

CSE547 HOMEWORK 4 (DISCRETE MATHEMATICS) SOLUTIONS

DEFINITIONS

Check the LIST OF DEFINITIONS (in Downloads) to verify the mistakes in case of NO answer.

PART 1: GENERAL DEFINITIONS

Power Set $\mathcal{P}(A) = \{X : A \subseteq X\}$.

n

Relative Complement $A - B = \{a : a \in A \cap a \notin B\}$.

y

(Cartesian) Product of two sets A and B.
 $A \times B = \{(a, b) : a \in A \cap b \in B\}$.

y

Domain of R Let $R \subseteq A \times A$, we define domain of R: $D_R = \{a \in A : (a, b) \in R\}$.

n

ONTO function $f : A \xrightarrow{\text{onto}} B$ iff $\forall b \in B \exists a \in A f(a) = b$.

n

Composition Let $f : A \rightarrow B$ and $g : B \rightarrow C$, we define a new function $h : A \rightarrow C$, called a COMPOSITION of f and g, as follows: for any $x \in A$, $h(x) = g(f(x))$.

y

Inverse function Let $f : A \rightarrow B$ and $g : B \rightarrow A$.
 g is called an INVERSE function to f iff $\forall a \in A ((f \circ g)(a) = g(f(a)) = a)$.

y

Sequence of elements of a set A is any function $f : N \rightarrow A$ or $f : N - \{0\} \rightarrow A$.

y

Generalized Intersection of a sequence $\{A_n\}_{n \in N}$ of sets: $\bigcap_{n \in N} A_n = \{x : \exists n \in N x \in A_n\}$.

n

Equivalence relation $R \subseteq A \times A$ is an equivalence relation in A iff it is reflexive, antisymmetric and transitive.

n

Partition A family of sets $\mathbf{P} \subseteq \mathcal{P}(A)$ is called a partition of the set A iff the following conditions hold.

1. $\forall X \in \mathbf{P} (X \neq \emptyset)$

2. $\forall X, Y \in \mathbf{P} (X \cup Y = \emptyset)$

3. $\bigcup \mathbf{P} = A$

n

Partition and Equivalence For any partition $\mathbf{P} \subseteq \mathcal{P}(A)$ of A , there is an equivalence relation on A such that its equivalence classes are some sets of the partition \mathbf{P} .

n

Mathematical Induction Let $P(n)$ be any property (predicate) defined on a set N of all natural numbers such that:

Base Case $n = 2$ $P(2)$ is true.

Inductive Step The implication $P(n) \Rightarrow P(n + 1)$ can be proved for any $n \in N$
THEN $\forall n \in NP(n)$ is a true statement.

n

PART 2: POSETS

Poset A set $A \neq \emptyset$ ordered by a relation R is called a poset. We write it as a tuple: (A, R) , (A, \preceq) , (A, \preceq) or $(A, \boxed{\preceq})$. Name poset stands for "partially ordered set".

y

Smallest (least) $a_0 \in A$ is a smallest (least) element in the poset (A, \preceq) iff $\exists a \in A (a_0 \preceq a)$.

y

Greatest (largest) $a_0 \in A$ is a greatest (largest) element in the poset (A, \preceq) iff $\forall a \in A (a \preceq a_0)$.

y

Maximal $a_0 \in A$ is a maximal element in the poset (A, \preceq) iff $\neg \forall a \in A (a_0 \preceq a \cap a_0 \neq a)$.

n

Minimal $a_0 \in A$ is a minimal element in the poset (A, \preceq) iff $\neg \exists a \in A (a \preceq a_0 \cap a_0 \neq a)$.

y

Lower Bound Let $B \subseteq A$ and (A, \preceq) is a poset. $a_0 \in A$ is a lower bound of a set B iff $\exists b \in B (a_0 \preceq b)$.

n

Upper Bound Let $B \subseteq A$ and (A, \preceq) is a poset. $a_0 \in A$ is an upper bound of a set B iff $\forall b \in B (b \preceq a_0)$.

y

Least upper bound of B (lub B) Given: a set $B \subseteq A$ and (A, \preceq) a poset.

