CSE547 HOMEWORK 4 (DISCRETE MATHEMATICS)

DEFINITIONS

The definitions listed below are correct, or have small mistakes.

Circle YES if the definition listed is correct.

Circle NOT and CORRECT it, if the definition is not correct.

LIST of all definitions is posted in DOWNLOADS.

PART 1: GENERAL DEFINITIONS

Power Set $\mathcal{P}(A) = \{X : \ A \subseteq X\}$.

y n

Relative Complement $A - B = \{a : a \in A \ \cap \ a \ \notin B\}$.

y n

(Cartesian) Product of two sets A and B.
$A \times B = \{(a, b) : \ a \in A \ \cap b \ \in B\}$.

y n

Domain of R Let $R \subseteq A \times A$, we define domain of R: $D_R = \{a \in A : \ (a, b) \in R\}$.

y n

ONTO function $f : \ A \overset{onto}{\longrightarrow} B$ iff $\forall b \in A \exists a \in B \ f(a) = b$.

y n

Composition Let $f : \ A \longrightarrow B$ and $g : \ B \longrightarrow C$, we define a new function $h : \ A \longrightarrow C$, called a COMPOSITION of f and g, as follows: for any $x \in A$, $h(x) = g(f(x))$.

y n

Inverse function Let $f : \ A \longrightarrow B$ and $g : \ B \longrightarrow A$.
g is called an INVERSE function to f iff $\forall a \in A \ ((f \circ g)(a) = g(f(a) = a)$.

y n

Sequence of elements of a set A is any function $f : \ N \longrightarrow A$ or $f : \ N - \{0\} \longrightarrow A$.

y n

Generalized Intersection of a sequence $\{A_n\}_{n \in N}$ of sets: $\bigcap_{n \in N} A_n = \{x : \ \exists n \in N \ x \in A_n\}$.

y n

Equivalence relation $R \subseteq A \times A$ is an equivalence relation in A iff it is reflexive, antisymmetric and transitive.

y n

Partition A family of sets $\mathbf{P} \subseteq \mathcal{P}(A)$ is called a partition of the set A iff the following conditions hold.

1. $\forall\ X \in \mathbf{P}\ (X\ = \emptyset)$

2. $\forall\ X, Y \in \mathbf{P}\ (X \cup Y = \emptyset)$

3. $\bigcup \mathbf{P} = A$

y **n**

Partition and Equivalence For any partition $\mathbf{P} \subseteq \mathcal{P}(A)$ of A, there is an equivalence relation on A such that its equivalence classes are some sets of the partition $\mathbf{P}$.

y **n**

Mathematical Induction Let $P(n)$ be any property (predicate) defined on a set N of all natural numbers such that:
Base Case $n = 2$ $P(2)$ is true.
Inductive Step The implication $P(n) \Rightarrow P(n+1)$ can be proved for any $n \in N$
THEN $\forall n \in N P(n)$ is a true statement.

y **n**

PART 2: POSETS

Poset A set $A \neq \emptyset$ ordered by a relation R is called a poset. We write it as a tuple: (A, R), $(A, \leq)$, $(A, \preceq)$ or (A, $\boxed{\leq}$). Name poset stands for "partially ordered set".

y **n**

Smallest (least) $a_0 \in A$ is a smallest (least) element in the poset $(A, \preceq)$ iff $\exists a \in A\ (a_0 \preceq a)$.

y **n**

Greatest (largest) $a_0 \in A$ is a greatest (largest) element in the poset $(A, \preceq)$ iff $\forall a \in A\ (a \preceq a_0)$.

y **n**

Maximal $a_0 \in A$ is a maximal element in the poset $(A, \preceq)$ iff $\neg\ \forall a \in A\ (a_0 \preceq a\ \cap\ a_0 \neq a)$.

y **n**

Minimal $a_0 \in A$ is a minimal element in the poset $(A, \preceq)$ iff $\neg\ \exists a \in A\ (a \preceq a_0\ \cap\ a_0 \neq a)$.

y **n**

Lower Bound Let $B \subseteq A$ and $(A, \preceq)$ is a poset. $a_0 \in A$ is a lower bound of a set B iff $\exists b \in B\ (a_0 \preceq b)$.

y **n**

Upper Bound Let $B \subseteq A$ and $(A, \preceq)$ is a poset. $a_0 \in A$ is an upper bound of a set B iff $\forall b \in B\ (b \preceq a_0)$.

y **n**

Least upper bound of B (lub B) Given: a set $B \subseteq A$ and $(A, \preceq)$ a poset.

