Assignment Six: Koebe's Iteration

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Circular Slit Map

Figure: Circular slit map.

Gram–Schmidt Orthonormalization

Input: Poly annulus M, $\partial M = \gamma_0 - \gamma_1 - \cdots - \gamma_n$;
Output: $n-1$ orthonormal non-exact harmonic 1-forms.

1. **for** $i = 0$ to n **do**
2. **while** true **do**
3. generate a random non-exact harmonic 1-form ω_i;
4. **for** $j = 0$ to $i - 1$ **do**
5. $w \leftarrow \langle \omega_i, \omega_j \rangle = \int_M \omega_i \wedge^* \omega_j$;
6. $\omega_i \leftarrow \omega_i - w * \omega_j$;
7. **endfor**
8. $w \leftarrow \langle \omega_i, \omega_i \rangle$;
9. **if** $w > 0.5$ **then** break;
10. **endwhile**
11. $\omega_i \leftarrow \omega_i / \sqrt{w}$
12. **endfor**

Circular Slit Map Algorithm

Input: Poly annulus M, $\partial M = \gamma_0 - \gamma_1 - \cdots - \gamma_n$;
Output:Circular slit map $\varphi : M \to \mathbb{C}$.

1. Compute $n-1$ exact harmonic 1-forms, using assignment 3,ω_i, such that $\omega_i = df_i$, f_i is 1 on γ_i, and zero on $\gamma_j, j \neq i$, f_i harmonic;
2. Gram–Schmidt orthonormalization;
3. Compute conjugate harmonic 1-forms, using Hodge star from assignment 3, obtain holomorphic 1-forms

$$\left\{\omega_1 + \sqrt{-1}^*\omega_1, \omega_2 + \sqrt{-1}^*\omega_2, \cdots, \omega_{n-1} + \sqrt{-1}^*\omega_n\right\}$$

4. Solving linear equation $\omega := \sum_i \lambda_i(\omega_i + \sqrt{-1}^*\omega_i)$

$$\int_{\gamma_0} \omega = 1, \int_{\gamma_1} \omega = -1, \int_{\gamma_j} \omega = 0, j = 2, \cdots, n.$$

Circular Slit Map Algorithm

5. Choose a base point $p_0 \in M$, define the polar map:

$$\varphi(p) = \exp\left(2\pi \int_{p_0}^{p} \omega\right).$$

Hole Filling Algorithm

Input: Poly annulus M, $\partial M = \gamma_0 - \gamma_1 - \cdots - \gamma_n$;
Output: A topologgical disk $\bar{M}$, such that all holes are filled.

1. $M_0 \leftarrow M$;
2. **for** $k = 0$ **to** n
3. Compute a circular slit map, map the surface to the circular slit domain $f_k : M_k \to \mathbb{C}$, γ_0 and γ_k are mapped to the exterior and interior circular boundary of $\mathbb{C}$;
4. Generate a mesh D_k using the inner boundary of $f_k(M_k)$ using Delaunay refinement mesh generation;
5. Fill the inner circle of $f_k(M_k)$ to obtain M_{k+1};

$$M_{k+1} \leftarrow f_k(M_k) \cup D_k.$$

6. **endfor**
7. $\bar{M} \leftarrow M_{n+1}$, return $\bar{M}$.