An element $x_0 \in B$ is a least upper bound of B , $x_0 = lub B$ iff x_0 is (if exists) the least (smallest) element in the set of all upper bounds of B , ordered by the poset order \preceq .

n

Greatest lower bound of B (glb B) Given: a set $B \subseteq A$ of a poset (A, \preceq) .

An element $x_0 \in A$ is a greatest lower bound of B , $x_0 = glb B$ iff x_0 is (if exists) the greatest element in the set of all lower bounds of B , ordered by the poset order \preceq .

y

PART 3: LATTICES and BOOLEAN ALGEBRAS

Lattice A poset (A, \preceq) is a lattice iff For all $a, b \in A$ $\text{lub}\{a, b\}$ or $\text{glb}\{a, b\}$ exist.

y

Lattice notation Observe that by definition elements $\text{lub}B$ and $\text{glb}B$ are always unique (if they exist). For $B = \{a, b\}$ we denote:

$$\text{lub}\{a, b\} = a \cup b \text{ and } \text{glb}\{a, b\} = a \cap b.$$

y

Lattice union (meet) The element $\text{lub}\{a, b\} = a \cup b$ is called a lattice union (meet) of a and b . By lattice definition for any $a, b \in A$ $a \cup b$ always exists.

n

Lattice intersection (joint) The element $\text{glb}\{a, b\} = a \cap b$ is called a lattice intersection (joint) of a and b . By lattice definition for any $a, b \in A$ $a \cap b$ always exists.

y

Lattice as an Algebra An algebra (A, \cup, \cap) , where \cup, \cap are two argument operations on A is called a lattice iff the following conditions hold for any $a, b, c \in A$ (they are called lattice AXIOMS):

11 $a \cup b = b \cup a$ and $a \cap b = b \cap a$

12 $(a \cup b) \cup c = a \cup (b \cup c)$ and $(a \cap b) \cap c = a \cap (b \cap c)$

13 $a \cap (a \cup b) = a$ and $a \cup (a \cap b) = a$.

y

Lattice axioms The conditions **11- 13** from above definition are called lattice axioms.

y

Lattice orderings Let the (A, \cup, \cap) be a lattice. The relations:

$$a \preceq b \text{ iff } a \cup b = b, \quad a \preceq b \text{ iff } a \cap b = a$$

are order relations in A and are called a lattice orderings.

y

Distributive lattice Axioms A lattice (A, \cup, \cap) is called a distributive lattice iff for all $a, b, c \in A$ the following conditions hold

14 $a \cup (b \cap c) = (a \cup b) \cap (a \cup c)$

15 $a \cap (b \cup c) = (a \cap b) \cup (a \cap c)$.

Conditions 14- 15 from above are called a **distributive lattice axioms**.

y

Lattice special elements The greatest element in a lattice (if exists) is denoted by 1 and is called a lattice UNIT. The least (smallest) element in A (if exists) is denoted by 0 and called a lattice zero.

y

Lattice with unit and zero If 0 (lattice zero) and 1 (lattice unit) exist in a lattice, we will write the lattice as: $(A, \cup, \cap, 0, 1)$ and call it a lattice with zero and unit.

y

Lattice Unit Definition Let (A, \cup, \cap) be a lattice. An element $x \in A$ is called a lattice unit iff for any $a \in A$ $x \cup a = a$ and $x \cap a = x$.

n

Lattice Unit Axioms If lattice unit x exists we denote it by 1 and we write the unit axioms as follows.

16 $1 \cap a = a$

17 $1 \cup a = 1.$

n

Lattice Zero Let (A, \cup, \cap) be a lattice. An element $x \in A$ is called a lattice zero iff for any $a \in A$ $x \cup a = x$ and $x \cap a = a.$

n

Lattice Zero Axioms If lattice zero exists we denote it by 0 and write the zero axioms as follows.