An element $x_0 \in B$ is a least upper bound of B, $x_0 = lubB$ iff x_0 is (if exists) the least (smallest) element in the set of all upper bounds of B, ordered by the poset order $\preceq$.

y **n**

Greatest lower bound of B (glb B) Given: a set $B \subseteq A$ of a poset $(A, \preceq)$.

An element $x_0 \in A$ is a greatest lower bound of B, $x_0 = glbB$ iff x_0 is (if exists) the greatest element in the set of all lower bounds of B, ordered by the poset order $\preceq$.

y **n**

PART 3: LATTICES and BOOLEAN ALGEBRAS

Lattice A poset $(A, \preceq)$ is a lattice iff For all $a, b \in A$ $lub\{a, b\}$ or $glb\{a, b\}$ exist.

y **n**

Lattice notation Observe that by definition elements $lubB$ and $glbB$ are always unique (if they exist). For $B = \{a, b\}$ we denote:
$lub\{a, b\} = a \cup b$ and $glb\{a, b\} = a \cap b$.

y **n**

Lattice union (meet) The element $lub\{a, b\} = a \cap b$ is called a lattice union (meet) of a and b. By lattice definition for any $a, b \in A$ $a \cap b$ always exists.

y **n**

Lattice intersection (joint) The element $glb\{a, b\} = a \cup b$ is called a lattice intersection (joint) of a and b. By lattice definition for any $a, b \in A$ $a \cup b$ always exists.

y **n**

Lattice as an Algebra An algebra $(A, \cup, \cap)$, where $\cup, \cap$ are two argument operations on A is called a lattice iff the following conditions hold.

For any $a, b, c \in A$ (they are called lattice AXIOMS):

l1 $a \cup b = b \cup a$ and $a \cap b = b \cap a$

l2 $(a \cup b) \cup c = a \cup (b \cup c)\ and\$(a \cap b) \cap c = a \cap (b \cap c)$

l3 $a \cap (a \cup b) = a$ and $a \cup (a \cap b) = a$.

y **n**

Lattice axioms The conditions **l1- l3** from above definition are called lattice axioms.

y **n**

Lattice orderings Let the $(A, \cup, \cap)$ be a lattice. The relations:

$a \preceq b$ iff $a \cup b = b$, $a \preceq b$ iff $a \cap b = a$
are order relations in A and are called a lattice orderings.

y **n**

Distributive lattice A lattice $(A, \cup, \cap)$ is called a distributive lattice iff for all $a, b, c \in A$ if the following conditions hold

l4 $a \cup (b \cap c) = (a \cup b) \cap (a \cup c)$

l5 $a \cap (b \cup c) = (a \cap b) \cup (a \cap c)$.

y **n**

Distributive lattice axioms Conditions l4- l5 from above are called a distributive lattice axioms.

y **n**

Lattice special elements The greatest element in a lattice (if exists) is denoted by 1 and is called a lattice UNIT. The least (smallest) element in A (if exists)is denoted by 0 and called a lattice zero.

y **n**

Lattice with unit and zero If 0 (lattice zero) and 1 (lattice unit) exist in a lattice, we will write the lattice as: $(A, \cup, \cap, 0, 1)$ and call is a lattice with zero and unit.

y **n**

Lattice Unit Definition Let $(A, \cup, \cap)$ be a lattice. An element $x \in A$ is called a lattice unit iff for any $a \in A$ $\quad x \cup a = a \quad$ and $\quad x \cap a = x$.

y **n**

Lattice Unit Axioms If lattice unit x exists we denote it by 1 and we write the unit axioms as follows.

l6 $1 \cap a = a$

l7 $1 \cup a = 1$.

y **n**

Lattice Zero Let $(A, \cup, \cap)$ be a lattice. An element $x \in A$ is called a lattice zero iff for any $a \in A$ $x \cup a = x \quad$ and $\quad x \cap a = a$.

y **n**

Lattice Zero Axioms If lattice zero exists we denote it by 0 and write the zero axioms as follows.

l8 $0 \cup a = 0$

l9 $0 \cap a = a$.

y **n**

Complement Definition Let $(A, \cup, \cap, 1, 0)$ be a lattice with unit and zero. An element $x \in A$ is called a complement of an element $a \in A$ iff $\quad a \cap x = 1 \quad$ and $\quad a \cup x = 0$.

