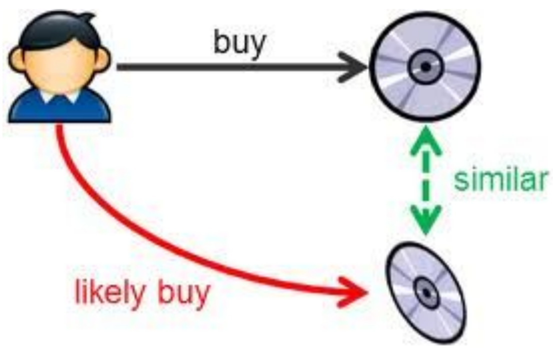
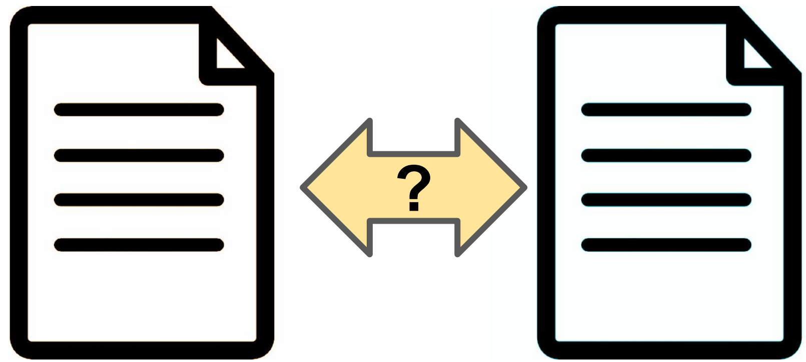
Similarity Search

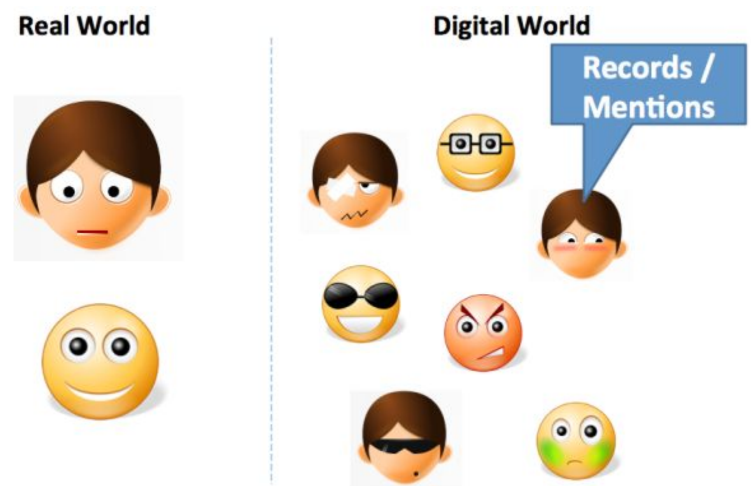
Stony Brook University
CSE545, Spring 2019

Finding Similar "Items"

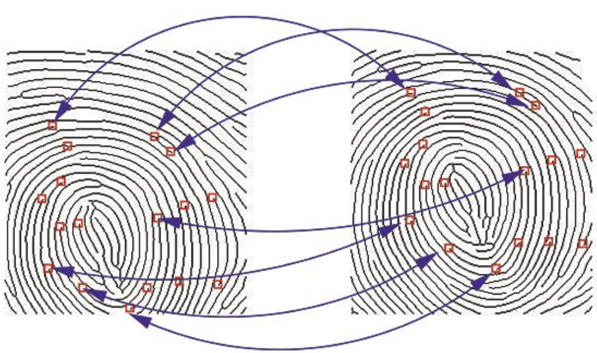


(http://blog.soton.ac.uk/hive/2012/05/10/recommendation-system-of-hive/)





(http://www.datacommunitydc.org/blog/2013/08/entity-resolution-for-big-data)



Finding Similar “Items”: What we will cover

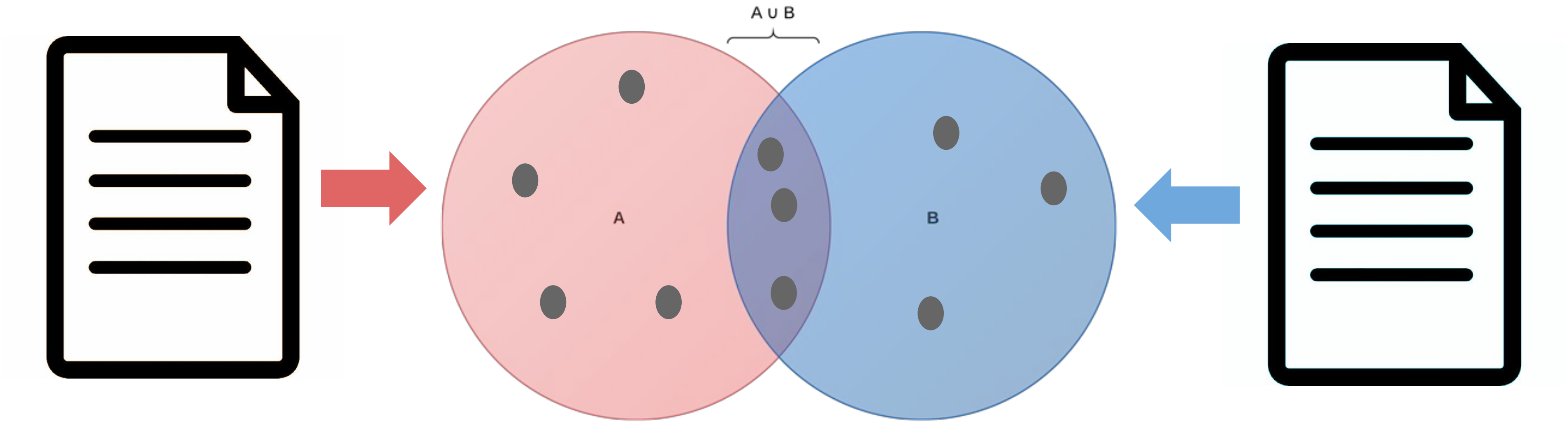
- Shingling
- Minhashing
- Locality-sensitive hashing
- Distance Metrics

Document Similarity

Challenge: How to represent the document in a way that can be efficiently encoded and compared?

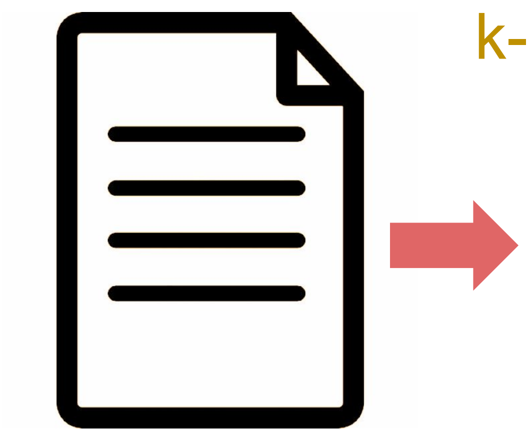
Shingles

Goal: Convert documents to sets



Shingles

Goal: Convert documents to sets

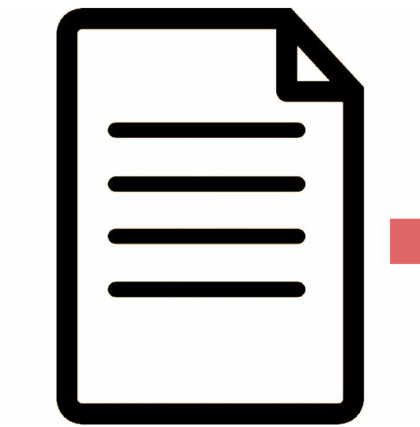


k-shingles (aka "character n-grams")
- sequence of *k* characters

E.g. k=2 doc="abcdabd"
singles(doc, 2) = {ab, bc, cd, da, bd}

Shingles

Goal: Convert documents to sets



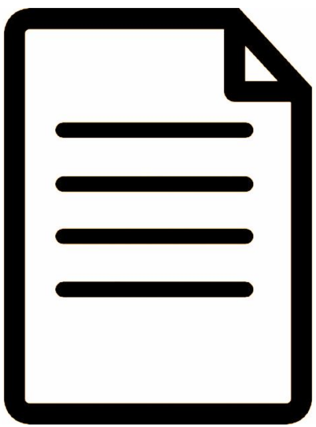
k-shingles (aka “character n-grams”)
- sequence of *k* characters

E.g. *k*=2 doc=”abcdabd”
singles(doc, 2) = {ab, bc, cd, da, bd}

- Similar documents have many common shingles
- Changing words or order has minimal effect.
- In practice use 5 < k < 10

Shingles

Goal: Convert documents to sets



Large enough that any given shingle appearing a document is highly unlikely
(e.g. < .1% chance)

Can hash large shingles to smaller
(e.g. 9-shingles into 4 bytes)

Can also use words (aka n-grams).