18 $0 \cup a = 0$

19 $0 \cap a = a.$

n

Complement Definition Let $(A, \cup, \cap, 1, 0)$ be a lattice with unit and zero. An element $x \in A$ is called a complement of an element $a \in A$ iff $a \cap x = 1$ and $a \cup x = 0.$

n

Complement axioms Let $(A, \cup, \cap, 1, 0)$ be a lattice with unit and zero. The complement of $a \in A$ is usually denoted by $-a$ and the above conditions that define the complement above are called complement axioms. The complement axioms are usually written as follows.

c1 $a \cup -a = 0$

c2 $a \cap -a = 1.$

n

Boolean Algebra A distributive lattice with zero and unit such that each element has a complement is called a Boolean Algebra.

y

Boolean Algebra Axioms A lattice $(A, \cup, \cap, 1, 0)$ is called a Boolean Algebra iff the operations \cap, \cup satisfy axioms **11 -15**, $0 \in A$ and $1 \in A$ satisfy axioms **16 - 19** and each element $a \in A$ has a complement $-a \in A$, i.e.

11o $\forall a \in A \exists -a \in A ((a \cup -a = 1) \cap (a \cap -a = 0)).$

y

PART 4: CARDINALITIES OF SETS, Finite and Infinite Sets.

Cardinality definition Sets A and B have the same cardinality iff $\exists f(f : A \xrightarrow{1-1, onto} B).$

y

Cardinality notations $|A| = |B|$ or $cardA = cardB$, or $A \sim B$ all denote that the sets A and B have the same cardinality.

y

Finite We say: a set A is finite iff $\exists n \in N(|A| = n).$

y

Infinite A set A is infinite iff A is NOT finite.

y

Cardinality Aleph zero We say that a set A has a cardinality \aleph_0 ($|A| = \aleph_0$) iff $|A| = |N|.$

y

Countable A set A is countable iff $|A| = \aleph_0.$

n

Uncountable A set A is uncountable iff A is NOT countable.

y

Cardinality Continuum We say that a set A has a cardinality \mathcal{C} ($|A| = \mathcal{C}$) iff $|A| = |R|$.

y

Cardinality $A \leq$ Cardinality B $|A| \leq |B|$ iff $A \sim C$ and $C \subseteq B$.

y

Cardinality $A <$ Cardinality B $|A| < |B|$ iff $|A| \leq |B|$ or $|A| \neq |B|$.

n

Cantor Theorem For any set A , $|A| \leq |\mathcal{P}(A)|$.

n

PART 5: ARITHMETIC OF CARDINAL NUMBERS

Sum ($\mathcal{N} + \mathcal{M}$) We define:

$\mathcal{N} + \mathcal{M} = |A \cup B|$, where A, B are such that $|A| = \mathcal{N}$, $|B| = \mathcal{M}$.

n

Multiplication ($\mathcal{N} \cdot \mathcal{M}$) We define:

$\mathcal{N} \cdot \mathcal{M} = |A \times B|$, where A, B are such that $|A| = \mathcal{N}$, $|B| = \mathcal{M}$.

y

Power ($\mathcal{M}^{\mathcal{N}}$) $\mathcal{M}^{\mathcal{N}} = \text{card}\{f : f : A \rightarrow B\}$, where A, B are such that $|A| = \mathcal{M}$, $|B| = \mathcal{N}$.

y

Power $2^{\mathcal{N}}$ We define:

$2^{\mathcal{N}} = \text{card}\{f : f : A \rightarrow \{0, 1\}\}$, where $|A| = \mathcal{N}$.

y

PART 4: ARITHMETIC OF n, \aleph_0, \mathcal{C}

Union 1 $\aleph_0 + \aleph_0 = \aleph_0$.

Union of two countable sets is a countable set.

n

Union 2 $\aleph_0 + n = \aleph_0$.

Union of a finite (cardinality n) and a countable set is an infinitely countable set.

n

Union 3 $\aleph_0 + \mathcal{C} = \mathcal{C}$.

Union of an infinitely countable set and an uncountable set is an uncountable set.

n

Cartesian Product 1 $\aleph_0 \cdot \aleph_0 = \aleph_0$.

Cartesian Product of two countable sets is a countable set.

n

Cartesian Product 2 $n \cdot \aleph_0 = \aleph_0$.

Cartesian Product of a finite set and an infinite set is an infinite set.

n

Cartesian Product 3 $\aleph_0 \cdot \mathcal{C} = \mathcal{C}$.