y **n**

Complement axioms Let $(A, \cup, \cap, 1, 0)$ be a lattice with unit and zero. The complement of $a \in A$ is usually denoted by $-a$ and the above conditions that define the complement above are called complement axioms. The complement axioms are usually written as follows.

c1 $a \cup -a = 0$

c2 $a \cap -a = 1$.

y **n**

Boolean Algebra A distributive lattice with zero and unit such that each element has a complement is called a Boolean Algebra.

y **n**

Boolean Algebra Axioms A lattice $(A, \cup, \cap, 1.0)$ is called a Boolean Algebra iff the operations $\cap, \cup$ satisfy axioms **l1 -l5**, $0 \in A$ and $1 \in A$ satisfy axioms **l6 - l9** and each element $a \in A$ has a complement $-a \in A$, i.e.

l1o $\forall a \in A \ \exists -a \in A \ ((a \cup -a = 1) \ \cap \ (a \cap -a = 0))$.

y **n**

PART 4: CARDINALITIES OF SETS, Finite and Infinite Sets.

Cardinality definition Sets A and B have the same cardinality iff $\exists f(\ f : \ A \overset{1-1, onto}{\longrightarrow} B)$.

y **n**

Cardinality notations $|A| = |B|$ or $cardA = cardB$, or $A \sim B$ all denote that the sets A and B have the same cardinality.

y **n**

Finite We say: a set A is finite iff $\exists n \in N(|A| = n)$.

y n

Infinite A set A is infinite iff A is NOT finite.

y n

Cardinality Aleph zero We say that a set A has a cardinality $\aleph_0$ ($|A| = \aleph_0$) iff $|A| = |N|$.

y n

Countable A set A is countable iff $|A| = \aleph_0$.

y n

Uncountable A set A is uncountable iff A is NOT countable.

y n

Cardinality Continuum We say that a set A has a cardinality $\mathcal{C}$ ($|A| = \mathcal{C}$) iff $|A| = |R|$.

y n

Cardinality $A \leq$ **Cardinality** B $|A| \leq |B|$ iff $A \sim C$ and $C \subseteq B$.

y n

Cardinality $A <$ **Cardinality** B $|A| < |B|$ iff $|A| \leq |B|$ or $|A| \neq |B|$.

y n

Cantor Theorem For any set A, $|A \leq \mathcal{P}(A)|$.

y n

PART 5: ARITHMETIC OF CARDINAL NUMBERS

Sum of cardinal numbers ($\mathcal{N} + \mathcal{M}$)

We define:

$\mathcal{N} + \mathcal{M} = |A \cup B|$, where A, B are such that $|A| = \mathcal{N}$, $|B| = \mathcal{M}$.

y n

Multiplication of cardinal numbers ($\mathcal{N} \cdot \mathcal{M}$)

We define:

$\mathcal{N} \cdot \mathcal{M} = |A \times B|$, where A, B are such that $|A| = \mathcal{N}$, $|B| = \mathcal{M}$.

y n

Power ($\mathcal{M}^{\mathcal{N}}$) $\mathcal{M}^{\mathcal{N}} = card\{f : \ f : \ A \longrightarrow B\}$, where A, B are such that $|A| = \mathcal{M}$, $|B| = \mathcal{N}$.

y n

Power $2^{\mathcal{N}}$ We define:

$2^{\mathcal{N}} = card\{f : \ f : \ A \longrightarrow \{0, 1\} \ \}$, where $|A| = \mathcal{N}$. **y n**

PART 6: ARITHMETIC OF n, $\aleph_0$, $\mathcal{C}$

Union 1 $\aleph_0 + \aleph_0 = \aleph_0$.

Union of two countable sets is a countable set.

y n

Union 2 $\aleph_0 + n = \aleph_0$.

Union of a finite (cardinality n) and a countable set is an infinitely countable set. **y n**

Union 3 $\aleph_0 + \mathcal{C} = \mathcal{C}$.

Union of an infinitely countable set and an uncountable set is an uncountable set.

y n

Cartesian Product 1 $\aleph_0 \cdot \aleph_0 = \aleph_0$.

Cartesian Product of two countable sets is a countable set.

y n

Cartesian Product 2 $n \cdot \aleph_0 = \aleph_0$.

Cartesian Product of a finite set and an infinite set is an infinite set.

y n

Cartesian Product 3 $\aleph_0 \cdot \mathcal{C} = \mathcal{C}$.

