cse547/ams547 Practice Final Problems Spring 2017

QUESTION 1

Part1 Prove that

$$\sum_{k=2}^{n} \frac{(-1)^k}{2k-1} = -\sum_{k=1}^{n-1} \frac{(-1)^k}{2k+1}$$

Part 2 Use **partial fractions** and Part 1 result (must use it!) to evaluate the sum

$$S = \sum_{k=1}^{n} \frac{(-1)^k k}{(4k^2-1)}$$

QUESTION 2 Give a direct proof from proper properties (use the list) of the following.
For all $x \in R, x > 0$

$$\lfloor \sqrt{\lfloor x \rfloor} \rfloor = \lfloor \sqrt{x} \rfloor$$

QUESTION 3

1. Prove that the series $\sum_{n=1}^{\infty} \frac{1}{(n+1)^2}$ **does not react** on D'Alambert's Criterium

2. Prove that the series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ converges.

QUESTION 4 Solve the recurrence: for $n > 0$)

$$a_0 = 1, \quad a_n = a_{n-1} + \lfloor \sqrt{a_{n-1}} \rfloor, \quad \text{for } n > 0$$

Hint assume first that $a_n = m^2$ for certain $m \in Z$ and find formulas for a_{n+2k+1} and a_{n+2k+2}.

QUESTION 5 Prove the following.

1. Let $m, n, k \in Z + -\{0\}$.
IF $k|mn$ and $k \perp m$ (it means k, m are relatively prime), THEN $k|n$.

2. When a number is relatively prime to each of several numbers, it is relatively prime to their product.

QUESTION 6

Write a proof of the following:

$spec(\sqrt{2})$ and $spec(2+\sqrt{2})$ are disjoint sets.

QUESTION 7

Find the sum of all multiples of x in the closed interval $[\alpha...\beta]$, when $x > 0$.
Justify methods used in each step of your calculation.

QUESTION 8

Denote by $N(\alpha, n)$ the number of elements in the $Spec(\alpha)$ that are $\leq n$, i.e.

$$N(\alpha, n) = |\{m \in Spec(\alpha) : \quad m \leq n\}|.$$

Write a detailed proof of

$$N(\alpha, n) = \left\lceil \frac{n+1}{\alpha} \right\rceil - 1.$$

No credit without each step explanations.

QUESTION 9 Show that the nth element of the sequence:

$$1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, ...$$

is $\lfloor \sqrt{2n} + \frac{1}{2} \rfloor$.

Hint: Let P(x) represent the position of the last occurrence of x in the sequence.
Use the fact that $P(x) = \frac{x(x+1)}{2}$.
Let the nth element be m. You need to find m.

QUESTION 10 Prove that
$\binom{x}{m}\binom{m}{k} = \binom{x}{k}\binom{x-k}{m-k}$
holds for all $m, k \in Z$ and $x \in R$. Consider all cases and Polynomial argument. No credit without all cases and pol. argument!

QUESTION 11 Prove the Hexagon property $(n, k \in N)$

$$\binom{n-1}{k-1}\binom{n}{k+1}\binom{n+1}{k} = \binom{n-1}{k}\binom{n+1}{k+1}\binom{n}{k-1}$$