Koebe Iteration Method

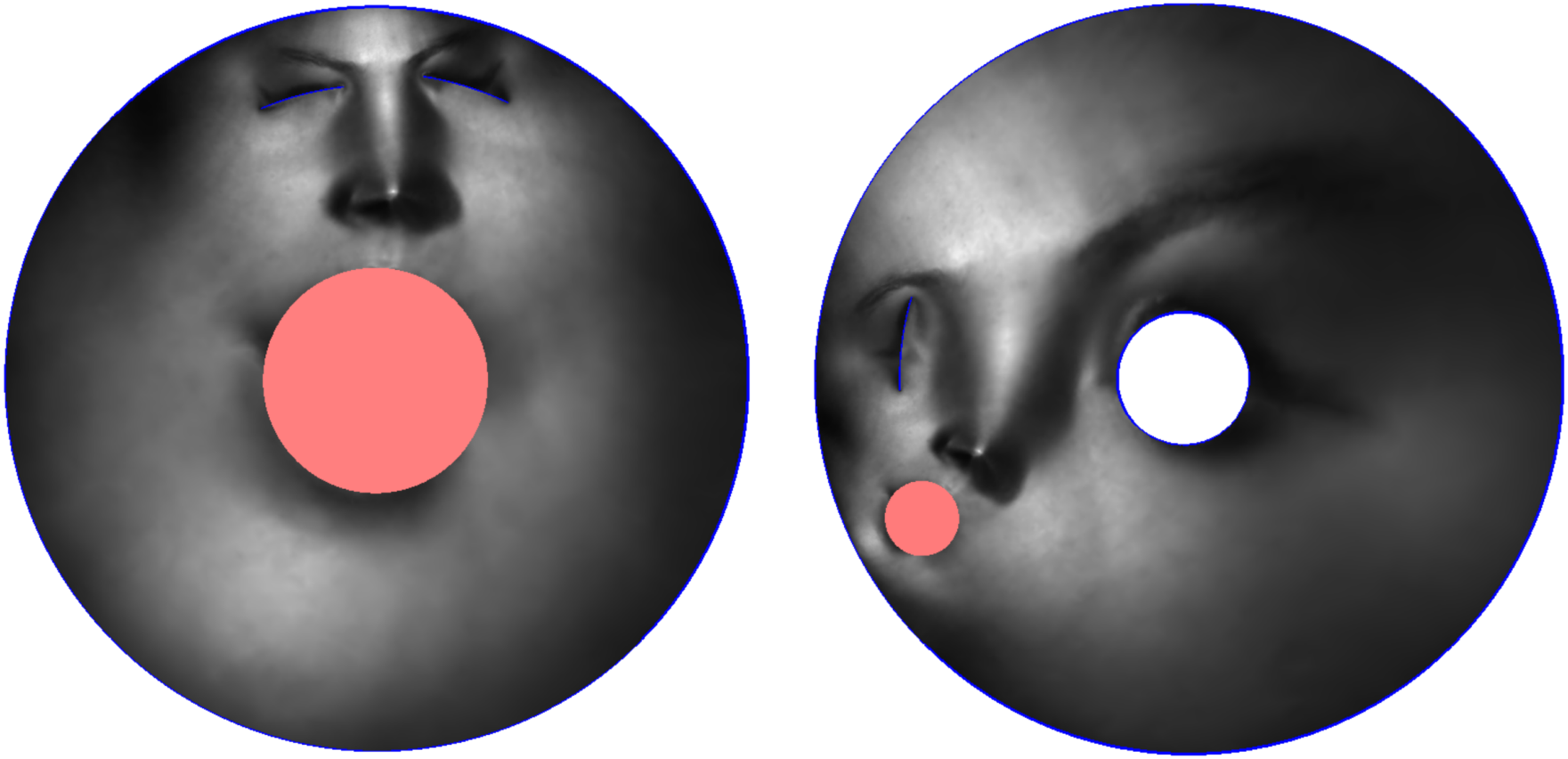


Figure: Hole filling and slit map.