Cartesian Product of an infinitely countable set and a set of the same cardinality as Real numbers has the same cardinality as the set of Real numbers.

y n

Cartesian Product 4 $\mathcal{C} \cdot \mathcal{C} = \mathcal{C}$.

Cartesian Product of two uncountable sets is an uncountable set.

y n

Power 1 $2^{\aleph_0} = \mathcal{C}$.

y n

Power 2 $\aleph_0^{\aleph_0} = \mathcal{C}$ means that

$card\{f : \quad f : N \longrightarrow N\} = \mathcal{C}$.

y n

Power 3 $\mathcal{C}^{\mathcal{C}} = 2^{\mathcal{C}}$ means that there are $2^{\mathcal{C}}$ of all functions that map R into R.

y n

Inequalities $n < \aleph_0 \leq \mathcal{C}$.

y n

QUESTIONS

Circle proper answer. WRITE a short JUSTIFICATION. NO JUSTIFICATION, NO CREDIT.

1. If $f : \quad A \longrightarrow^{1-1}_{onto} B$ and $g : B \longrightarrow^{1-1}_{onto} A$, then g is an inverse to f.

   JUSTIFY:

   y n

2. Let $f : \quad N \times N \longrightarrow N$ be given by a formula $f(n,m) = n + m^2$. f is a $1-1$ function.

   JUSTIFY:

   y n

3. Let $A = \{a, \{\emptyset\}, \emptyset\}$, $B = \{\emptyset, \{\emptyset\}, \emptyset\}$. There is a function $f: \ A \longrightarrow^{1-1}_{onto} \ B$.

JUSTIFY:

y **n**

4. If $f : A \longrightarrow^{1-1} \ B$ and $g : B \longrightarrow^{onto} \ A$, then $f \circ g$ and $g \circ f$ are *onto*.

JUSTIFY:

y **n**

5. $f : R - \{0\} \longrightarrow^{1-1} \ R$ is given by a formula: $f(x) = \frac{1}{x}$ and $g : R - \{0\} \longrightarrow \ R - \{0\}$ given by $g(x) = \frac{1}{x}$.
g is inverse to f.

JUSTIFY:

y **n**

6. $\{(1, 2), (a, 1)\}$ is a binary relation in $\{1, 2, 3, \}$.

JUSTIFY:

y **n**

7. The function $f: \ N \longrightarrow \ \mathcal{P}(N)$ given by formula: $f(n) = \{m \in N : \ m \leq n\}$ is a $1{-}1$ function.

JUSTIFY:

y **n**

8. The function $f: \ N \times N \longrightarrow \ \mathcal{P}(N)$ given by formula: $f(n, m) = \{m \in N : \ m + n = 1\}$ is is a sequence.

JUSTIFY:

y **n**

9. The function $f: \ N \times N \longrightarrow \ \mathcal{P}(N)$ given by formula: $f(n, m) = \{m \in N : \ m + n = 1\}$ is is 1-1.

JUSTIFY:

y **n**

10. The $f: \ N \longrightarrow \ \mathcal{P}(N)$ given by formula: $f(n) = \{m \in N : \ m + n = 1\}$ is a family of sets.

JUSTIFY:

y **n**

11. Let P be a predicate. If $P(0)$ is true and for all $k \leq n$, $P(k)$ is true implies $P(n+1)$ is true, then $\forall n \in N \ P(n)$ is true.

JUSTIFY:

y **n**

12. Let $A_n = \{x \in R : \ n \leq x \leq n+1\}$. Consider $\{A_n\}_{n\in N}$. $\bigcap_{n\in N} A_n = \emptyset$.

JUSTIFY:

y **n**

13. Let $A_n = \{x \in R : \ n+1 \leq x \leq n+2\}$. Consider $\{A_n\}_{n\in N}$. $\bigcup_{n\in N} A_n = R$.

JUSTIFY:

y **n**

14. $x \in \bigcup_{t\in T} A_t$ iff $\exists t \in T (x \in A_t)$.

JUSTIFY:

y **n**

15. Let $A_n = \{x \in N : 0 < x < n\}$. The family $\{A_n\}_{n\in N}$ form a partition of N.

JUSTIFY:

y **n**

16. Let $A_t = \{x \in \{1,2,3\} : x > t\}$ for $t \in \{0,1,2\}$. $\bigcap_{t\in T} A_t = \{1\}$.

JUSTIFY:

y **n**

17. There is an equivalence relation on N with infinite number of equivalence classes.

JUSTIFY:

y **n**

18. There is an equivalence relation on $A = \{x \in R : 1 \leq x < 4\}$ with equivalence classes: $[1] = \{x \in R : 1 \leq x < 2\}$, $[2] = \{x \in R : 2 \leq x < 3\}$, and $[3] = \{x \in R : 3 \leq x < 4\}$.

