Resolution

There are many other methods for deciding whether a given set of clauses is satisfiable or not. Some of these methods are based on an inference rule called *resolution* (or more specifically *ground* resolution):

$$\frac{C' \vee P \qquad D' \vee \neg P}{C \vee D}$$

where $C' \vee P$ and $D' \vee \neg P$ are meant to denote arbitrary clauses containing the literals P and $\neg P$, respectively. In particular, C' and/or D' may be "empty."

In general, the empty clause represents a contradiction and hence is denoted by $\perp$. The conclusion $C' \vee D'$ of the inference is also called a *resolvent* of the two premises.

Theorem. [Soundness]

> *A truth assignment that satisfies the two premises of a resolution inference also satisfies the conclusion.*

Proof. Let σ be a truth assignment that satisfies $C' \vee P$ and $D' \vee \neg P$. If P is true under assignment σ, then σ satisfies D', otherwise it satisfies C'. We may conclude that σ satisfies $C' \vee D'$.

Completeness

Various resolution calculi can be shown to be "refutationally complete" in the following sense.

Theorem. [Refutation Completeness]

> *If a set of ground clauses is unsatisfiable, then a contradiction (i.e., the empty clause) can be derived.*

We are not proving this result now but will consider a more general version of resolution for first-order clauses, and its completeness, later on.

Factoring

Completeness does raise some subtle points about presentation of clauses–in particular, of repeated occurrences of literals–and the correct formulation and implementation of the resolution rule.

For instance, we could slightly modify resolution to an inference rule

$$\frac{C' \vee P \vee \cdots \vee P \qquad D' \vee \neg P \vee \cdots \vee \neg P}{C' \vee D'}$$

with the additional condition that C' and D' do not contain P or $\neg P$.

A different approach is to combine the original resolution rule, without additional conditions on C' and D', with *ground factoring*:

$$\frac{C' \vee L \vee L}{C' \vee L}$$

where L is a (positive or negative) literal.