Koebe Iteration Method

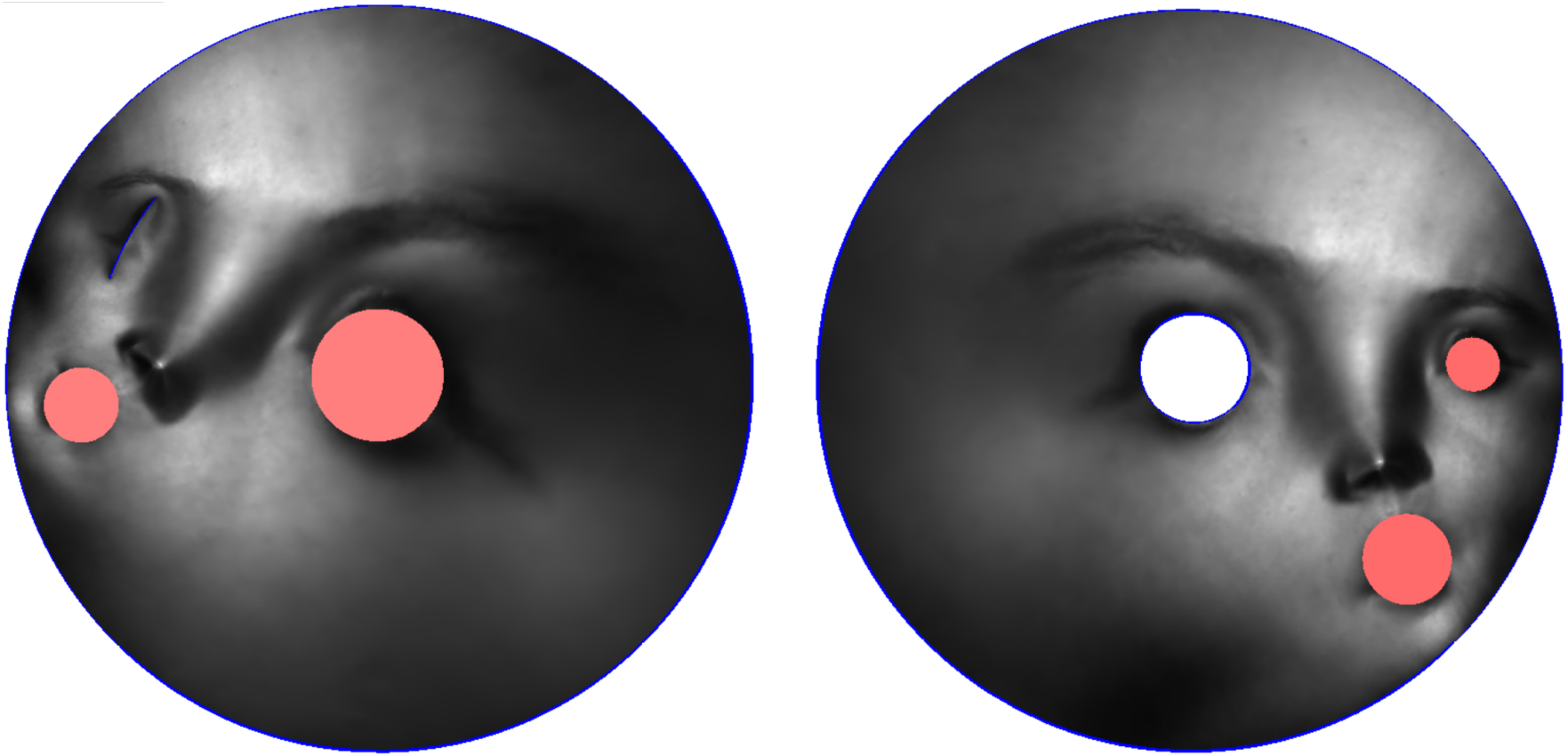


Figure: Hole filling and slit map.

Koebe Iteration Method

Figure: All holes are filled.