JUSTIFY:

y **n**

19. Each element of a partition of a set A = { 1,2,3} is an equivalence class of a certain equivalence relation.

JUSTIFY:

y **n**

20. Set of all equivalence classes of a given equivalence relation is a partition.

JUSTIFY:

y **n**

21. Let $R \subset A \times A$ The set $[a] = \{b \in A : (a,b) \in R\}$ is an equivalence class with a representative a.

JUSTIFY:

y **n**

22. Let $A = \{a, b, c, d\}$. There are 4^3 words of length 3 in A^*.

JUSTIFY:

y **n**

23. If a set A has n elements ($n \in N$), then every subset of A is finite.

JUSTIFY:

y **n**

24. Let $\sum$ be an alphabet $\sum = \{\%, \$, \&\}$. Denote $\sum^k = \{w \in \sum^* : lenghth(w) = k\}$. The set $\sum^3$ has 29 elements.

JUSTIFY:

y **n**

25. There is an order relation that is also an equivalence relation and a function.

JUSTIFY:

y **n**

26. $R = \{(N, \{1, 2, 3, \}), (Z, \{1, 2, 3, \}), (1, N), (-1, N), (3, Z)\}$ is a function defined on a set $\{N, Z, 1, -1, 3\}$ with values in the set Z.

JUSTIFY:

y **n**

27. If $f: \ R \longrightarrow \ R$ and $g: R \longrightarrow^{1-1} \ R$, then $g \circ f$ and $f \circ g$ exists.

JUSTIFY:

y **n**

28. $\{(1, 2), (a, 1), (a, a)\}$ is a transitive binary relation defined in $A = \{1, 2, a\}$.

JUSTIFY:

y **n**

29. $f: \ N \longrightarrow \mathcal{P}(R)$ is given by the formula: $f(n) = \{x \in R; \ \ x \leq \frac{-n^3+1}{\sqrt{n+3+6}}\}$ is a sequence.

JUSTIFY:

y **n**

30. There is an order relation R defined in $A \neq \emptyset$ such that (A, R) is a poset.

JUSTIFY:

y **n**

31. Let $A = \{\emptyset, N, \{1\}, \{a, b, 3\}\}$. There are no more then 50 words of length 4 in A^*.

JUSTIFY:

y **n**

32. There is an equivalence relation on Z with infinitely countably many equivalence classes.

JUSTIFY:

y **n**

33. A is uncountable iff $|A| = |R|$ where R is the set of real numbers.

JUSTIFY:

y **n**

34. A is infinite iff some subsets of A are infinite.

JUSTIFY:

y **n**

35. There exists an equivalence relation on N with $\aleph_0$ equivalence classes.

JUSTIFY:

y **n**

36. A is finite iff some subsets of A are finite.

JUSTIFY:

y **n**

37. If A is a countable set, the any subset of A is countable.

JUSTIFY:

y **n**

38. If A is uncountable set, then any subset of A is uncountable.

JUSTIFY:

y **n**

39. $\{x \in Q : 1 \leq x \leq 2\}$ has the same cardinality as $\{x \in Q : 5 \leq x \leq 10\}$.

JUSTIFY:

y **n**

40. If A is infinite set and B finite set, then $((A \cup B) \cap A)$ is infinite set.

JUSTIFY:

y **n**

41. The set of all squares centered in the origin has the same cardinality as R.

y **n**

JUSTIFY:

42. If A, B are infinitely countable sets, then $A \cap B$ is a countable set.

JUSTIFY:

y **n**

43. A is uncountable iff there is a subset B of A such that $|B| = |A|$.

JUSTIFY:

y **n**

44. A is uncountable iff $|A| = C$.

JUSTIFY:

y **n**

45. $\aleph_0 + \aleph_0 = \aleph_0$ means that the union of two infinitely countable sets is an infinitely countable set.

