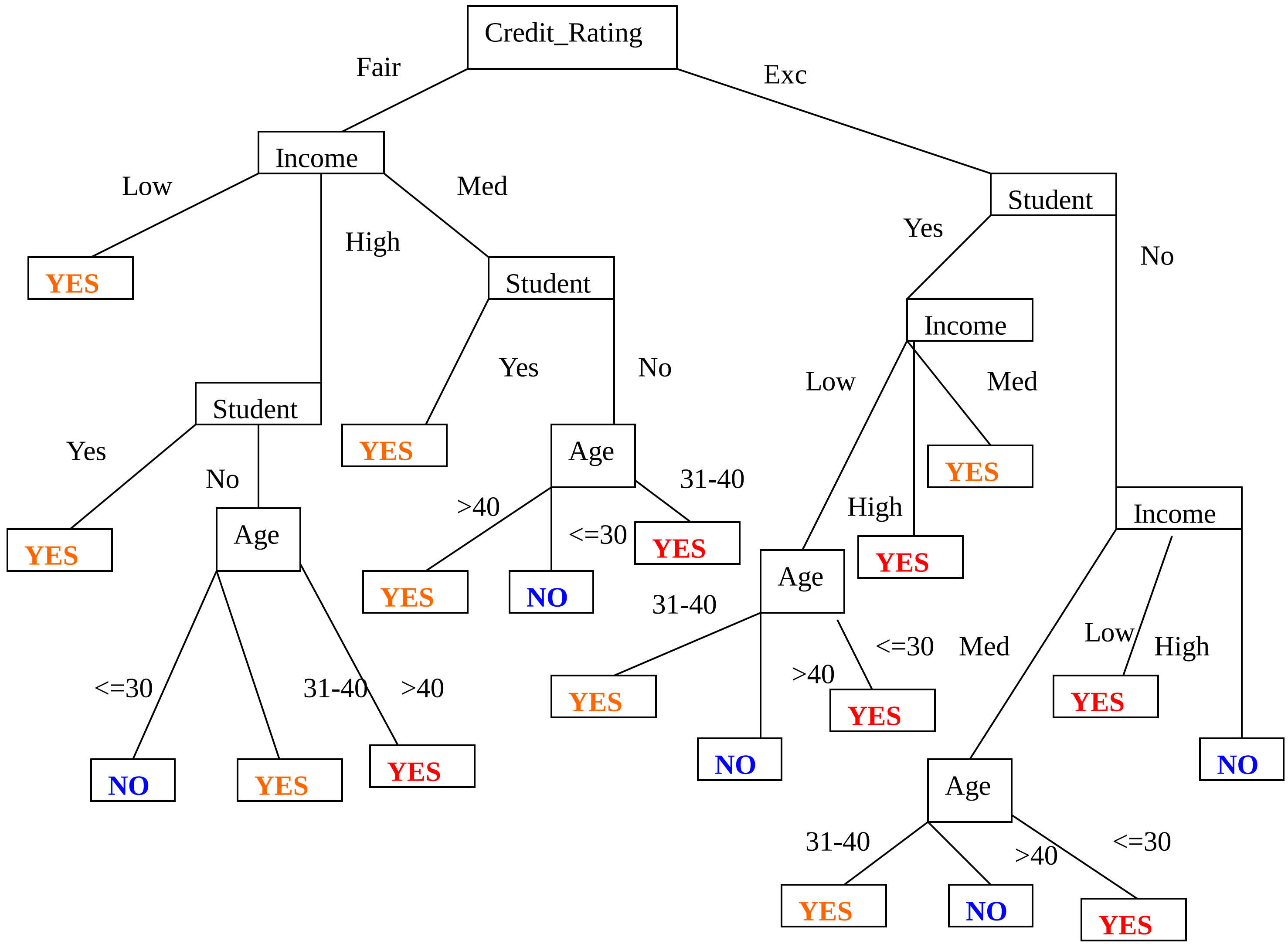
Tree 2 with root attribute Credit Rating



Credit_Rating
Fair
Exc
Income
Low
Med
High
YES
Student
Yes
No
Student
Yes
No
Age
Class
31-40
Yes
Age
Class
<=30
No
31-40
Yes
Age
Class
>40
Yes
Age
Class
>40
Yes
<=30
No
Student
Yes
No
Income
Low
Med
High
Income
High
Med
Low
Age
Class
<=30
No
Age
Class
31-40
Yes
>40
No
Age
Class
>40
Yes
<=30
No
Age
Class
31-40
Yes
No Records
Majority Voting
YES
No Records
Majority Voting
YES



Credit_Rating
Fair
Exc
Income
Low
High
Med
YES
Student
Yes
No
YES
Student
Yes
No
YES
Age
Class
<=30
No
31-40
Yes
Age
Class
>40
Yes
<=30
No
Student
Yes
No
Income
Low
High
Med
YES
YES
Age
Class
>40
Yes
<=30
No
Income
High
Low
Med
NO
YES
Age
Class
31-40
Yes
>40
No



Final Tree 2 with root attribute Credit Rating

The Decision tree with root attribute *Credit_Rating* has produced 13 rules, two more than with root attribute *Income*

