Volumetric Harmonic Brain Mapping Using a Variational Method

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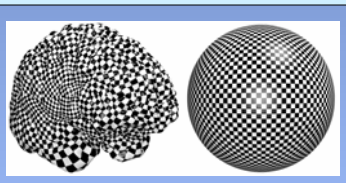
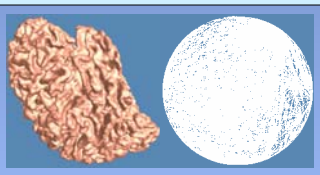
INTRODUCTION

Brain surface conformal mapping research has been successful. We extended our algorithm [1] to find a 3D volumetric harmonic map from a 3D brain volumetric model to a solid sphere. We use mesh structure to represent both surface and volumetric data. We proposed a new algorithm, sphere carving algorithm, which can build 3D tetrahedral mesh from a set of images with topology preservation. Experimental results on both synthetic data and real data are reported.

METHODS



We extend our cortical surface conformal mapping work to volumetric 3D volumetric harmonic map from a 3D brain volumetric model to a solid sphere.



Harmonic Map

- The map minimizes the stretching energy
- Geodesics are harmonic maps from a circle to the surface
- In general, harmonic maps may not exist, or unique
- Depends on the Riemannian metrics. Independent of the embeddings
- For 3-manifold, the existence of harmonic maps, the uniqueness of harmonic maps, the diffeomorphic properties of harmonic maps are extremely difficult theoretic problems
- Between convex 3-disk, harmonic map exists and is most likely diffeomorphism

Volumetric Harmonic Map Algorithm Details

- Harmonic energy

$$E(f) = < f, f > = \sum_{\{u,v\} \in K} k(u,v) \| f(u) - f(v) \|^2$$

$$k(u,v) = \frac{1}{12} \sum_{i=1}^{n} l_i \cot(\theta_i)$$

- Minimize Harmonic Energy

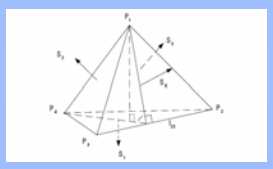
Volumetric Harmonic Map Method

- *Initialization:* We conformally map the volume surface to a sphere surface. The internal parts are mapped to the sphere center.
- *Energy Minimization:* steepest descent method with a fixed boundary condition.

Sphere Carving Algorithm

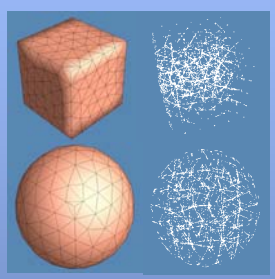
- Input (a sequence of volume images and a desired surface genus number)
- Output (a tetrahedral mesh whose surface has the desired genus number)

Build a solid handle body tetrahedral mesh consisted of tetrahedra, such that the sphere totally enclose the 3D data. Let the boundary of the solid sphere be S. We cut the model until we get the object 3D tetrahedral model.

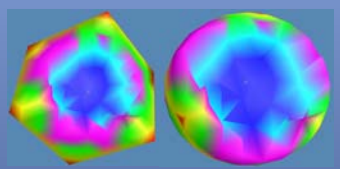


RESULTS

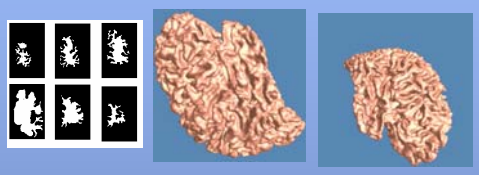
Volumetric Harmonic Map on Synthetic Data



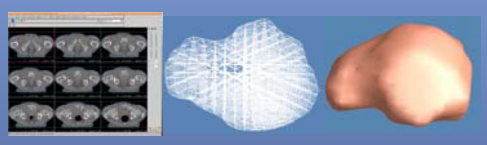
Volumetric Parameterization



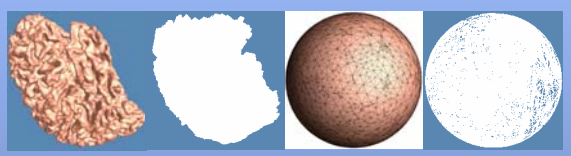
Sphere Carving Algorithm on Brain Model



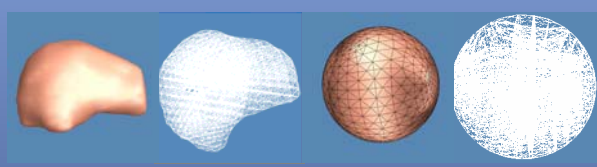
Sphere Carving Algorithm on Prostate Model



Volumetric Brain Harmonic Map



More Volumetric Harmonic Map Examples



CONCLUSION

This paper introduces a volumetric harmonic map method. First we map the volumetric boundary conformally onto a sphere. Then with this boundary condition, we compute its harmonic map in the object interior with a heat flow method. Our work is general enough to be easily generalized to higher dimensional cases,or to other organ systems than brain, e.g. for representing cardiac motion. We suggest that 3D harmonic mapping of volume can provide a canonical coordinate system for feature identification and registration for medical imaging.

References: [1] Gu X. Wang Y., Chan T.F., Thompson P.M., and Yau S.-T., **Genus zero surface conformal mapping and its application to brain surface mapping,** *Information Processing in Medical Imaging*, pages 172-184, Ambleside, UK, July 2003

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