JUSTIFY:

y **n**

46. $|\mathcal{P}(N)| = \aleph_0$

JUSTIFY:

y **n**

47. $card(N \cap \{1, 3\}) = card(Q \cap \{1, 2\})$

JUSTIFY:

y **n**

48. A relation in N defined as follows: $n \approx m$ iff $n + m \in EVEN$ has $\aleph_0$ equivalence classes. in N.

JUSTIFY:

y **n**

49. $cardA < card\mathcal{P}(A)$

JUSTIFY:

y **n**

50. A is infinite set iff there is $f : N \longrightarrow^{1-1}_{onto} A$.

JUSTIFY:

y **n**

51. $\mathcal{P}(A) = \{B : B \subset A\}$

JUSTIFY:

y **n**

52. $|Q \cup N| = \aleph_0$

JUSTIFY:

y **n**

53. $|R \times Q| = \mathcal{C}$

JUSTIFY:

y **n**

54. $|N| \leq \aleph_0$

JUSTIFY:

y **n**

55. Any non empty POSET has at least one MAX element.

JUSTIFY:

y **n**

56. If $(A, \preceq)$ is a finite poset (i.e. A is a finite set), then a unique maximal is the largest element and a unique minimal is the least element.

JUSTIFY:

y **n**

57. There is a non empty POSET that has no Max element.

JUSTIFY:

y **n**

58. Any lattice is a POSET.

JUSTIFY:

y **n**

59. It is possible to order N in such a way that $(N, \leq)$ has $\aleph_0$ MAX elements and no MIN elements.

JUSTIFY:

y **n**

60. In any poset $(A, \preceq)$, the greatest and least elements are unique.

JUSTIFY:

y **n**

61. If a non empty poset in finite, then unique MAX element is the smallest.

JUSTIFY:

y **n**

62. Each non empty lattice has 0 and 1.

JUSTIFY:

y **n**

63. In any poset $(A, \preceq)$, if a greatest and a least elements exist, then they are unique.

JUSTIFY:

y **n**

64. Each distributive lattice has zero and unit elements.

JUSTIFY:

y **n**

65. It is possible to to order the set of Natural numbers N in such a way that the poset $(N, \preceq)$ has a unique maximal element (minimal element) and no largest element (least element).

JUSTIFY:

y **n**

66. It is possible to to order the set of rational numbers Q in such a way that the poset $(Q, \preceq)$ has a unique minimal element and no smallest (least) element.

JUSTIFY:

y **n**

67. In any poset, the largest element is a unique maximal element and the least element is the unique minimal element.

JUSTIFY:

y **n**

68. If $(A, \cup, \cap)$ is an infinite lattice (i.e. the set A is infinite), then 1 or 0 might or might not exist.

JUSTIFY:

y **n**

69. There is a poset $(A, \preceq)$ and a set $B \subseteq A$ and that B has none upper bounds.

JUSTIFY:

y **n**

70. There is a poset $(A, \preceq)$ and a set $B \subseteq A$ and that B has none infinite number of lower bounds.

JUSTIFY:

y **n**

71. If $(A, \cup, \cap)$ is a finite lattice (i.e. A is a finite set), then 1 and 0 always exist.

JUSTIFY:

y **n**

72. Any finite lattice is distributive.

JUSTIFY:

y **n**

73. Every Boolean algebra is a lattice.

JUSTIFY:

y **n**

74. Any infinite Boolean algebra has unit (greatest) and zero (smallest) elements.

JUSTIFY:

y **n**

75. A non- generate Finite Boolean Algebras always have 2^n elements ($n \geq 1$.

JUSTIFY:

y **n**

76. Sets A and B have the same cardinality iff $\exists f(f : A \stackrel{1-1}{\longrightarrow} B)$.

JUSTIFY:

y **n**

77. We say: a set A is finite iff $\exists n \in N(|A| = n)$.

JUSTIFY:

y **n**

78. A set A is infinite iff A is NOT finite.

JUSTIFY:

y **n**

79. $\aleph_0$ (Aleph zero) is a cardinality of only N (Natural numbers).

JUSTIFY:

y **n**

80. A set A is countable iff $|A| = \aleph_0$.

JUSTIFY:

y **n**

81. $\mathcal{C}$ (Continuum) is a cardinality of Real Numbers, i.e. $\mathcal{C} = |\mathcal{R}|$.

y **n**

82. For any set A, $|A| < |\mathcal{P}(A)|$.

JUSTIFY:

y **n**

83. $\mathcal{M}^{\mathcal{N}}$ is the cardinality of all functions that map a set A (of cardinality $\mathcal{N}$) into a set B (of cardinality $\mathcal{M}$).

JUSTIFY:

y **n**