1. Credit(x, Fair) ^ Income(x,Low) -> buysComp(x,Yes).
2. Credit(x,Fair) ^ Income(x, High) ^ Student(x,Yes) -> buysComp(x, Yes).
3. Credit(x,Fair) ^ Income(x, High) ^ Student(x, No) ^ Age(<=30) -> buysComp(x, No).
4. Credit(x,Fair) ^ Income(x, High) ^ Student(x, No) ^ Age(31-40) -> buysComp(x, Yes).
5. Credit(x, Fair) ^ Income(x, Med) ^ Student(x, Yes) -> buysComp(x, Yes).
6. Credit(x, Fair) ^ Income(x, Med) ^ Student(x, No) ^ Age(>40) -> buysComp(x, Yes).
7. Credit(x, Fair) ^ Income(x, Med) ^ Student(x, No) ^ Age(<=30) -> buysComp(x, No).
8. Credit(x, Exc) ^ Student(x, Yes) ^ Income(x, Low) ^ Age(31-40) -> buysComp(x, Yes).
9. Credit(x, Exc) ^ Student(x, Yes) ^ Income(x, Low) ^ Age(>40) -> buysComp(x, No).
10. Credit(x, Exc) ^ Student(x, Yes) ^ Income(x, Med) -> buysComp(x, Yes).
11. Credit(x, Exc) ^ Student(x, No) ^ Income(x, Med) ^ Age(x, 31-40) -> buysComp(x, Yes).
12. Credit(x, Exc) ^ Student(x, No) ^ Income(x, Med) ^ Age(x, >40) -> buysComp(x, No).
13. Credit(x, Exc) ^ Student(x, No) ^ Income(x, High) -> buysComp(x, No).
14. Credit(x,Fair) ^ Income(x, High) ^ Student(x, No) ^ Age(>40) -> buysComp(x, Yes). Majority Voting
15. Credit(x, Fair) ^ Income(x, Med) ^ Student(x, No) ^ Age(31-40) -> buysComp(x, Yes). Majority Voting
16. Credit(x, Exc) ^ Student(x, Yes) ^ Income(x, Low) ^ Age(<=30) -> buysComp(x, Yes). Majority Voting
17. Credit(x, Exc) ^ Student(x,No) ^ Income(x, Med) ^ Age(x<=30) -> buysComp(x, Yes). Majority Voting
18. Credit(x, Exc) ^ Student(x, Yes) ^ Income(x, High) -> buysComp(x, Yes). Majority Voting
19. Credit(x, Exc) ^ Student(x, No) ^ Income(x, Low) -> buysComp(x, Yes). Majority Voting

Random Tuples to Check Predictive Accuracy based on three sets of rules

Obj	Age	Income	Student	Credit_R	Class
1	<=30	High	Yes	Fair	Yes
2	31-40	Low	No	Fair	Yes
3	31-40	High	Yes	Exc	No
4	>40	Low	Yes	Fair	Yes
5	>40	Low	Yes	Exc	No
6	<=30	Low	No	Fair	No

Predictive accuracy:

1. Against Lecture Notes: 4/6 = 66.66%
2. Against Tree 1 rules with root att. *Income*: 3/6 = 50%
3. Against Tree 2 rules with root att. *Credit*: 4/6 = 66.66%
4. Against OLD Tree 2 rules with root att. *Credit*: 5/6 = 83.33%

Calculation of Information gain at each level of tree with root att. *Income*

1. <u>Original Table:</u>

Class P: *buys_computer* = yes; Class N: *buys_computer* = No

I(P,N) = -P/P+N log $_2$ (P/P+N) – N/P+N log$_2$N/P+N-------(equation 1)

I(P,N) = I(9,5) = (-9/9+5) log $_2$(9/9+5) – (5/9+5) log$_2$ (5/9+5)

= 0.940

2. <u>Index:1</u>

Income	Pi	Ni	I(Pi,Ni)
Low	3	1	0.8111
Med	4	2	0.9234
High	2	2	1

E(Income) = 4/14 I(3,1) + 6/14 I(4,2) + 4/14 I(2,2)-----------(eq.2)

I(3,1) = 0.8111 (Using equation 1)

I(4,2) = 0.9234 (Using equation 1)

I(2,2) = 1

Contd…..

Information gain calculation for Index 1 contd:

Substituting the values in eq.2 we get,

E(Income) = 0.2317 + 0.3957 + 0.2857 = 0.9131

Gain (Income) = I(P,N) – E(Income)

= 0.940 – 0.9131 = 0.027

2. <u>Index 2</u>

Credit	Pi	Ni	I(Pi,Ni)
Fair	2	1	0.913
Exc	2	1	0.913

I(P,N) = I(4,2) = 0.9234 (Using equation 1)

E(Credit) = 3/6 I(2,1) + 3/6 I(2,1) -----(3)

I(2,1) = 0.913 (Using equation 1)

E(Credit) = 0.913 (Substituting value of I(2,1) in (3)

Gain(Credit) = I(P,N) – E(Credit) = 0.9234 – 0.913

= 0.01

Similarly we can calculate Information gain of tables at each stage.