- Similar documents have many common shingles
- Changing words or order has minimal effect.
- **In practice use $5 < k < 10$**

Shingles

Problem: Even if hashing, sets of shingles are large
(e.g. 4 bytes => 4x the size of the document).

Minhashing

Goal: Convert sets to shorter ids, signatures

Minhashing - Background

Goal: Convert sets to shorter ids, “signatures”

Characteristic Matrix, X:

Element	S_1	S_2	S_3	S_4
a	1	0	0	1
b	0	0	1	0
c	0	1	0	1
d	1	0	1	1
e	0	0	1	0

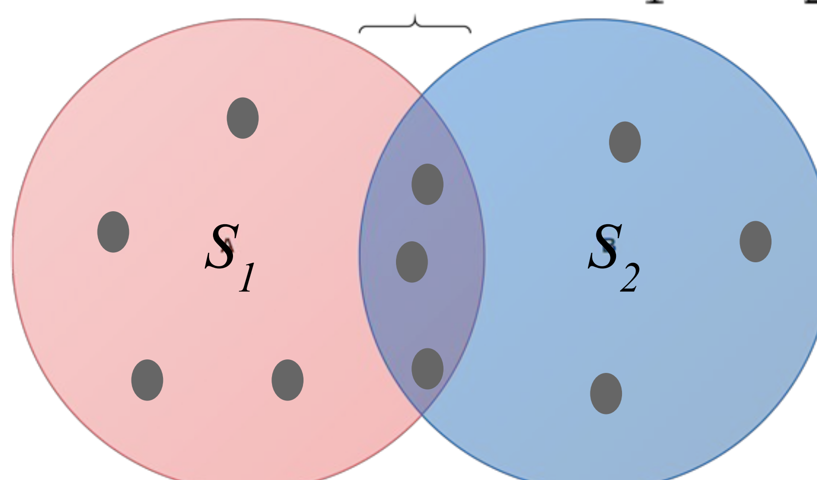
....

(Leskovec at al., 2014; http://www.mmds.org/)

often very sparse! (lots of zeros)

Jaccard Similarity:

$$sim(S_1, S_2) = \frac{S_1 \cap S_2}{S_1 \cup S_2}$$



Minhashing - Background

Characteristic Matrix:

	S_1	S_2
ab	1	1
bc	0	1
de	1	0
ah	1	1
ha	0	0
ed	1	1
ca	0	1

Jaccard Similarity:

$$sim(S_1, S_2) = \frac{S_1 \cap S_2}{S_1 \cup S_2}$$

Minhashing - Background

Characteristic Matrix:

	S_1	S_2	
ab	1	1	* *
bc	0	1	*
de	1	0	*
ah	1	1	**
ha	0	0	
ed	1	1	**
ca	0	1	*

Jaccard Similarity:

$$sim(S_1, S_2) = \frac{S_1 \cap S_2}{S_1 \cup S_2}$$

Minhashing - Background

Characteristic Matrix:

	S_1	S_2	
ab	1	1	* *
bc	0	1	*
de	1	0	*
ah	1	1	**
ha	0	0	
ed	1	1	**
ca	0	1	*

Jaccard Similarity:

$$sim(S_1, S_2) = \frac{S_1 \cap S_2}{S_1 \cup S_2}$$

$sim(S_{1,}\ S_2) = 3 / 6$

both have / # at least one has

Shingles

Problem: Even if hashing shingle contents,
sets of shingles are large

e.g. 4 byte integer per shingle: assume all unique shingles,
=> 4x the size of the document
(since there are as many shingles as characters and 1byte per char).

Minhashing

Goal: Convert sets to shorter ids, “signatures”

Characteristic Matrix: X

	S_1	S_2	S_3	S_4
ab	1	0	1	0
bc	1	0	0	1
de	0	1	0	1
ah	0	1	0	1
ha	0	1	0	1
ed	1	0	1	0
ca	1	0	1	0

Approximate Approach:

1) Instead of keeping whole characteristic matrix, just keep first row where 1 is encountered.

2) Shuffle and repeat to get a “signature” for each set.

(Leskovec at al., 2014; http://www.mmds.org/)

Minhashing

Goal: Convert sets to shorter ids, “signatures”

Characteristic Matrix: X

	1	3	1	2
	S_1	S_2	S_3	S_4
ab	1	0	1	0
bc	1	0	0	1
de	0	1	0	1
ah	0	1	0	1
ha	0	1	0	1
ed	1	0	1	0
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(Leskovec at al., 2014; http://www.mmds.org/)

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Minhashing

Goal: Convert sets to shorter ids, "signatures"

Characteristic Matrix: X

	S_1	S_2	S_3	S_4
	1	3	1	2
ab	1	0	1	0
bc	1	0	0	1
de	0	1	0	1
ah	0	1	0	1
ha	0	1	0	1
ed	1	0	1	0
ca	1	0	1	0

(Leskovec at al., 2014; http://www.mmds.org/)

Approximate Approach:

1) Instead of keeping whole characteristic matrix, just keep first row where 1 is encountered.

2) Shuffle and repeat to get a "signature".

	S_1	S_2	S_3	S_4
	2	1	2	1
ah	0	1	0	1
ca	1	0	1	0
ed	1	0	1	0
de	0	1	0	1
ab	1	0	1	0
bc	1	0	0	1

...

Minhashing

Goal: Convert sets to shorter ids, "signatures"

Characteristic Matrix: X

	S_1 (1)	S_2 (3)	S_3 (1)	S_4 (2)
ab	1	0	1	0
bc	1	0	0	1
de	0	1	0	1
ah	0	1	0	1
ha	0	1	0	1
ed	1	0	1	0
ca	1	0	1	0

(Leskovec at al., 2014; http://www.mmds.org/)

Approximate Approach:

1) Instead of keeping whole characteristic matrix, just keep first row where 1 is encountered.

2) Shuffle and repeat to get a "signature".

	S_1 (2)	S_2 (1)	S_3 (2)	S_4 (1)
ah	0	1	0	1
ca	1	0	1	0
ed	1	0	1	0
de	0	1	0	1
ab	1	0	1	0
bc	1	0	0	1

...

signatures

S_1	S_2	S_3	S_4
1	3	1	2
2	1	2	1
...	...	...	...

Minhashing

Goal: Convert sets to shorter ids, "signatures"

Characteristic Matrix: X

	S_1	S_2	S_3	S_4
ab	1	0	1	0
bc	1	0	0	1
de	0	1	0	1
ah	0	1	0	1
ha	0	1	0	1
ed	1	0	1	0
ca	1	0	1	0

(Leskovec at al., 2014; http://www.mmds.org/)

Approximate Approach:

1) Instead of keeping whole characteristic matrix, just keep first row where 1 is encountered.

2) Shuffle and repeat to get a "signature" for each set.

Idea: We don't need to actually shuffle we can just use hash functions.