Cartesian Product of an infinitely countable set and a set of the same cardinality as Real numbers has the same cardinality as the set of Real numbers.

y

Cartesian Product 4 $\mathcal{C} \cdot \mathcal{C} = \mathcal{C}$.

Cartesian Product of two uncountable sets is an uncountable set.

n

Power 1 $2^{\aleph_0} = \mathcal{C}$.

y

Power 2 $\aleph_0^{\aleph_0} = \mathcal{C}$ means that

$$\text{card}\{f : f : N \rightarrow N\} = \mathcal{C}.$$

y

Power 3 $\mathcal{C}^{\mathcal{C}} = 2^{\mathcal{C}}$ means that there are $2^{\mathcal{C}}$ of all functions that map R into R.

y

Inequalities $n < \aleph_0 \leq \mathcal{C}$.

n

QUESTIONS

Circle proper answer. WRITE a short JUSTIFICATION. NO JUSTIFICATION, NO CREDIT.

Here are YES/NO answers with FEW JUSTIFICATIONS as examples

1. If $f : A \xrightarrow{1-1}_{\text{onto}} B$ and $g : B \xrightarrow{1-1}_{\text{onto}} A$, then g is an inverse to f .

JUSTIFY: The statement guarantee only that INVERSE function EXISTS.

n

2. Let $f : N \times N \rightarrow N$ be given by a formula $f(n, m) = n + m^2$. f is a 1 - 1 function.

JUSTIFY: $f(1, 2) = 5 = f(4, 1)$

n

3. Let $A = \{a, \{\emptyset\}, \emptyset\}$, $B = \{\emptyset, \{\emptyset\}, \emptyset\}$. There is a function $f : A \xrightarrow{1-1}_{\text{onto}} B$.

JUSTIFY: $|A| = 3$, $|B| = 2$

n

4. If $f : A \xrightarrow{1-1} B$ and $g : B \xrightarrow{\text{onto}} A$, then $f \circ g$ and $g \circ f$ are onto.

JUSTIFY: $g \circ f$ no; take $|A| = 2$, $|B| = 3$

n

5. $f : R - \{0\} \xrightarrow{1-1} R$ is given by a formula: $f(x) = \frac{1}{x}$ and $g : R - \{0\} \rightarrow R - \{0\}$ given by $g(x) = \frac{1}{x}$.
 g is inverse to f .

JUSTIFY: f is not "onto"; inverse does not exist.

n

6. $\{(1, 2), (a, 1)\}$ is a binary relation in $\{1, 2, 3, \}$.
 JUSTIFY: $a \notin \{1, 2, 3, \}$. **n**
7. The function $f : N \longrightarrow \mathcal{P}(N)$ given by formula: $f(n) = \{m \in N : m \leq n\}$ is a 1-1 function.
 JUSTIFY: $n_1 \neq n_2$ then obviously $f(n_1) \neq f(n_2)$ **y**
8. The function $f : N \times N \longrightarrow \mathcal{P}(N)$ given by formula: $f(n, m) = \{m \in N : m + n = 1\}$ is a sequence.
 JUSTIFY: Domain of f is not N . **n**
9. The function $f : N \times N \longrightarrow \mathcal{P}(N)$ given by formula: $f(n, m) = \{m \in N : m + n = 1\}$ is 1-1.
 JUSTIFY: $f(n, m) = \emptyset$ for all n, m such that $m + n \neq 1$. **n**
10. The $f : N \longrightarrow \mathcal{P}(N)$ given by formula: $f(n) = \{m \in N : m + n = 1\}$ is a family of sets.
 JUSTIFY: Values of f are sets. **y**
11. Let P be a predicate. If $P(0)$ is true and for all $k \leq n$, $P(k)$ is true implies $P(n + 1)$ is true, then $\forall n \in N$ $P(n)$ is true.
 JUSTIFY: Principle of mathematical Induction. **y**
12. Let $A_n = \{x \in R : n \leq x \leq n + 1\}$. Consider $\{A_n\}_{n \in N}$. $\bigcap_{n \in N} A_n = \emptyset$.
 JUSTIFY: $A_n \cap A_{n+1} = \emptyset$ **n**
13. Let $A_n = \{x \in R : n + 1 \leq x \leq n + 2\}$. Consider $\{A_n\}_{n \in N}$. $\bigcup_{n \in N} A_n = R$.
 JUSTIFY: $\bigcup_{n \in N} \{x \in R : n + 1 \leq x \leq n + 2\} = [1, \infty) \neq R$ **n**
14. $x \in \bigcup_{t \in T} A_t$ iff $\exists t \in T (x \in A_t)$
 JUSTIFY: definition **y**
15. Let $A_n = \{x \in N : 0 < x < n\}$. The family $\{A_n\}_{n \in N}$ form a partition of N .
 JUSTIFY: $A_0 = \{x \in N : 0 < x < 0\} = \emptyset$. **n**
16. Let $A_t = \{x \in \{1, 2, 3\} : x > t\}$ for $t \in \{0, 1, 2\}$. $\bigcap_{t \in T} A_t = \{1\}$.
 JUSTIFY: $A_0 = \{1, 2, 3\}$, $A_1 = \{2, 3\}$, $A_2 = \{3\}$ and $\bigcap_{t \in T} A_t = \emptyset$. **n**
17. There is an equivalence relation on N with infinite number of equivalence classes.
 JUSTIFY: Equality on N . **y**

