CSE451 EXERCISE 4 SOLUTIONS

QUESTION 1 **Use the fact** that $v : VAR \longrightarrow \{F, \perp, T\}$ be such that $v^*((a \cap b) \Rightarrow \neg b) = \perp$ under **Ł** semantics **to evaluate** $v^*(((b \Rightarrow \neg a) \Rightarrow (a \Rightarrow \neg b)) \cup (a \Rightarrow b))$. Use shorthand notation.

Ł Negation

¬	F	⊥	T
	T	⊥	F

Ł Disjunction

∪	F	⊥	T
F	F	⊥	T
⊥	⊥	⊥	T
T	T	T	T

Ł Conjunction

∩	F	⊥	T
F	F	F	F
⊥	F	⊥	⊥
T	F	⊥	T

Ł-Implication

⇒	F	⊥	T
F	T	T	T
⊥	⊥	T	T
T	F	⊥	T

Solution : $((a \cap b) \Rightarrow \neg b) = \perp$ in two cases.

C1 $(a \cap b) = \perp$ and $\neg b = F$.

C2 $(a \cap b) = T$ and $\neg b = \perp$.

Case C1: $\neg b = F$, i.e. $b = T$, and hence $(a \cap T) = \perp$ iff $a = \perp$. We get that v is such that $v(a) = \perp$ and $v(b) = T$.

We evaluate: $v^*(((b \Rightarrow \neg a) \Rightarrow (a \Rightarrow \neg b)) \cup (a \Rightarrow b)) = (((T \Rightarrow \neg \perp) \Rightarrow (\perp \Rightarrow \neg T)) \cup (\perp \Rightarrow T)) = ((\perp \Rightarrow \perp) \cup T) = T$.

Case C2: $\neg b = \perp$, i.e. $b = \perp$, and hence $(a \cap \perp) = T$ what is impossible, hence v from case C1 is the only one.

QUESTION 2 Prove using proper logical equivalences (list them at each step) that

$$\neg((A \Rightarrow \neg B) \cup (B \Rightarrow \neg A)) \equiv (A \cap B).$$

Solution : $\neg((A \Rightarrow \neg B) \cup (B \Rightarrow \neg A)) \equiv^{deMorg} (\neg(A \Rightarrow \neg B) \cap \neg(B \Rightarrow \neg A)) \equiv^{negimpl} ((A \cap \neg\neg B) \cap (B \cap \neg\neg A)) \equiv^{dneg} ((A \cap B) \cap (B \cap A)) \equiv^{assoc,comm} (A \cap B)$.

QUESTION 3 We define an EQUIVALENCE of LANGUAGES as follows:

Given two languages:
$\mathcal{L}_1 = \mathcal{L}_{CON_1}$ and $\mathcal{L}_2 = \mathcal{L}_{CON_2}$, for $CON_1 \neq CON_2$.
We say that they are **logically equivalent**, i.e.

$$\mathcal{L}_1 \equiv \mathcal{L}_2$$

if and only if the following conditions **C1, C2** hold.

C1: For every formula A of $\mathcal{L}_1$, there is a formula B of $\mathcal{L}_2$, such that

$$A \equiv B,$$

C2: For every formula C of $\mathcal{L}_2$, there is a formula D of $\mathcal{L}_1$, such that

$$C \equiv D.$$

Prove that $\mathcal{L}_{\{\neg, \cap, \Rightarrow\}} \equiv \mathcal{L}_{\{\uparrow\}}$.

HINT: use $\neg a = a \uparrow a, \quad a \cup b = (a \uparrow a) \uparrow (b \uparrow b)$.