Minhashing

Characteristic Matrix:

	S_1	S_2	S_3	S_4
ab	1	0	1	0
bc	1	0	0	1
de	0	1	0	1
ah	0	1	0	1
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ed	1	0	1	0
ca	1	0	1	0

(Leskovec at al., 2014; http://www.mmds.org/)

Minhash function: h

- Based on permutation of rows in the characteristic matrix, h maps sets to first row where set appears.

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	S_1	S_2	S_3	S_4
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ah	0	1	0	1
ha	0	1	0	1
ed	1	0	1	0
ca	1	0	1	0

permuted order
1 ha
2 ed
3 ab
4 bc
5 ca
6 ah
7 de

(Leskovec at al., 2014; http://www.mmds.org/)

Minhashing

Minhash function: h

- Based on permutation of rows in the characteristic matrix, h maps sets to first row where set appears.

Characteristic Matrix:

		S_1	S_2	S_3	S_4
3	ab	1	0	1	0
4	bc	1	0	0	1
7	de	0	1	0	1
6	ah	0	1	0	1
1	ha	0	1	0	1
2	ed	1	0	1	0
5	ca	1	0	1	0

permuted order
1 ha
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Characteristic Matrix:

		S_1	S_2	S_3	S_4
3	ab	1	0	1	0
4	bc	1	0	0	1
7	de	0	1	0	1
6	ah	0	1	0	1
1	ha	0	1	0	1
2	ed	1	0	1	0
5	ca	1	0	1	0

permuted order
1 ha
2 ed
3 ab
4 bc
5 ca
6 ah
7 de

$h(S_1)$ = ed #permuted row 2
$h(S_2)$ = ha #permuted row 1
$h(S_3)$ =

(Leskovec at al., 2014; http://www.mmds.org/)

Minhashing

Minhash function: h

- Based on permutation of rows in the characteristic matrix, h maps sets to first row where set appears.

Characteristic Matrix:

		S_1	S_2	S_3	S_4
3	ab	1	0	1	0
4	bc	1	0	0	1
7	de	0	1	0	1
6	ah	0	1	0	1
1	ha	0	1	0	1
2	ed	1	0	1	0
5	ca	1	0	1	0

permuted order
1 ha
2 ed
3 ab
4 bc
5 ca
6 ah
7 de

$h(S_1)$ = ed #permuted row 2
$h(S_2)$ = ha #permuted row 1
$h(S_3)$ = ed #permuted row 2
$h(S_4)$ =

(Leskovec at al., 2014; http://www.mmds.org/)

Minhashing

Minhash function: h

- Based on permutation of rows in the characteristic matrix, h maps sets to first row where set appears.

Characteristic Matrix:

		S_1	S_2	S_3	S_4
3	ab	1	0	1	0
4	bc	1	0	0	1
7	de	0	1	0	1
6	ah	0	1	0	1
1	ha	0	1	0	1
2	ed	1	0	1	0
5	ca	1	0	1	0

permuted order
1 ha
2 ed
3 ab
4 bc
5 ca
6 ah
7 de

$h(S_1)$ = ed #permuted row 2
$h(S_2)$ = ha #permuted row 1
$h(S_3)$ = ed #permuted row 2
$h(S_4)$ = ha #permuted row 1

(Leskovec at al., 2014; http://www.mmds.org/)

Minhashing

Characteristic Matrix:

		S_1	S_2	S_3	S_4
3	ab	1	0	1	0
4	bc	1	0	0	1
7	de	0	1	0	1
6	ah	0	1	0	1
1	ha	0	1	0	1
2	ed	1	0	1	0
5	ca	1	0	1	0

(Leskovec at al., 2014; http://www.mmds.org/)

Minhash function: h

- Based on permutation of rows in the characteristic matrix, h maps sets to rows.

Signature matrix: *M*

- Record first row where each set had a 1 in the given permutation

	S_1	S_2	S_3	S_4
h_1	2	1	2	1

$h_1(S_1)$ = ed #permuted row 2
$h_1(S_2)$ = ha #permuted row 1
$h_1(S_3)$ = ed #permuted row 2
$h_1(S_4)$ = ha #permuted row 1

Minhashing

Characteristic Matrix:

		S_1	S_2	S_3	S_4
3	ab	1	0	1	0
4	bc	1	0	0	1
7	de	0	1	0	1
6	ah	0	1	0	1
1	ha	0	1	0	1
2	ed	1	0	1	0
5	ca	1	0	1	0

(Leskovec at al., 2014; http://www.mmds.org/)

Minhash function: h

- Based on permutation of rows in the characteristic matrix, h maps sets to rows.

Signature matrix: M

- Record first row where each set had a 1 in the given permutation

	S_1	S_2	S_3	S_4
h_1	2	1	2	1

2 $h_1(S_1)$ = ed #permuted row

1 $h_1(S_2)$ = ha #permuted row

$h_1(S_3)$ = ed #permuted row

Minhashing

Characteristic Matrix:

		S_1	S_2	S_3	S_4
3	ab	1	0	1	0
4	bc	1	0	0	1
7	de	0	1	0	1
6	ah	0	1	0	1
1	ha	0	1	0	1
2	ed	1	0	1	0
5	ca	1	0	1	0

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	S_1	S_2	S_3	S_4
h_1	2	1	2	1

2 $h_1(S_1)$ = ed #permuted row

$h_1(S_2)$ = ha #permuted row

1

$h_1(S_3)$ = ed #permuted row

Minhashing

Characteristic Matrix:

			S_1	S_2	S_3	S_4
4	3	ab	1	0	1	0
2	4	bc	1	0	0	1
1	7	de	0	1	0	1
3	6	ah	0	1	0	1
6	1	ha	0	1	0	1
7	2	ed	1	0	1	0
5	5	ca	1	0	1	0

Minhash function: h

- Based on permutation of rows in the characteristic matrix, h maps sets to rows.

Signature matrix: M

- Record first row where each set had a 1 in the given permutation

	S_1	S_2	S_3	S_4
h_1	2	1	2	1
h_2				

(Leskovec at al., 2014; http://www.mmds.org/)

Minhashing

Minhash function: h

- Based on permutation of rows in the characteristic matrix, h maps sets to rows.

Characteristic Matrix:

			S_1	S_2	S_3	S_4
4	3	ab	1	0	1	0
2	4	bc	1	0	0	1
1	7	de	0	1	0	1
3	6	ah	0	1	0	1
6	1	ha	0	1	0	1
7	2	ed	1	0	1	0
5	5	ca	1	0	1	0

Signature matrix: M

- Record first row where each set had a 1 in the given permutation

	S_1	S_2	S_3	S_4
h_1	2	1	2	1
h_2	2	1	4	1

(Leskovec at al., 2014; http://www.mmds.org/)

Minhashing

Characteristic Matrix:

				S_1	S_2	S_3	S_4
1	4	3	ab	1	0	1	0
3	2	4	bc	1	0	0	1
7	1	7	de	0	1	0	1
6	3	6	ah	0	1	0	1
2	6	1	ha	0	1	0	1
5	7	2	ed	1	0	1	0
4	5	5	ca	1	0	1	0

Minhash function: h

- Based on permutation of rows in the characteristic matrix, h maps sets to rows.

Signature matrix: *M*

- Record first row where each set had a 1 in the given permutation

	S_1	S_2	S_3	S_4
h_1	2	1	2	1
h_2	2	1	4	1
h_3				

(Leskovec at al., 2014; http://www.mmds.org/)

Minhashing

Characteristic Matrix:

				S_1	S_2	S_3	S_4
1	4	3	ab	1	0	1	0
3	2	4	bc	1	0	0	1
7	1	7	de	0	1	0	1
6	3	6	ah	0	1	0	1
2	6	1	ha	0	1	0	1
5	7	2	ed	1	0	1	0
4	5	5	ca	1	0	1	0

Minhash function: *h*

- Based on permutation of rows in the characteristic matrix, *h* maps sets to rows.

