

CSE547 HW3 Chapter5-3

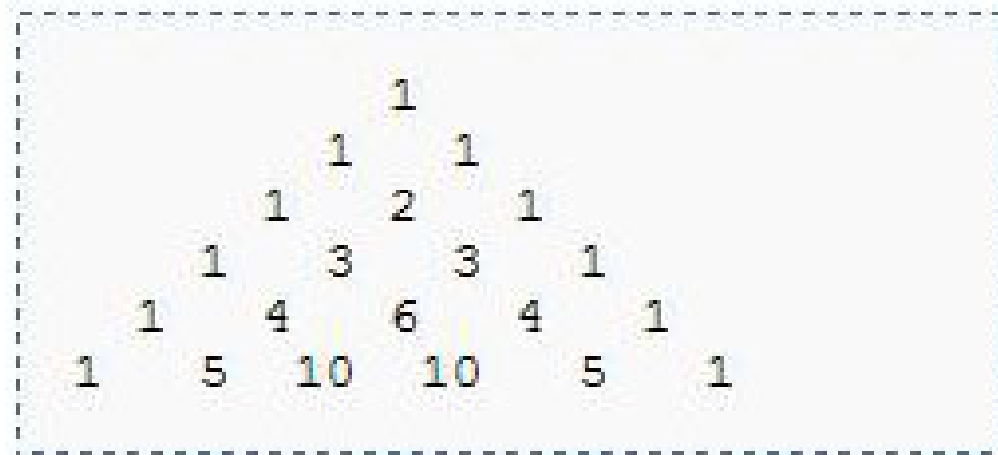
Problem 3

- Prove the hexagon property.

$$\binom{n-1}{k-1} \binom{n}{k+1} \binom{n+1}{k} = \binom{n-1}{k} \binom{n+1}{k+1} \binom{n}{k-1}$$

Pascal's Triangle

- **Pascal's triangle** is a geometric arrangement of the binomial coefficients in a triangle. It is named after Blaise Pascal.



Hexagon Property

- From the Pascal's triangle, we can form a hexagon surrounded by a number in it. Multiplying alternate number from the hexagon gives the same product. (text p155)

A piece of the Pascal's triangle table

n			$\binom{n}{2}$	$\binom{n}{3}$	$\binom{n}{4}$	
8	1	...	28	56	70	...
9	1	...	36	84	126	...
10	1	...	45	120	210	...

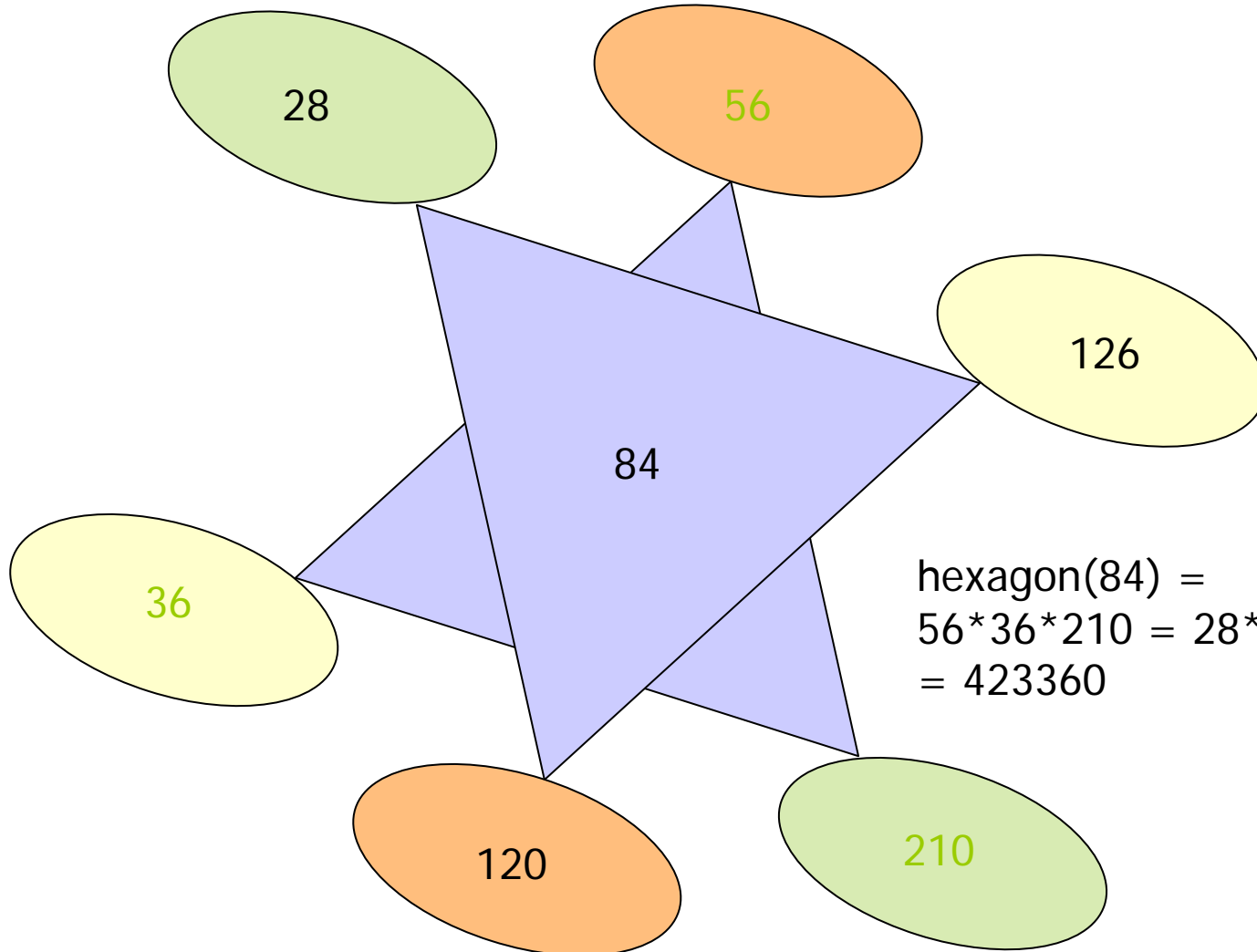
example

$$\binom{8}{2} = \frac{8 * 7}{2 * 1} = 28$$

$$\binom{9}{2} = \frac{9 * 8}{2 * 1} = 36$$

$$\binom{10}{2} = \frac{10 * 9}{2 * 1} = 45$$

Hexagon Property (Cont.)



$$\begin{aligned}\text{hexagon}(84) &= \\ 56 * 36 * 210 &= 28 * 120 * 126 \\ &= 423360\end{aligned}$$

Definition

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Integers $n \geq k \geq 0$

Solution

$$\begin{aligned} \binom{n-1}{k-1} \binom{n}{k+1} \binom{n+1}{k} &= \frac{(n-1)!}{(k-1)!(n-1-(k-1))!} \frac{n!}{(k+1)!(n-k-1)!} \frac{(n+1)!}{k!(n+1-k)!} \\ &= \frac{(n-1)!}{(k-1)!(n-k)!} \frac{n!}{(k+1)!(n-k-1)!} \frac{(n+1)!}{k!(n-k+1)!} \\ &= \frac{(n-1)!}{k!(n-k-1)!} \frac{(n+1)!}{(k+1)!(n-k)!} \frac{n!}{(k-1)!(n-k+1)!} \\ &= \binom{n-1}{k} \binom{n+1}{k+1} \binom{n}{k-1} \end{aligned}$$