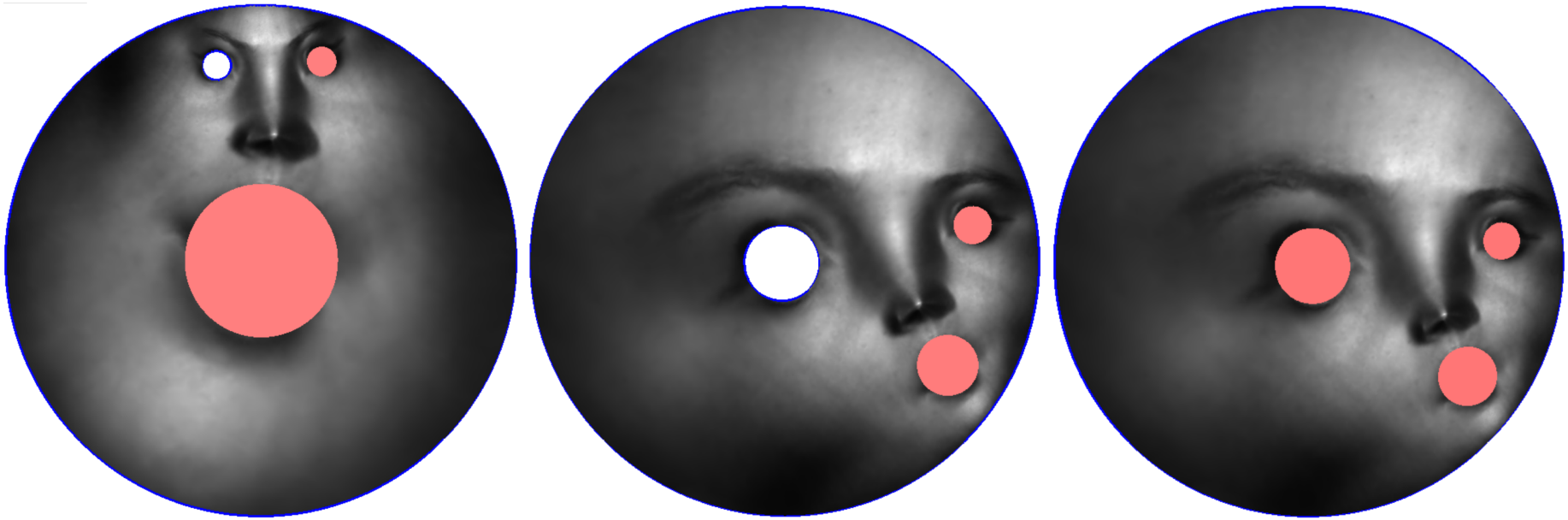
Koebe Iteration Algorithm

Input:A poly annulus M, $\partial M = \gamma_0 - \gamma_1 - \cdots - \gamma_n$
Output: A circular domain map $\varphi : M \to D$

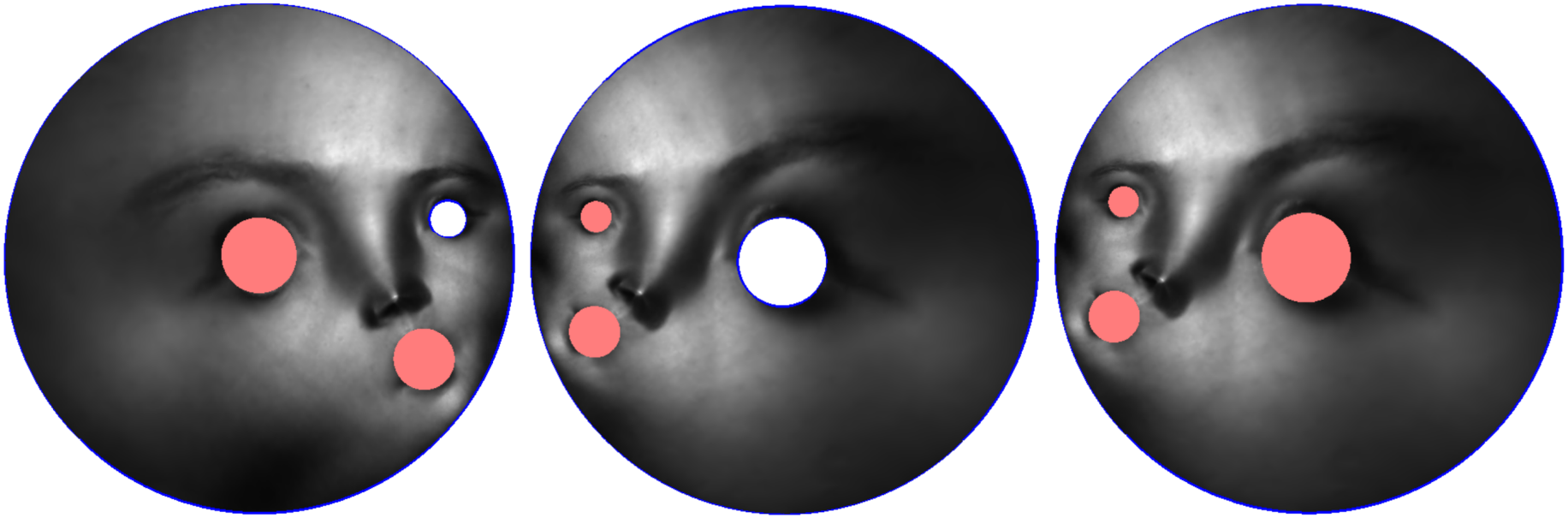
1. Puch a hole at the k-th inner boundary;
2. Compute a circular slit map, to map the surface onto a canonical planar annulus;
3. Fill the inner circular hole;
4. Repeat step 4 through 6, each time punch a different hole, until the process convergences.

