

CSE 541 – Logic in Computer Science Sample Exam Problems

The following is a sample of problems from previous exams.

1. Kripke models

Let $\mathcal{M} = (W, R, L)$ be a Kripke model where

$$\begin{aligned} W &= \{s_0, s_1, s_2, s_3\}, \\ R &= \{(s_0, s_1), (s_1, s_1), (s_1, s_2), (s_2, s_1), (s_2, s_0)\}, \text{ and} \\ L(s_0) &= L(s_1) = \{p\}, L(s_2) = \emptyset. \end{aligned}$$

- (a) List all worlds that satisfy $\Box\Diamond p$:
- (b) List all worlds that satisfy $\Diamond\Box p$:

2. Multi-agent systems

Let G be a set of three agents, $G = \{1, 2, 3\}$. Give $KT45^3$ formulas for the following statements.

- (a) Agent 1 knows p but doesn't know q .
- (b) Agent 2 knows whether agent 3 knows p .
- (c) Not everyone knows whether p .

3. Modal logic

Determine whether the following statements are true and accordingly give either a proof or a counterexample.

- (a) For all Kripke models \mathcal{M} , if $p \rightarrow \Box p$ is true in all worlds of \mathcal{M} , then $\Box p \rightarrow \Box\Box p$ is also true in all worlds of \mathcal{M} .
- (b) For all Kripke models \mathcal{M} , if $\Box p \rightarrow p$ is true in all worlds of \mathcal{M} , then $\Box p \rightarrow \Box\Box p$ is also true in all worlds of \mathcal{M} .

4. Frames

Recall that a binary relation R is serial if for every x there exists a y such that $R(x, y)$; and Euclidean if for all x, y , and z with $R(x, y)$ and $R(x, z)$, also $R(y, z)$.

- (a) Give a frame (W, R) , where R is serial but not Euclidean, and a formula ϕ satisfied by this frame (i.e., $(W, R, L), x \models \phi$ for all labelling functions L and worlds $x \in W$).
- (b) Give a frame (W, R) , where R is Euclidean but not serial, and a formula ϕ satisfied by this frame.

5. Computation tree logic

- (a) Show that $A[(pU r) \vee (qU r)]$ and $A[(p \vee q)U r]$ are not equivalent.
- (b) Show that $E[GFp]$ and $EG EFp$ are not equivalent.
- (c) Explain why $(AFp) \vee (AF\neg p)$ and $(EFp) \vee (EF\neg p)$ are equivalent.

6. Linear time and branching time

- (a) Give a transition system to show the difference between $A[Fp \rightarrow Fq]$ and $AF p \rightarrow AF q$.
- (b) Show the difference between $Gp \rightarrow Fq$ and $G(p \rightarrow Fq)$.
- (c) Find a CTL equivalent of $E[Fp \wedge Gq]$.