Signature matrix: *M*

- Record first row where each set had a 1 in the given permutation

	S_1	S_2	S_3	S_4
h_1	2	1	2	1
h_2	2	1	4	1
h_3	1	2	1	2

(Leskovec at al., 2014; http://www.mmds.org/)

Minhashing

Characteristic Matrix:

				S_1	S_2	S_3	S_4
1	4	3	ab	1	0	1	0
3	2	4	bc	1	0	0	1
7	1	7	de	0	1	0	1
6	3	6	ah	0	1	0	1
2	6	1	ha	0	1	0	1
5	7	2	ed	1	0	1	0
4	5	5	ca	1	0	1	0

(Leskovec at al., 2014; http://www.mmds.org/)

Minhash function: h

- Based on permutation of rows in the characteristic matrix, h maps sets to rows.

Signature matrix: *M*

- Record first row where each set had a 1 in the given permutation

	S_1	S_2	S_3	S_4
h_1	2	1	2	1
h_2	2	1	4	1
h_3	1	2	1	2
...				
...				

Minhashing

Property of signature matrix:
The probability for any h_i (i.e. any row), that $h_i(S_1) = h_i(S_2)$ is the same as Sim(S_1, S_2)

Characteristic Matrix:

				S_1	S_2	S_3	S_4
1	4	3	ab	1	0	1	0
3	2	4	bc	1	0	0	1
7	1	7	de	0	1	0	1
6	3	6	ah	0	1	0	1
2	6	1	ha	0	1	0	1
5	7	2	ed	1	0	1	0
4	5	5	ca	1	0	1	0

	S_1	S_2	S_3	S_4
h_1	2	1	2	1
h_2	2	1	4	1
h_3	1	2	1	2
...				
...				

(Leskovec at al., 2014; http://www.mmds.org/)

Minhashing

Characteristic Matrix:

Property of signature matrix:
The probability for any h_i (i.e. any row), that $h_i(S_1) = h_i(S_2)$ is the same as Sim(S_1, S_2)

Thus, similarity of signatures S_1, S_2 is the fraction of minhash functions (i.e. rows) in which they agree.

				S_1	S_2	S_3	S_4
1	4	3	ab	1	0	1	0
3	2	4	bc	1	0	0	1
7	1	7	de	0	1	0	1
6	3	6	ah	0	1	0	1
2	6	1	ha	0	1	0	1
5	7	2	ed	1	0	1	0
4	5	5	ca	1	0	1	0

	S_1	S_2	S_3	S_4
h_1	2	1	2	1
h_2	2	1	4	1
h_3	1	2	1	2
...				
...				

(Leskovec at al., 2014; http://www.mmds.org/)

Minhashing

Property of signature matrix:
The probability for any h_i (i.e. any row), that $h_i(S_1) = h_i(S_2)$ is the same as Sim(S_1, S_2)

Thus, similarity of signatures S_1, S_2 is the fraction of minhash functions (i.e. rows) in which they agree.

Characteristic Matrix:

				S_1	S_2		S_4
1	4	3	ab	1	0		0
3	2						
7							
6	3	6					
2	6	1	ha	0	1	0	1
5	7	2	ed	1	0	1	0
4	5	5	ca	1	0	1	0

Estimate with a random sample of permutations (i.e. ~100)

	S_1	S_2	S_3	S_4
h_1	2	1	2	1
h_2	2	1	4	1
h_3	1	2	1	2
…				
…				

(Leskovec at al., 2014; http://www.mmds.org/)

Minhashing

Property of signature matrix:
The probability for any h_i (i.e. any row), that $h_i(S_1) = h_i(S_2)$ is the same as Sim(S_1, S_2)

Thus, similarity of signatures S_1, S_2 is the fraction of minhash functions (i.e. rows) in which they agree.

Characteristic Matrix:

				S_1	S_2	S_3	S_4
1	4	3	ab	1	0	[illegible]	0
3	2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
6	3	6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
2	6	1	ha	0	1	0	1
5	7	2	ed	1	0	1	0
4	5	5	ca	1	0	1	0

Estimate with a random sample of permutations (i.e. ~100)

	S_1	S_2	S_3	S_4
h_1	2	1	2	1
h_2	2	1	4	1
h_3	1	2	1	2

Estimated Sim(S_1, S_3) = agree / all = 2/3

(Leskovec at al., 2014; http://www.mmds.org/)

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Solution: Use “random” hash functions.

- Setup:
  - Pick ~100 hash functions, hashes
  - Store M[i][s] = a potential minimum $h_i(r)$ *#initialized to infinity (num hashs x num sets)*

Minhashing

Solution: Use "random" hash functions.

Setup:

hashes = [func(i) for i in rand(1, num=100)] *#100 hash functions, seeded random*

for i in hashes: for s in sets:

M[i][s] = np.inf *#represents a potential minimum $h_i(r)$; initially infinity*

Algorithm ("efficient minhashing"):

```
for r in rows of cm: #cm is characteristic matrix
 compute h_i(r) for all i in hashes #precompute 100 values
 for each set s in sets:
   if cm[r][s] == 1:
     for i in hashes: #check which hash produces smallest value
       if h_i(r) < M[i][s]: M[i][s] = h_i(r)
```

Minhashing

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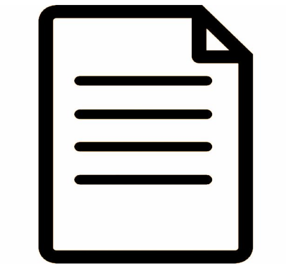
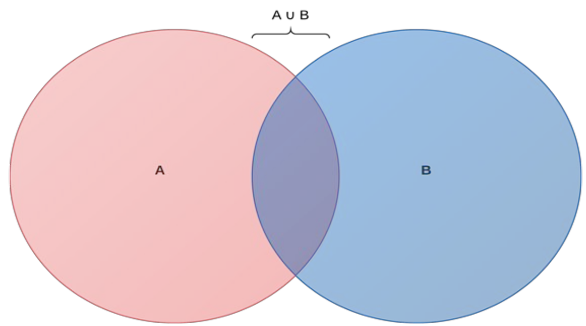
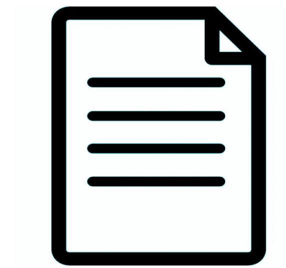
(1m documents isn't even "big data")

Document Similarity







Duplicate web pages (useful for ranking

Plagiarism

Cluster News Articles

Anything similar to documents: movie/music/art tastes, product characteristics

Locality-Sensitive Hashing

Goal: find pairs of minhashes ***likely*** to be similar (in order to then test more precisely for similarity).

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If we wanted the similarity for all pairs of documents, could anything be done?