Koebe Iteration Method

Koebe Iteration Method



Koebe Iteration Method



Koebe Iteration Method

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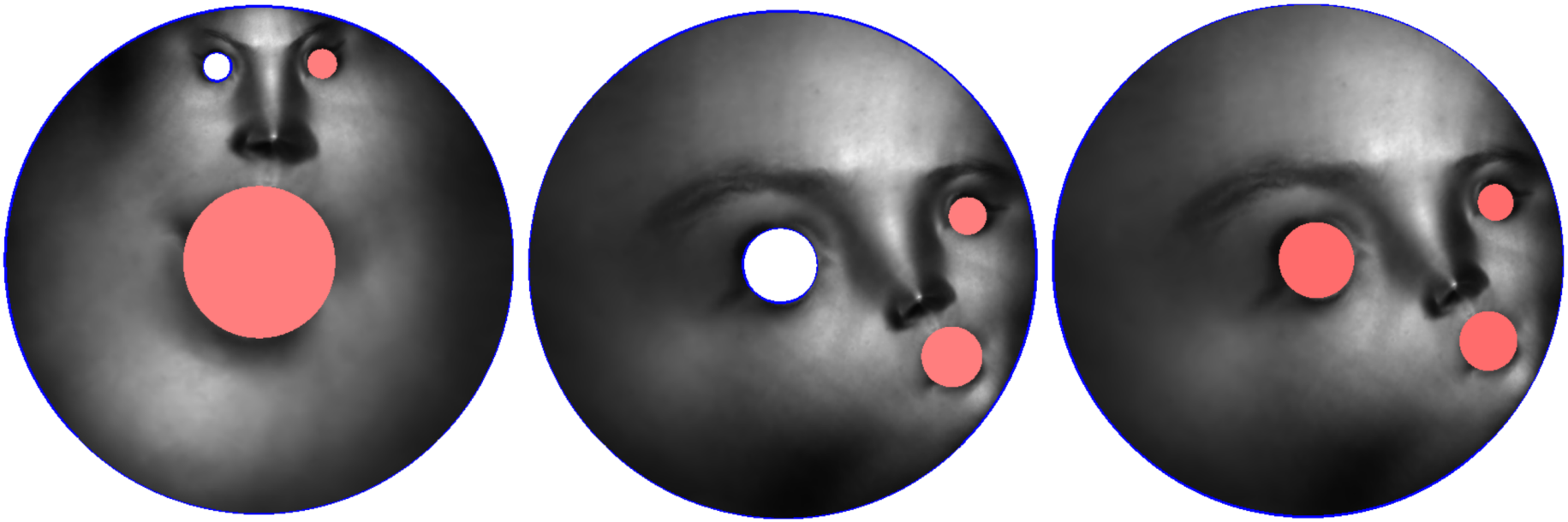
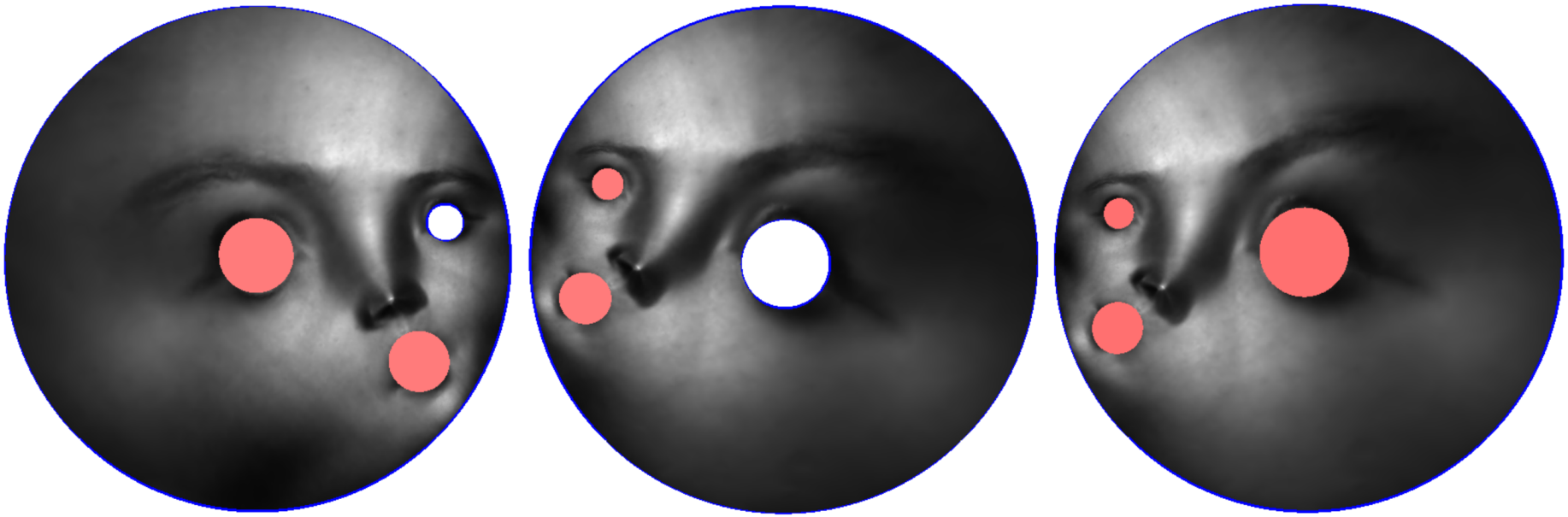


