CSE 352 Artificial Intelligence
Homework 4, Part 1 (15pts)

Circle a proper answer in Y/N questions (5pts). Write JUSTIFICATION (10pts) when it is asked for.

QUESTIONS

1. For any propositional functions A(x), B(x), the formula
$\forall x(A(x) \cup B(x)) \Rightarrow (\forall x A(x) \cup \forall x B(x))$ is a predicate tautology.
y n

2. For any propositional function A(x) the formula
$\forall x A(x) \Rightarrow \exists A(x)$ is a predicate tautology.
y n

3. For any predicates A(x), B, (this means that B does not contain the variable x) the formula
$\forall x(A(x) \Rightarrow B) \Rightarrow (\exists x A(x) \Rightarrow B)$ is a predicate tautology.
y n

4. For any predicates A(x), B(x), the formula
$\forall x(A(x) \Rightarrow B(x)) \Rightarrow (\forall x A(x) \Rightarrow \forall x B(x))$ is a predicate tautology.
y n

5. For any predicates A(x), B, (this means that B does not contain the variable x) the formula
$\forall x(A(x) \cap B) \Rightarrow (\forall x A(x) \cap B)$ is a predicate tautology.
y n

6. For any predicates A(x), B, (this means that B does not contain the variable x) the formula
$\forall x(A(x) \cup B) \Rightarrow (\forall x A(x) \cup B)$ is a predicate tautology.
y n

7. For any propositional functions A(x), B(x), the formula
$\forall x(A(x) \cup B(x)) \Rightarrow (\forall x A(x) \cup \forall x B(x))$ is a predicate tautology.
y n

8. For any predicates A(x), B(x), the formula
$\exists x(A(x) \cap B(x)) \Rightarrow (\exists x A(x) \cap \exists x B(x))$ is a predicate tautology.
y n

9. For any predicates A(x), B, the formula
$\exists x(B \Rightarrow A(x)) \equiv (B \Rightarrow \exists x A(x))$
y n

10. For any predicates A(x), B(x), the formula
$(\exists x A(x) \cap \exists x B(x)) \Rightarrow \exists x(A(x) \cap B(x))$ is a predicate tautology.
y n

11. $\neg \forall n \in N \exists r \in R(x < \frac{1}{n}) \equiv \exists n(n \in N \cap \forall x(x \notin R \cup x \geq \frac{1}{n}))$
JUSTIFY: (use definition of quantifiers with restricted domain and proper tautologies) **y n**

12. The argument: *All cats can tell the mouse from the flower. All clever cats can tell the mouse from the flower. Hence all cats are clever.*
is VALID
JUSTIFY: (All sentences are UNIVERSAL, so you can reduce argument to propositional logic) **y** **n**

Solution Follow the steps below.

1. Write your predicates (in $X \neq \emptyset$).

2. Write corresponding logic formulas (remember to translate from restricted domain quantifiers to $\forall x$, $\exists x$).

1. All cats can tell the mouse from the flower.

2. All clever cats can tell the mouse from the flower.

3. All cats are clever.

3. Reduce the argument to propositional logic and solve.

13. The argument: *Some cats can tell the mouse from the flower. All clever cats can tell the mouse from the flower. Some cats are clever.*
is VALID.
JUSTIFY: (Be careful - you can't use the reduction to propositional logic!) **y** **n**

Solution: Follow the steps below.

1. Write your predicates (in $X \neq \emptyset$)

2. Write corresponding logic formulas (remember to translate from restricted domain quantifiers to $\forall x$, $\exists x$.)

1. Some cats can tell the mouse from the flower.

2. All clever cats can tell the mouse from the flower.

3. Some cats are clever.

Solve the argument. Use definition of the TRUTH SETS:

$\forall x A(x) = \text{TRUE}$ iff $\{x \in X : A(x)\} = X$

$\exists x A(x) = \text{TRUE}$ iff $\{x \in X : A(x)\} \neq \emptyset$

14. The following statement is false.
$\forall x \in R(x^2 < 0) \Rightarrow \exists x \in R(x^2 > 0)$

y **n**

15. $\forall (x \leq 0) \exists (y = 1)(x + y = 4) \equiv \forall x \exists y (x \leq 0 \Rightarrow (y = 1 \cap x + y = 4))$
JUSTIFY: **y** **n**