18. There is an equivalence relation on $A = \{x \in R : 1 \leq x < 4\}$ with equivalence classes: $[1] = \{x \in R : 1 \leq x < 2\}$, $[2] = \{x \in R : 2 \leq x < 3\}$, and $[3] = \{x \in R : 3 \leq x < 4\}$.
- JUSTIFY: $\{[1], [2], [3]\}$ is a partition of A . **y**
19. Each element of a partition of a set $A = \{1, 2, 3\}$ is an equivalence class of a certain equivalence relation.
- JUSTIFY: True for any set $A \neq \emptyset$. **y**
20. Set of all equivalence classes of a given equivalence relation is a partition.
- JUSTIFY: Partition Theorem. **y**
21. Let $R \subseteq A \times A$ The set $[a] = \{b \in A : (a, b) \in R\}$ is an equivalence class with a representative a .
- JUSTIFY: ONLY when R is an equivalence relation. **n**
22. Let $A = \{a, b, c, d\}$. There are 4^3 words of length 3 in A^* .
- JUSTIFY: Counting the functions theorem. **y**
23. If a set A has n elements ($n \in N$), then every subset of A is finite.
- JUSTIFY: Any subset of a finite set is a finite set. **y**
24. Let Σ be an alphabet $\Sigma = \{\%, \$, \&\}$. Denote $\Sigma^k = \{w \in \Sigma^* : \text{length}(w) = k\}$. The set Σ^3 has 27 elements.
- JUSTIFY: $3^3 = 27$ **n**
25. There is an order relation that is also an equivalence relation and a function.
- JUSTIFY: Equality on any set. **y**
26. $R = \{(N, \{1, 2, 3, \}), (Z, \{1, 2, 3, \}), (1, N), (-1, N), (3, Z)\}$ is a function defined on a set $\{N, Z, 1, -1, 3\}$ with values in the set Z .
- JUSTIFY: Elements of the range (values) of R are SUBSETS, not elements of Z . **n**
27. If $f : R \rightarrow R$ and $g : R \rightarrow^{1-1} R$, then $g \circ f$ and $f \circ g$ exists.
- JUSTIFY: Corresponding domains and ranges agree. **y**
28. $\{(1, 2), (a, 1), (a, a)\}$ is a transitive binary relation defined in $A = \{1, 2, a\}$.
- JUSTIFY: $(a, 2) \notin R$. **n**
29. $f : N \rightarrow \mathcal{P}(R)$ is given by the formula: $f(n) = \{x \in R; x \leq \frac{-n^3+1}{\sqrt{n+3+6}}\}$ is a sequence.
- JUSTIFY: Domain of f is N . **y**

