CSE547 HW3 Chapter5-3

Problem 3

- Prove the hexagon property.

$$\binom{n-1}{k-1}\binom{n}{k+1}\binom{n+1}{k} = \binom{n-1}{k}\binom{n+1}{k+1}\binom{n}{k-1}$$

Pascal’s Triangle

- **Pascal's triangle** is a geometric arrangement of the binomial coefficients in a triangle. It is named after Blaise Pascal.

```
          1
        1   1
      1   2   1
    1   3   3   1
  1   4   6   4   1
1   5  10  10   5   1
```

Hexagon Property

- From the Pascal's triangle, we can form a hexagon surrounded by a number in it. Multiplying alternate number from the hexagon gives the same product. (text p155)

A piece of the Pascal's triangle table

n			$\binom{n}{2}$	$\binom{n}{3}$	$\binom{n}{4}$	
8	1	…	28	56	70	…
9	1	…	36	84	126	…
10	1	…	45	120	210	…

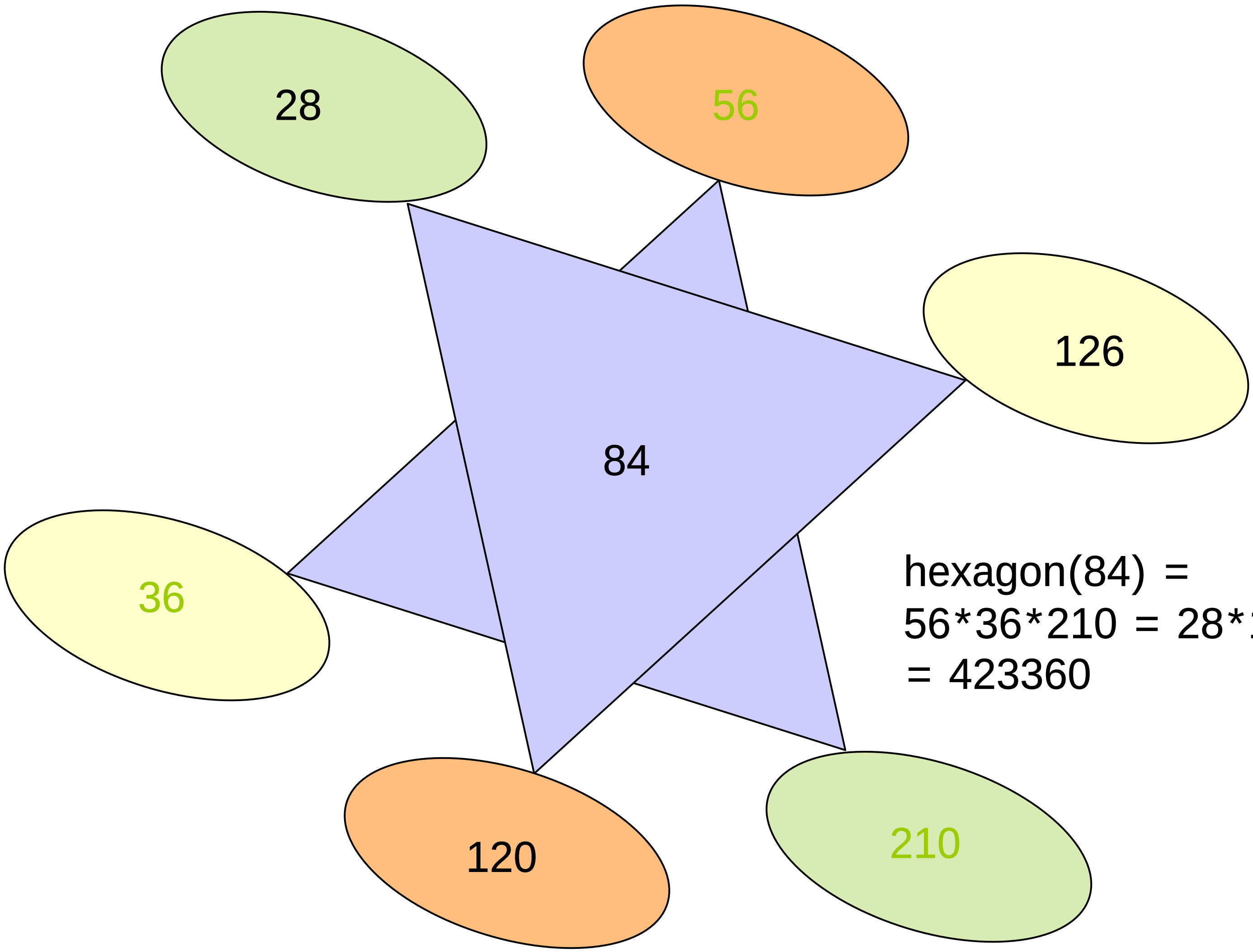
example

$$\binom{8}{2} = \frac{8*7}{2*1} = 28$$

$$\binom{9}{2} = \frac{9*8}{2*1} = 36$$

$$\binom{10}{2} = \frac{10*9}{2*1} = 45$$

Hexagon Property (Cont.)



hexagon(84) =
56*36*210 = 28*120*126
= 423360

Definition

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Integers $n \geq k \geq 0$

Solution

$$\binom{n-1}{k-1}\binom{n}{k+1}\binom{n+1}{k} = \frac{(n-1)!}{(k-1)!(n-1-(k-1))!}\frac{n!}{(k+1)!(n-k-1)!}\frac{(n+1)!}{k!(n+1-k)!}$$

$$= \frac{(n-1)!}{(k-1)!(n-k)!}\frac{n!}{(k+1)!(n-k-1)!}\frac{(n+1)!}{k!(n-k+1)!}$$

$$= \frac{(n-1)!}{k!(n-k-1)!}\frac{(n+1)!}{(k+1)!(n-k)!}\frac{n!}{(k-1)!(n-k+1)!}$$

$$= \binom{n-1}{k}\binom{n+1}{k+1}\binom{n}{k-1}$$