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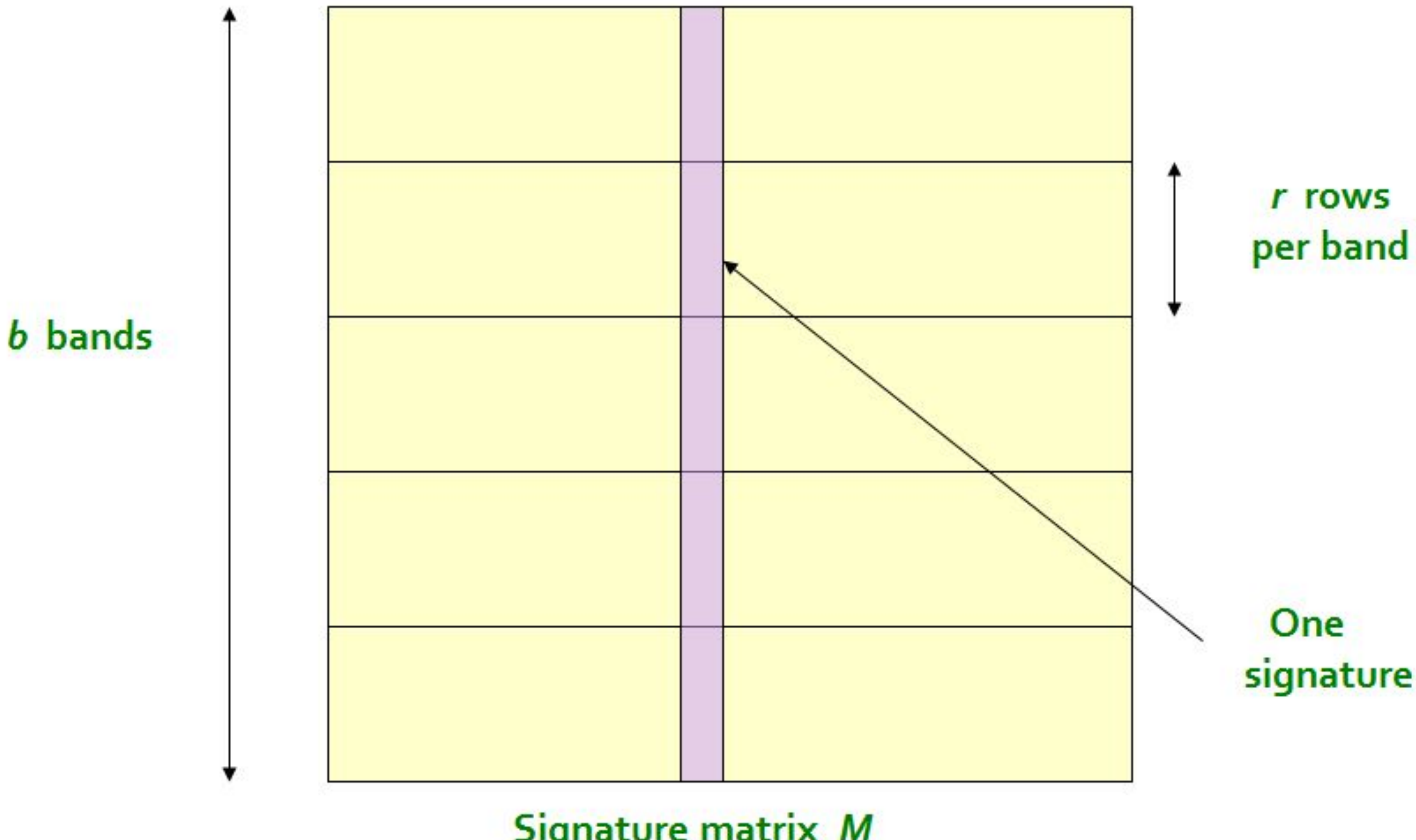
Approach from MinHash: Hash columns of signature matrix

⇒ Candidate pairs end up in the same bucket.

Locality-Sensitive Hashing

Step 1: Divide signature matrix into *b* bands



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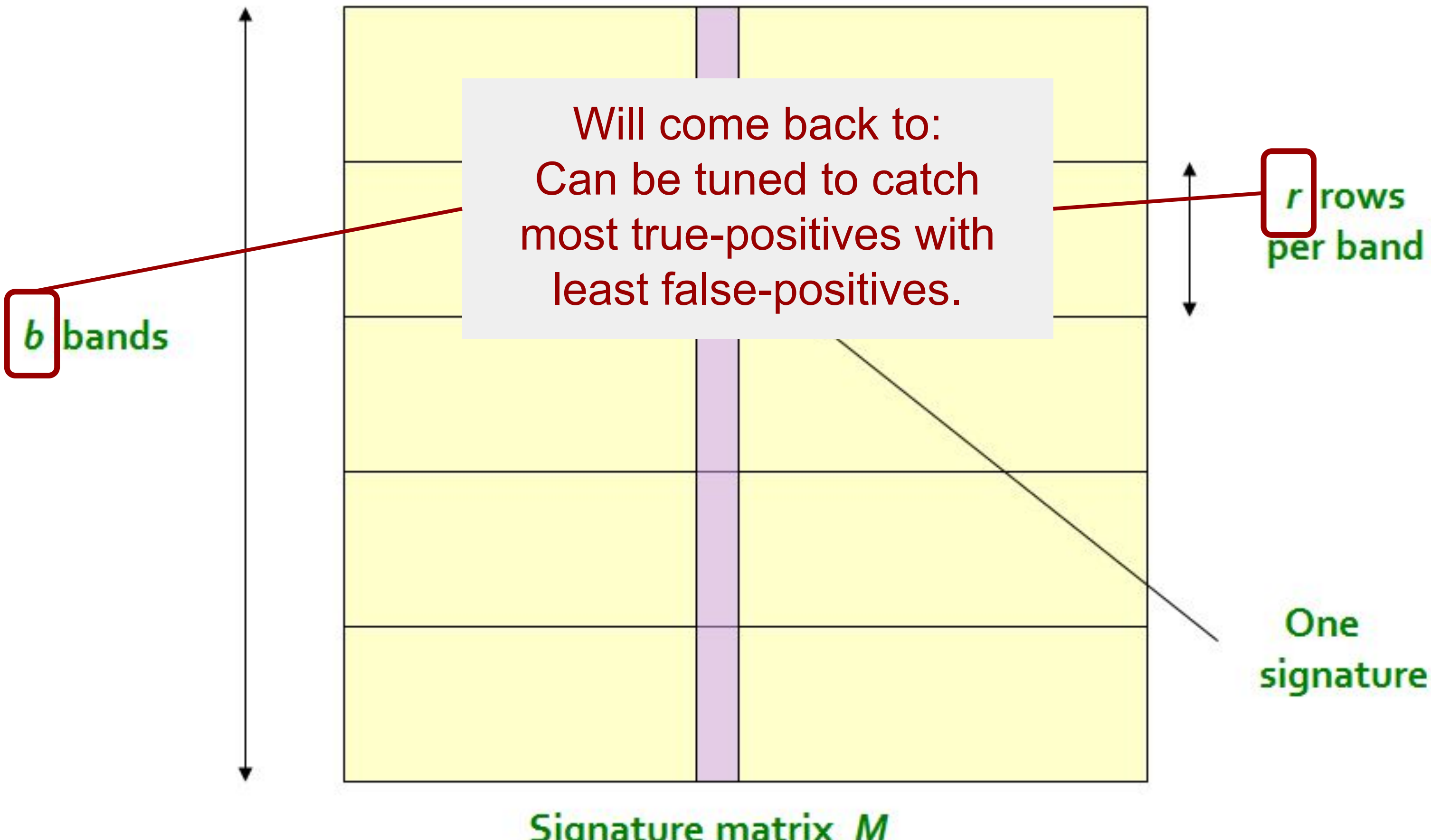

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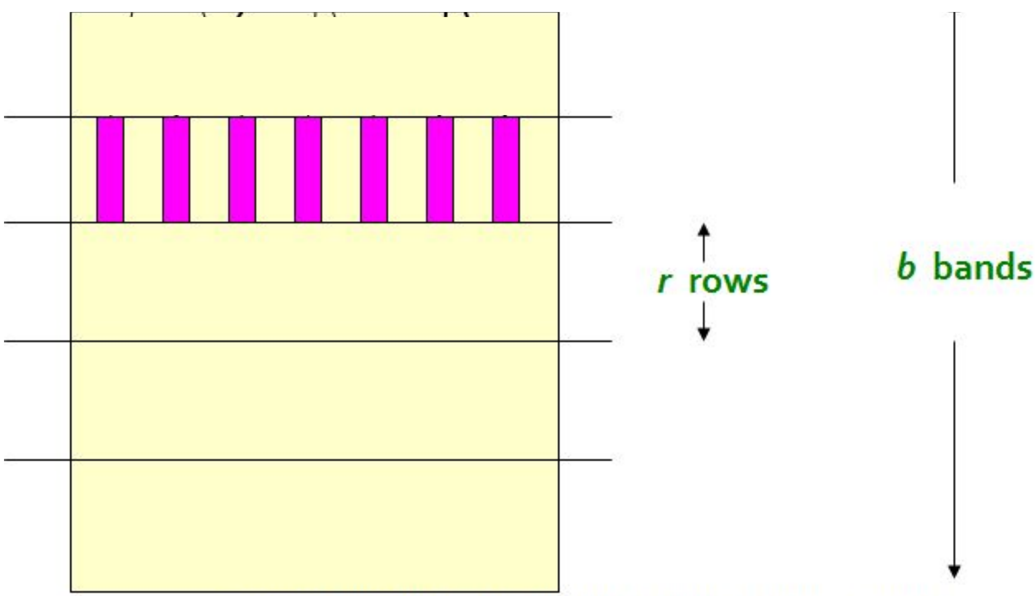

