EVALUATE : (Book page 39)

$$\boxed{S_n = \sum_{1 \le j < k \le n} \frac{1}{k-j}}$$

+

$P(k,j)$:

$1 \le j < k \le n$

$$S_n = \sum_{P(j,k)} a_{k,j}$$

$$\boxed{a_{k,j} = \frac{1}{k-j}}$$

$$S_1 = \sum_{1 \le j < k < 1} a_{k,j}$$ contrad

$\boxed{S_1}$ is undefined

Book defines

$$\boxed{S_1 = 0}$$

$$S_2 = \sum_{1 \le j < k \le 2} a_{k,j}$$

$1 \le j < k \le 2$ only ther

$j=1$, $k=2$

$$S_2 = a_{2,1} = \frac{1}{2-1} = 1$$

$$\boxed{S_2 = 1}$$

$$\boxed{S_n = \sum_{1 \le j < k \le n} a_{k,j}} \quad \text{for } a_{k,j} = \frac{1}{k-j}$$

$$S_3 = \sum_{1 \le j < k \le 3} a_{k,j}$$

$\boxed{1 \le j < k \le 3}$ true for $\begin{cases} k = 3 \\ j = 1, 2 \end{cases}$ $\begin{cases} k = 2 \\ j = 1 \end{cases}$

$$= a_{3,2} + a_{3,1} + a_{2,1}$$

$$= \frac{1}{3-2} + \frac{1}{3-1} + \frac{1}{2-1} = \frac{1}{1} + \frac{1}{2} + 1 = 5/2$$

$$\boxed{S_3 = 5/2}$$

$\boxed{\begin{array}{l} P(k,j) = \\ 1 \le j < k \le \end{array}}$

$$\boxed{S_3 = \sum_{1 \le j < k \le 3} \frac{1}{k-j} \quad = 3}$$

Now we want to express $P(k,j)$

$$\boxed{P(k,j) \equiv P_1(k) \cap P_2(j)}$$

$$\boxed{\sum_{P(k,j)} a_{k,j} \overset{\text{def.}}{=} \sum_{P_1(k)} \sum_{P_2(j)} a_{k,j} = \sum_{P_2(j)} \sum_{P_1(k)} a_{k,j}}$$

STEP 1

Obiwiously:

$$(*)\quad \boxed{1 \le j < k \le n \;\equiv\; 1 < k \le n \wedge 1 \le j < k}$$

$$P(k,j) \;\equiv\; P_1(k) \wedge P_2(j)$$

For $n = 1$ $\quad F = F$

$n > 1$ $\quad T = T$

We GET from $(*)$

$$\boxed{S_n = \sum_{1<k\le n} \sum_{1\le j<k} \frac{1}{k-j}}$$

Put $\boxed{j := k-j}$

Evaluate boundaries

$$= \sum_{1<k\le n} \sum_{1\le k-j<k} \frac{1}{j}$$

$1 \le k-j < k$

$1-k \le -j < 0$

$k-1 \ge j > 0$

$\boxed{0 < j \le k-1}$

$=$

$\boxed{1 \le j \le k-1}$

$$= \sum_{1<k\le n} \sum_{j=1}^{k-1} \frac{1}{j}$$

$$= \sum_{1<k\le n} H_{k-1}$$

Put

$k := k+1$

Boundaries

$1 < k+1 \le n$

$0 < k \le n-1$

$1 \le k \le n-1$

USE $\boxed{H_n = \sum_{j=1}^{n} 1/j}$

$$= \sum_{k=1}^{n-1} H_k$$

$$\boxed{S_n = \sum_{k=1}^{n-1} H_k}$$

NOT CLOSED

CHECK OUR RESULT

$$S_n = \boxed{\sum_{k=1}^{n-1} H_k} \stackrel{?}{=} \boxed{\sum_{1\le j<k\le n} \frac{1}{k-j}} = S_n$$

$$S_1 = \sum_{k=1}^{0} H_1 \qquad S_1 = \sum_{1\le j<k\le n} \frac{1}{k-j}$$

UNDEFINED + UNDEFINED.

p120

$$S_2 = \sum_{k=1}^{1} H_k = H_1 = 1 \quad + \qquad S_1 = 1$$

$$H_n = \sum_{k=1}^{n} \frac{1}{k} \qquad S_3 = 5/2$$

$$S_3 = \sum_{k=1}^{2} H_k = H_1 + H_2$$

$$\boxed{H_n = \sum_{j=1}^{n} \frac{1}{j}}$$

$$S_3 = H_1 + H_2 = 1 + 1\tfrac{1}{2} = 5/2 \quad +$$

$$\boxed{H_1 = 1}$$

$$H_1 = \frac{1}{1} = 1$$

$$H_2 = \sum_{j=1}^{2} \frac{1}{j} = \frac{1}{1} + \frac{1}{2}$$

$$\boxed{H_2 = 1\tfrac{1}{2}}$$

we define

$$\boxed{S_1 = 0}$$

BOOK: $\boxed{S_n = \sum_{k=0}^{n} H_k}$ NOT CORRECT!

$n=0$ UNDEFI[NED] H_0 !

STEP 2

CONTINUE

$$\boxed{S_n = \sum_{1 \le j < k \le n} \frac{1}{k-j}}$$

Express BOUNDARIES DIFFERENTLY

$$\boxed{P(k,j) \equiv R_1(k) \cap R_2(j)}$$

$$S_n = \sum_{P(k,j)} a_{k,j} = \sum_{R_2(j)} \sum_{R_1(k)} a_{k,j}$$

We write

$$\boxed{1 \le j < k \le n \equiv \underbrace{(1 \le j < n)}_{R_2} \cap \underbrace{(j < k \le n)}_{R_1}}$$

$$S_n = \sum_{1 \le j < k \le n} \frac{1}{k-j} = \sum_{1 \le j < n} \sum_{j < k \le n} \frac{1}{k-j}$$

put $k := k + j$

Boundaries

$j < k \le n$

$j < k + j \le n$

$0 < k \le n - j$

$\boxed{1 \le k \le n-j}$

$$= \sum_{1 \le j < n} \sum_{1 \le k \le n-j} \frac{1}{k}$$

$$= \sum_{1 \le j < n} H_{n-j}$$

next page

$$S_n = \sum_{1 \le j < n} H_{n-j}$$

Put

$j := n - j$

Boundaries

$1 \le j < n$

$1 \le n - j < n$

$1 - n \le -j < 0$

$n - 1 \geq j > 0$

$$\boxed{0 < j \le n-1}$$

$$\boxed{1 \le j \le n-1}$$

$$S_n = \sum_{1 \le j \le n-1} H_j$$

Book WRONG

$$\boxed{S_n = \sum_{j=0}^{n-1} H_j}$$

H_0 NOT exist

NOT Closed

$$\boxed{S_n = \sum_{j=1}^{n-1} H_j}$$

Similar formula as in STEP 1

$$S_n = \sum_{j=1}^{n-1} H_j \stackrel{?}{=} \sum_{1 \le j < k \le n} \frac{1}{k-1} = S_n$$

$n = 1$

S_1 undefined

$+$

$n = 1$

S_1 undefined

$$S_2 = \sum_{j=1}^{1} H_j = H_1$$

$+$

$S_2 = 1$

$S_2 = 1$

$S_3 = 5/2$

$$S_3 = \sum_{j=1}^{2} H_j = H_1 + H_2 = 1 + 1\tfrac{1}{2} = 5/2$$

$+$