30. There is an order relation R defined in $A \neq \emptyset$ such that (A, R) is a poset.
JUSTIFY: Definition of Poset. **y**
31. Let $A = \{\emptyset, N, \{1\}, \{a, b, 3\}\}$. There are no more than 50 words of length 4 in A^* .
JUSTIFY: $|A| = 4$, $3^4 > 50$. **n**
32. There is an equivalence relation on Z with infinitely countably many equivalence classes.
JUSTIFY: Equality on Z . **y**
33. A is uncountable iff $|A| = |R|$ where R is the set of real numbers.
JUSTIFY: $A = \mathcal{P}(R)$ is uncountable and by Cantor theorem $|R| < |\mathcal{P}(R)|$. **n**
34. A is infinite iff some subsets of A are infinite.
JUSTIFY: All subsets of a finite set are finite. **y**
35. There exists an equivalence relation on N with \aleph_0 equivalence classes.
JUSTIFY: Equality; $[n] = \{n\}$. **y**
36. A is finite iff some subsets of A are finite.
JUSTIFY: all subsets are finite; $\{1\} \subseteq N$ and N is infinite. **n**
37. If A is a countable set, then any subset of A is countable.
JUSTIFY: Theorem **y**
38. If A is uncountable set, then any subset of A is uncountable.
JUSTIFY: $N \subseteq R$. **n**
39. $\{x \in Q : 1 \leq x \leq 2\}$ has the same cardinality as $\{x \in Q : 5 \leq x \leq 10\}$.
JUSTIFY: both sets are of cardinality \aleph_0 . **y**
40. If A is infinite set and B is a finite set, then $((A \cup B) \cap A)$ is infinite set.
JUSTIFY: $((A \cup B) \cap A) = A$. **y**
41. The set of all squares centered in the origin has the same cardinality as R .
JUSTIFY: All such circles are uniquely defined by the radius r and $r \in R$. **y**
42. If A, B are infinitely countable sets, then $A \cap B$ is a countable set.
JUSTIFY: $A \cap B$ is finite or infinitely countable. **y**

43. A is uncountable iff there is a subset B of A such that $|B| = |A|$.
 JUSTIFY: $N \subseteq Q$, $|N| = |Q|$ and Q is NOT uncountable. **n**
44. A is uncountable iff $|A| = \mathcal{C}$.
 JUSTIFY: $\mathcal{P}(R)$ is uncountable and $|\mathcal{P}(R)| \neq \mathcal{C}$. **n**
45. $\aleph_0 + \aleph_0 = \aleph_0$ means that the union of two infinitely countable sets is an infinitely countable set.
 JUSTIFY: The fact that the union of two infinitely countable sets is an infinitely countable set is true (theorem), but does not reflect the definition of sum of cardinal numbers; two DISJOINT infinitely countable sets. **n**
46. $|\mathcal{P}(N)| = \aleph_0$
 JUSTIFY: $|\mathcal{P}(N)| = \mathcal{C}$. **n**
47. $\text{card}(N \cap \{1, 3\}) = \text{card}(Q \cap \{1, 2\})$
 JUSTIFY: both sets have 2 elements. **y**
48. A relation in \mathbb{N} defined as follows: $n \approx m$ iff $n + m \in \text{EVEN}$ has \aleph_0 equivalence classes. in \mathbb{N} .
 JUSTIFY: two equivalence classes. **n**
49. $\text{card}A < \text{card}\mathcal{P}(A)$
 JUSTIFY: Cantor Theorem **y**
50. A is infinite set iff there is $f : N \xrightarrow{1-1}_{\text{onto}} A$.
 JUSTIFY: this is definition of the infinitely countable set. **n**
51. $\mathcal{P}(A) = \{B : B \subset A\}$
 JUSTIFY: $B \subseteq A$ **n**
52. $|Q \cup N| = \aleph_0$
 JUSTIFY: $Q \cup N = Q$. **y**
53. $|R \times Q| = \mathcal{C}$
 JUSTIFY: $\mathcal{C} \cdot \aleph_0 = \mathcal{C}$. **y**