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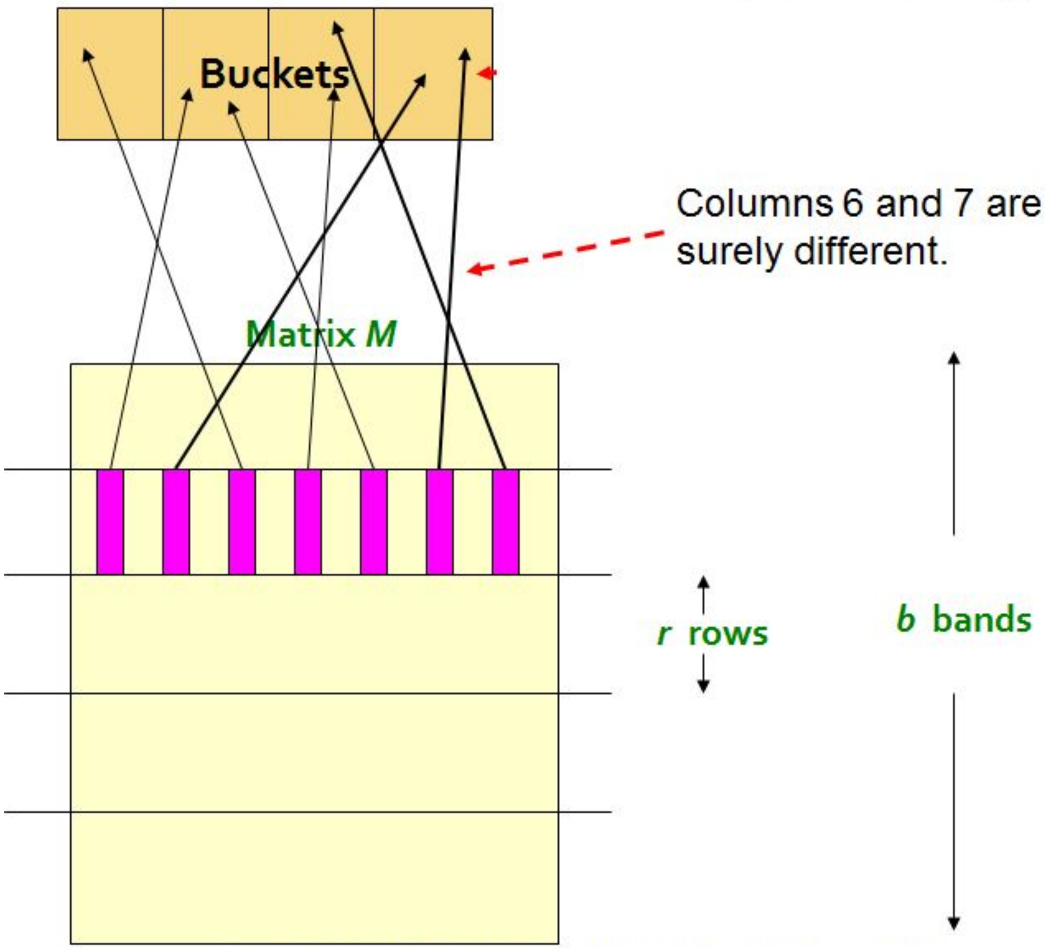

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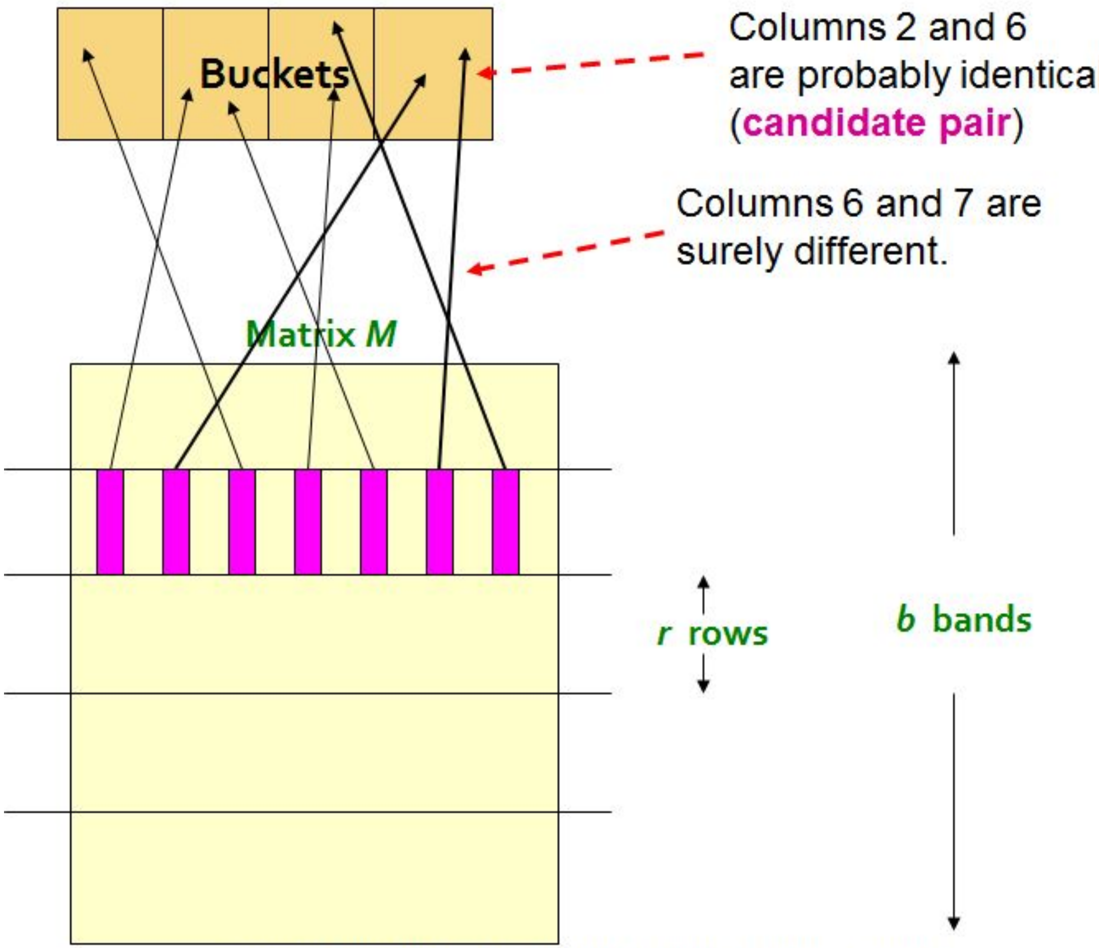

