Instruction for Assignment One: Algebraic Topology

David Gu

Computer Science Department
Stony Brook University

gu@cs.stonybrook.edu

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Algebraic Topology: Theoretic Proofs

Fundamental Group

Problem (Rotation Group)

Compute the fundamental group and the universal covering space of $SO(3)$.

Hint: Consider the unit tangent bundle of the sphere.

Fundamental Group

Problem (Surface Fundamental Group Algorithm)

Prove the algorithm based on the cut graph gives the generators and relators of the surface fundamental group.

Hint: Seifert-Van Kampen theorem and CW-cell decomposition.

Homology Group

Problem (Homology Group)

Let M be a simplicial complex, construct the combinatorial Laplace operator:

$$\Delta_k = \partial_k^T \partial_k + \partial_{k+1} \partial_{k+1}^T,$$

the eigen space of the zero eigven values gives $H_k(M, \mathbb{Z})$.