54. $|N| \leq \aleph_0$
- JUSTIFY: $|A| \leq |A|$. **y**
55. Any non empty POSET has at least one MAX element.
- JUSTIFY: (N, \leq) has no max element for \leq natural order. **n**
56. If (A, \preceq) is a finite poset (i.e. A is a finite set), then a unique maximal is the largest element and a unique minimal is the least element.
- JUSTIFY: Theorem **y**
57. There is a non empty POSET that has no Max element.
- JUSTIFY: (N, \leq) has no max element for \leq natural order. **y**
58. Any lattice is a POSET.
- JUSTIFY: definition **y**
59. It is possible to order N in such a way that (N, \leq) has \aleph_0 MAX elements and no MIN elements.
- JUSTIFY: diagram (lecture) **y**
60. In any poset (A, \preceq) , the greatest and least elements are unique.
- JUSTIFY: Theorem **y**
61. If a non empty poset is finite, then unique MAX element is the smallest.
- JUSTIFY; in a finite poset unique MAX element is the greatest. **n**
62. Each non empty lattice has 0 and 1.
- JUSTIFY: (Z, \leq) **n**
63. In any poset (A, \preceq) , if a greatest and a least elements exist, then they are unique.
- JUSTIFY: Theorem **y**
64. Each distributive lattice has zero and unit elements.
- JUSTIFY: diagram **n**
65. It is possible to to order the set of Natural numbers N in such a way that the poset (N, \preceq) has a unique maximal element (minimal element) and no largest element (least element).
- JUSTIFY: diagram **n**

66. It is possible to order the set of rational numbers Q in such a way that the poset (Q, \preceq) has a unique minimal element and no smallest (least) element.
- JUSTIFY: diagram **n**
67. In any poset, the largest element is a unique maximal element and the least element is the unique minimal element.
- JUSTIFY: Theorem **y**
68. If (A, \cup, \cap) is an infinite lattice (i.e. the set A is infinite), then 1 or 0 might or might not exist.
- JUSTIFY: always true **y**
69. There is a poset (A, \preceq) and a set $B \subseteq A$ and that B has none upper bounds.
- JUSTIFY: $(N, \leq), B = N - \{0\}$. **y**
70. There is a poset (A, \preceq) and a set $B \subseteq A$ and that B has infinite number of lower bounds.
- JUSTIFY: $(N, \geq), B = \{0, 1\}$. **y**
71. If (A, \cup, \cap) is a finite lattice (i.e. A is a finite set), then 1 and 0 always exist.
- JUSTIFY: Theorem **y**
72. Any finite lattice is distributive.
- JUSTIFY: example in the lecture of 5element non-distributive lattice **n**
73. Every Boolean algebra is a lattice.
- JUSTIFY: definition **y**
74. Any infinite Boolean algebra has unit (greatest) and zero (smallest) elements.
- JUSTIFY: by definition every Boolean algebra has unit (greatest) and zero (smallest) elements **y**
75. A non-generated Finite Boolean Algebras always have 2^n elements ($n \geq 1$).
- JUSTIFY: Theorem **y**
76. Sets A and B have the same cardinality iff $\exists f(f : A \xrightarrow{1-1} B)$.
- JUSTIFY: f must be also "onto". **n**
77. We say: a set A is finite iff $\exists n \in N(|A| = n)$.
- JUSTIFY: definition **y**
78. A set A is infinite iff A is NOT finite.
- JUSTIFY: definition **y**

79. \aleph_0 (Aleph zero) is a cardinality of only N (Natural numbers).

JUSTIFY: definition

y

80. A set A is countable iff $|A| = \aleph_0$.

JUSTIFY: A set A is countable iff is FINITE or $|A| = \aleph_0$.

n

81. \mathcal{C} (Continuum) is a cardinality of Real Numbers, i.e. $\mathcal{C} = |\mathcal{R}|$.

JUSTIFY: definition

y

82. For any set A , $|A| < |\mathcal{P}(A)|$.

JUSTIFY: Cantor Theorem

y

83. $\mathcal{M}^{\mathcal{N}}$ is the cardinality of all functions that map a set A (of cardinality \mathcal{N}) into a set B (of cardinality \mathcal{M}).

JUSTIFY: definition

y