CSE 541 – Logic in Computer Science
Sample Exam Problems

The following is a sample of problems from previous exams.

1. **Predicate Logic Models**

   Consider the following sentences.

   ϕ_1: $\forall x \neg P(x,x)$
   ϕ_2: $\forall x \forall y \forall z\, [P(x,y) \wedge P(y,z) \rightarrow P(x,z)]$
   ϕ_3: $\forall x \forall y\, [P(x,y) \rightarrow \exists z\, (P(x,z) \wedge P(z,y))]$
   ϕ_4: $\neg \forall x \exists y P(x,y)$

   (a) Give a model $\mathcal{M}_1$ such that $\mathcal{M}_1 \models \phi_1 \wedge \phi_2 \wedge \phi_3 \wedge \phi_4$.

   (b) Give a model $\mathcal{M}_2$ such that $\mathcal{M}_2 \models \phi_1 \wedge \phi_2 \wedge \phi_3 \wedge \neg\phi_4$.

2. **Natural deduction**

   Give natural deduction proofs for the following sequents.

   (a) $P \rightarrow Q \vdash (P \rightarrow \neg Q) \rightarrow \neg P$

   (b) $(P \vee Q) \vee (R \rightarrow S) \vdash (S \vee Q) \vee (R \rightarrow P))$
   Hint. Use the $\vee$e-rule and LEM.

3. **Natural deduction**

   Use the standard natural deduction rules to prove the following sequents.

   (a) $\forall y\, P(c,y), \forall x \forall y\, [P(x,y) \rightarrow P(f(x), f(y))] \vdash \exists z[P(c,z) \wedge P(z, f(f(c)))]$
   (where P is a binary predicate symbol, f a unary function symbol, and c a constant).

   (b) $\exists x\, P(x,x), \forall x[\exists y\, P(x,y) \rightarrow Q(x)] \vdash \exists x\, Q(x)$

4. **Unification**

   (a) Determine whether the unification problem

   $$\{x =^? f(y, g(y)), g(f(z,a)) =^? g(y)\}$$

   is solvable. Give a most general unifier or else explain why there is no unifier. (Note that a is a constant, wheras x, y, and z are variables.)

(b) Give terms s, t, and u such that (i) s and t have exactly one unifier, (ii) t and u have infinitely many unifiers, and (iii) s and u are not unifiable.

$s =$

$t =$

$u =$

5. **Clause Logic**

Recall that a clause is a disjunction of literals (atomic formulas or negations thereof). Find *unsatisfiable* sets of clauses as specified. If there is no unsatisfiable set of clauses that meets the stated restrictions, explain why.

(a) Each clause in S_1 must contain both positive and negative literals. For example, $p(x) \vee \neg q(y)$ qualifies but $p(x)$ and $\neg p(x) \vee \neg q(y)$ do not.

(b) No clause in S_2 must contain both positive and negative literals. For example, $p(x) \vee q(y)$ qualifies but $\neg p(x) \vee q(y)$ does not.

6. **Skolemization**

(a) Let ϕ be the sentence $\neg[\exists x\, P(x, x) \vee \neg\exists y \exists z \forall x (P(x, y) \vee P(x, z))]$.

i. Convert ϕ to prenex form.

ii. Skolemize the formula you obtained in the first part.

iii. Use resolution to determine whether the skolemized formula from the second part is unsatisfiable.

7. **Resolution**

Use resolution to determine whether the following set of clauses is unsatisfiable:

$$\neg P(x) \vee Q(x) \vee R(x, f(x)), \tag{1}$$
$$\neg P(x) \vee Q(x) \vee S(f(x)) \tag{2}$$
$$\neg R(a, x) \vee T(x), \tag{3}$$
$$\neg T(x) \vee \neg Q(x), \tag{4}$$
$$\neg T(x) \vee \neg S(x), \tag{5}$$
$$P(a), \tag{6}$$
$$T(a). \tag{7}$$

8. **Compactness**

(a) Give a sentence ϕ_k that expresses that "there are at least k distinct elements."

(b) Prove that there is no set S of predicate logic formulas that is satisfied exactly by those models that have a finite domain.