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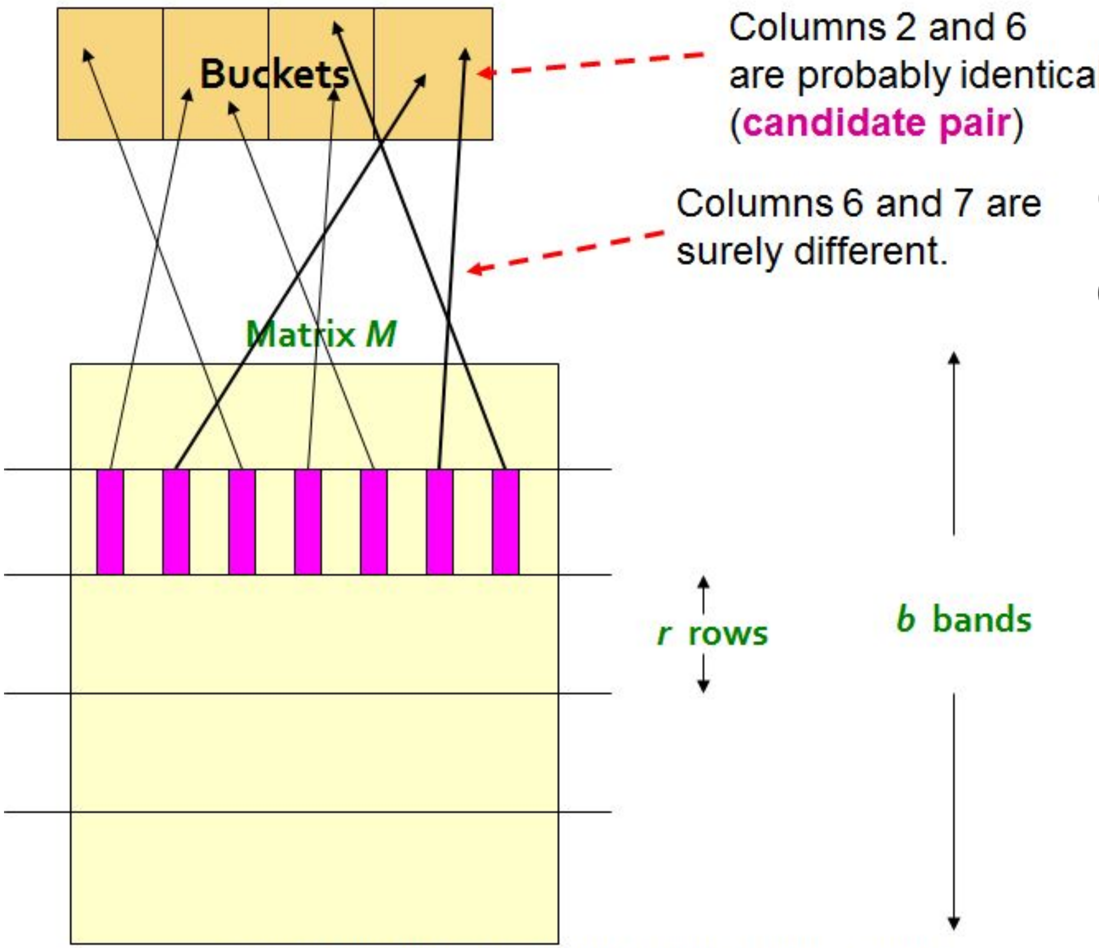

Criteria for being candidate pair:

- They end up in same bucket for at least 1 band.

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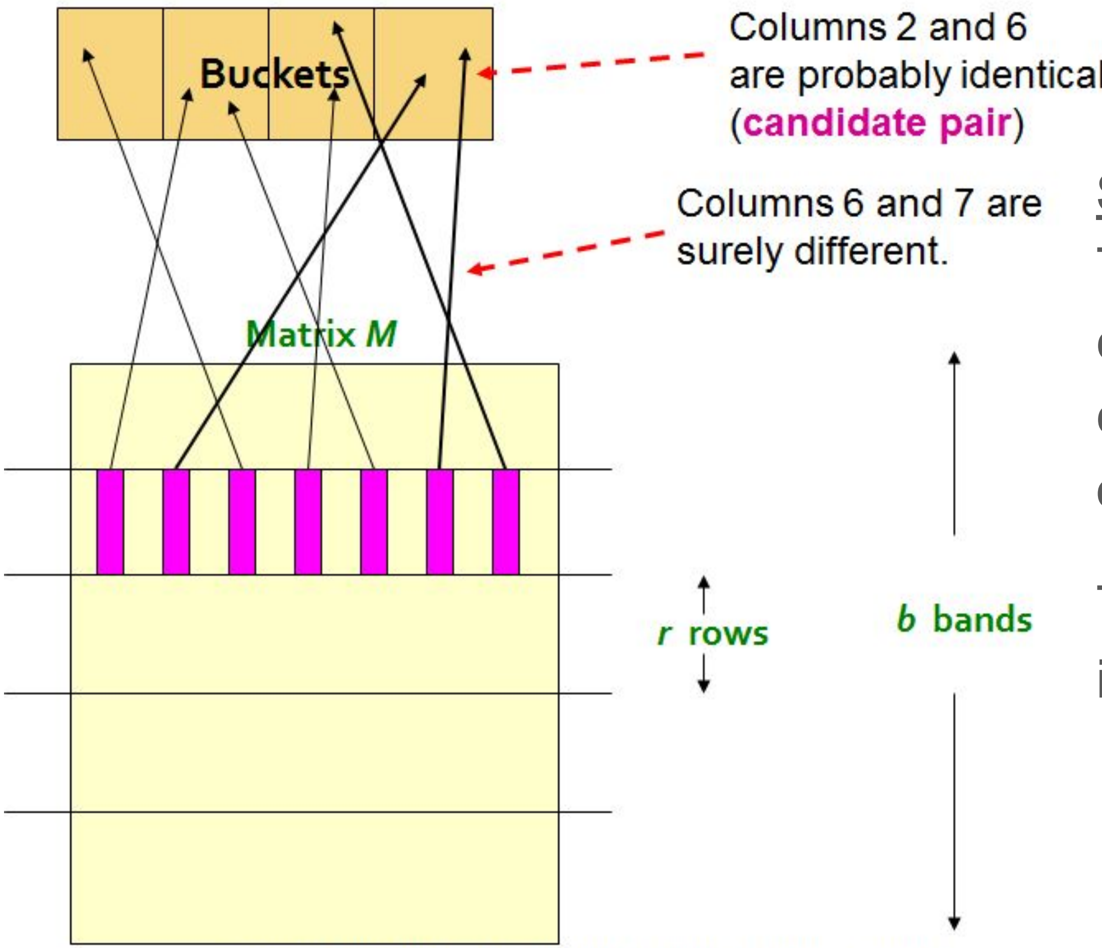
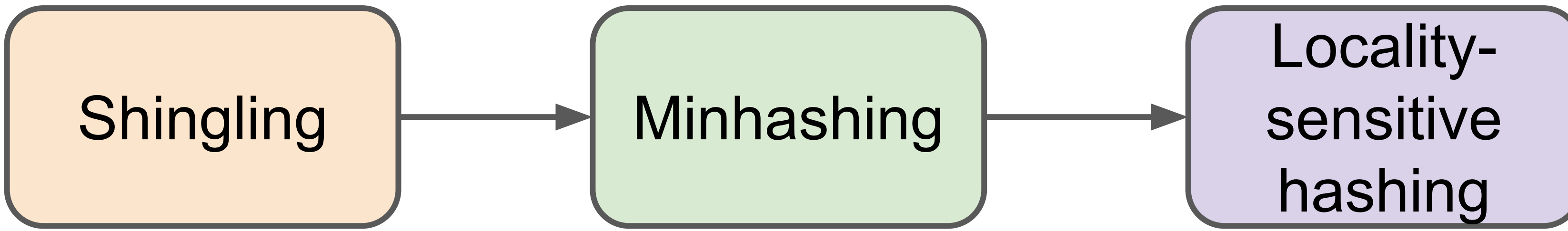


Simplification:
There are enough buckets compared to rows per band that columns must be identical in order to hash into same bucket.

Thus, we only need to check if identical within a band.