Koebe Iteration Method



Koebe Iteration Method

Koebe Iteration Method

Figure: Final result.

Instruction

Dependencies

1. 'Delaunay', Delaunay refinement for mesh generation.
2. 'Hodge', compute exact harmonic forms, random harmonic forms.
3. 'MeshLib', a mesh library based on halfedge data structure.
4. 'freeglut', a free-software/open-source alternative to the OpenGL Utility Toolkit (GLUT) library.

Directory Structure

- 3rdparty/Delaunay, headers, dlls, libs for Delaunay mesh generation;
- 3rdparty/Hodge, headers, dlls, libs for Hodge decomposition;
- koebeiteration/include, the header files for Koebe Iteration;
- koebeiteration/src, the source files for koebe iteration;
- data,Some data models and batch scripts;
- CMakeLists.txt, CMake configuration file;
- resources, snapshot for circular slit mapping results;
- textures, texture images needed.

Configuration

Before you start, read README.md carefully, then go three the following procedures, step by step.

1. Install [CMake](https://cmake.org/download/).
2. Download the source code of the C++ framework.
3. Configure and generate the project for Visual Studio.
4. Open the .sln using Visual Studio, and complie the solution.
5. Finish your code in your IDE.
6. Run the executable program.

3. Configure and generate the project

1. open a command window
2. cd Assignment_6_Koebe_Iteration_skeleton
3. mkdir build
4. cd build
5. cmake ..
6. open CCGHomework.sln inside the build directory.

5. Finish your code in your IDE

- You need to modify the file: SlitMap.h, CircularSlitMap.cpp and HoleFiller.cpp
- search for comments "insert your code"
- Modify functions:
  1. $CSlitMap\langle M\rangle :: \_slit\_map(intc1, intc2)$
  2. $MeshLib :: polar\_map(CHodgeDecompositionMesh * pMesh)$
  3. Gram-Schmidt orthonormalization

$$
\begin{aligned}
&MeshLib :: calc\_holo\_1\_form\_open\_mesh(\\
&conststd :: string\&input\_mesh,\\
&std :: vector\langle CHodgeDecompositionMesh*\rangle\&g\_meshes,\\
&std :: vector\langle CHodgeDecompositionMesh*\rangle\&h\_meshes,\\
&std :: string\&output\_mesh\_name)
\end{aligned}
$$

5. Finish your code in your IDE

- Modify functions:

$$\begin{aligned} &\mathit{void}\ \mathit{MeshLib}::\mathit{punch\_hole}(\mathit{CDTMesh\&}\ \mathit{original\_mesh},\\ &\qquad\qquad \mathit{CDTMesh\&}\ \mathit{filled\_mesh},\\ &\qquad\qquad \mathit{std}::\mathit{string\&}\ \mathit{punched\_mesh\_name},\\ &\qquad\qquad \mathit{int}\ \mathit{id}) \end{aligned}$$

6. Run the executable program

Dynamic Linking Libraries

1. Construct a directory bin/;
2. Copy Delaunay.dll from 3rdparty/Delaunay/lib/windows/x64 to bin/;
3. Copy Hodge.dll from 3rdparty/Hodge/lib/windows/x64 to bin/;
4. Copy KoebeIteration.exe to bin/

Command

Go to data/boy_3_holes folder, click on koebe_iteration_test.bat.