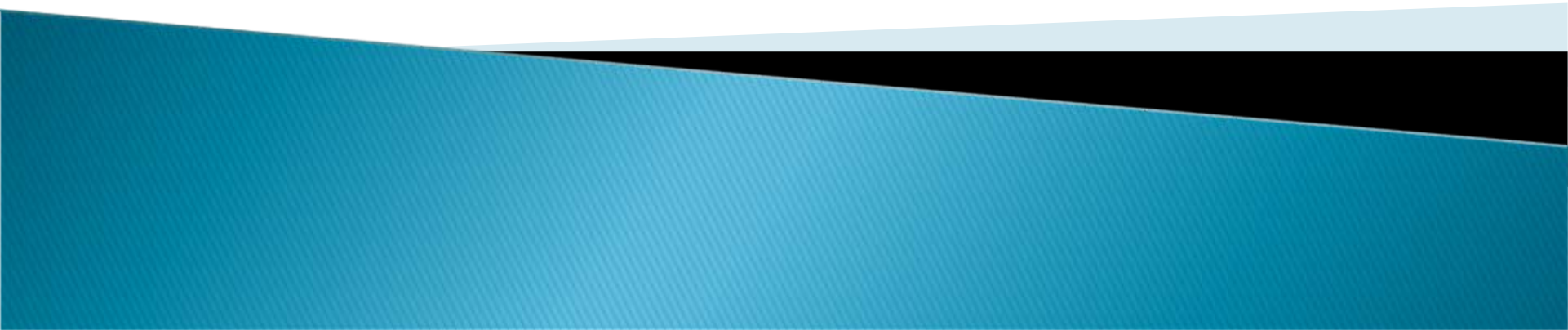


# Chapter 5, Problem 7



# Problem

- ▶ Is it true also when  $k < 0$  ?

$$r^k \left( r - \frac{1}{2} \right)^k = \frac{(2r)^{2k}}{2^{2k}}$$

# Observation1 ( $k > 0$ )

- ▶ Each term in the denominator of expanded  $r$  to the  $-k$  falling adds  $2r$  with an even number, increasingly.

$$\begin{aligned} r^{-k} &= \frac{1}{(r+1)(r+2)\dots(r+k)} \\ &= \frac{1}{\left(\frac{2r+2}{2}\right)\left(\frac{2r+4}{2}\right)\dots\left(\frac{2r+2k}{2}\right)} \\ &= \frac{2^k}{(2r+2)(2r+4)\dots(2r+2k)} \end{aligned}$$

# Observation2 ( $k > 0$ )

- ▶ Each term in the denominator of expanded  $(r-1/2)$  to the  $-k$  falling adds with an odd number, increasingly.

$$\begin{aligned}
 \left(r - \frac{1}{2}\right)^{-k} &= \frac{1}{\left(r - \frac{1}{2} + 1\right)\left(r - \frac{1}{2} + 2\right)\dots\left(r - \frac{1}{2} + k\right)} \\
 &= \frac{1}{\left(\frac{2r - 1 + 2}{2}\right)\left(\frac{2r - 1 + 4}{2}\right)\dots\left(\frac{2r - 1 + 2k}{2}\right)} \\
 &= \frac{1}{\left(\frac{2r + 1}{2}\right)\left(\frac{2r + 3}{2}\right)\dots\left(\frac{2r + 2k - 1}{2}\right)} \\
 &= \frac{1}{2^k (2r + 1)(2r + 3)\dots(2r + 2k - 1)}
 \end{aligned}$$

$$\begin{aligned}
& \frac{2^k}{(2r+2)(2r+4)\dots(2r+2k)} \\
\times & \frac{2^k}{(2r+1)(2r+3)\dots(2r+2k-1)} \\
= & \frac{2^{2k}}{(2r+1)(2r+2)(2r+3)\dots(2r+2k-1)(2r+2k)}
\end{aligned}$$

- ▶ This result equals to  $2r$  to the  $-2k$  falling times  $2^{2k}$ .

$$2^{2k}$$

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$$(2r + 1)(2r + 2)(2r + 3) \dots (2r + 2k - 1)(2r + 2k)$$

▶ Thus,

$$r^{-k} \left(r - \frac{1}{2}\right)^{-k} = (2r)^{-2k} 2^{2k} = \frac{(2r)^{-2k}}{2^{-2k}}, k > 0$$

- ▶ As the problem ask for in case  $k < 0$ , we can set a  $k'$  whose domain is negative integers; therefore we can replace  $-k$  with  $k'$
- ▶ We can rewrite the formula as

$$r^{k'} \left(r - \frac{1}{2}\right)^{k'} = \frac{(2r)^{2k'}}{2^{2k'}}, k' < 0$$

# The Result

- ▶ As the domain of  $k'$  is as same as the domain of  $k$ (that is less than zero) in the problem, we got the solution:

$$r^k \left(r - \frac{1}{2}\right)^k = \frac{(2r)^{2k}}{2^{2k}} \quad \text{Is also true when } k < 0.$$



# Verifying the property

- ▶ In case  $k = -1$ ,

$$r^{-1} \left(r - \frac{1}{2}\right)^{-1} = \frac{4}{(2r+1)(2r+2)} = \frac{(2)^{-2}}{2^{-2}}$$

- ▶ In case  $k = -2$ ,

$$r^{-2} \left(r - \frac{1}{2}\right)^{-2} = \frac{16}{(2r+1)(2r+2)(2r+3)(2r+4)} = \frac{(2)^{-4}}{2^{-4}}$$