(Leskovec at al., 2014; http://www.mmds.org/)

Document Similarity Pipeline



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- 100,000 documents
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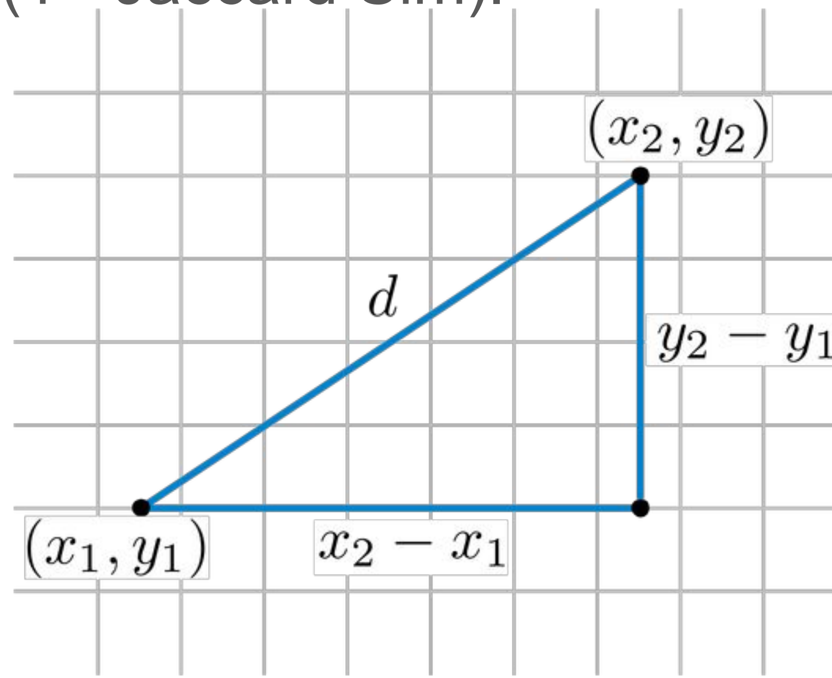
What if wanting 40% Jaccard Similarity?

Distance Metrics

Pipeline gives us a way to find *near-neighbors* in *high-dimensional space* based on Jaccard Distance (1 - Jaccard Sim).



(http://rosalind.info/glossary/euclidean-distance/)

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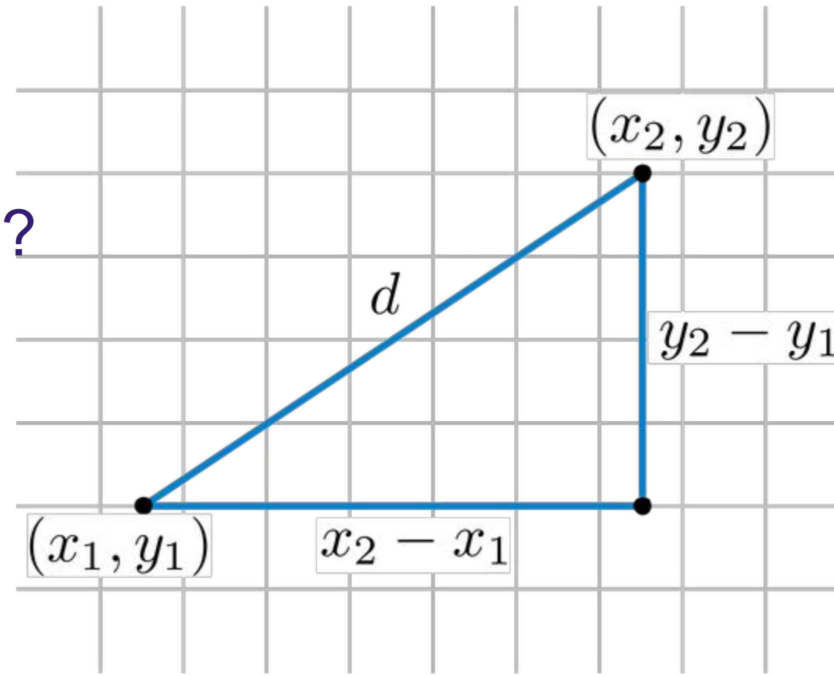
Typical properties of a distance metric, *d(point1,point2)*?



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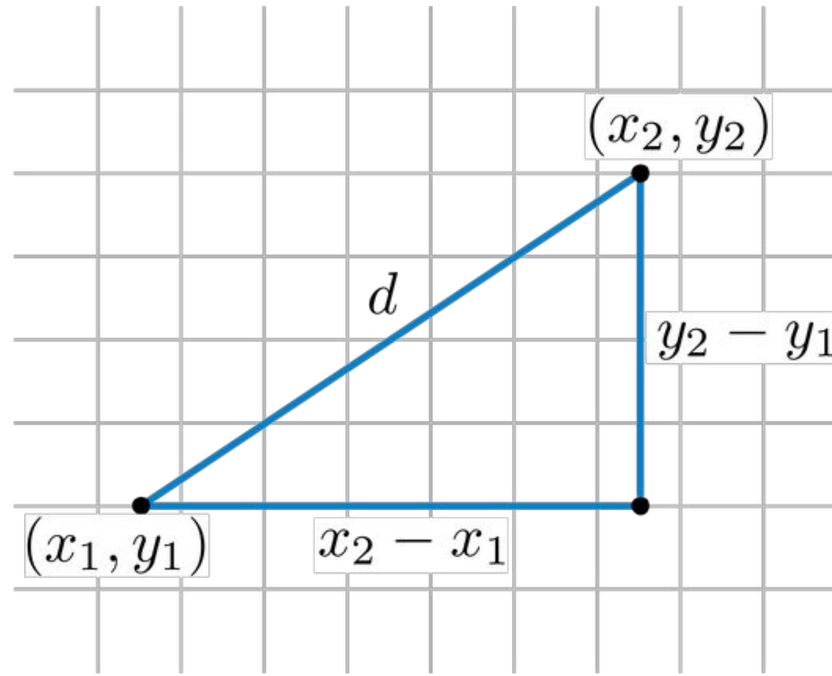
Typical properties of a distance metric, *d*:

d(a, a) = 0

d(a, b) = d(b, a)

d(a, b) ≤ d(a,c) + d(c,b)



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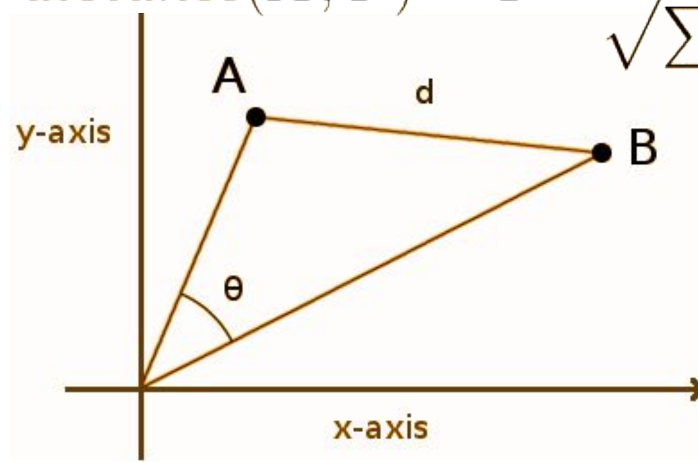

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E.g. for euclidean distance:

- Choose random lines (analogous to hash functions in minhashing)
- Project the two points onto each line; match if two points within an interval

Side Note on Generating Hash Functions:

What hash functions to use?

Start with 2 decent hash functions

e.g. $h_a(x)$ = ascii(string) % large_prime_number
$h_b(x) = (3*$ascii(string) + $16)$ % large_prime_number

Add together multiplying the second times i:

$h_i(x) = h_a(x) + i*h_b(x)$ % |BUCKETS|
e.g. $h_5(x) = h_a(x) + 5*h_b(x)$ % 100

https://www.eecs.harvard.edu/~michaelm/postscripts/rsa2008.pdf

Popular choices: md5 (fast, predistable); mmh3 (easy to seed; fast)