# Fast Spectrum Allocation in Coordinated Dynamic Spectrum Access Based Cellular Networks

Anand Prabhu Subramanian, Himanshu Gupta, Samir R. Das
State University of New York at Stony Brook
{anandps,hgupta,samir}@cs.sunysb.edu

Milind M. Buddhikot
Alcatel-Lucent Bell Labs
mbuddhikot@alcatel-lucent.com

***Abstract*—Existing capacity constrained cellular networks that operate in fixed spectrum bands can be enhanced with capacity-on-demand services using the Coordinated Dynamic Spectrum Access (CDSA) model. In this model, a centralized spectrum broker coordinates access to spectrum in a given region and assigns short term spectrum leases to competing wireless service providers and/or end users. In contrast to existing multi-year cellular spectrum licenses that span large regions, a spectrum broker can grant spectrum leases that are for small regions (e.g.: per base station) and valid for short durations (e.g.: tens of minutes). Fast spectrum allocation algorithms are crucial to the design of scalable spectrum brokers that can provide such real-time spectrum access.**

**In this paper, we address this challenge. Specifically, we formulate the spectrum allocation problem as two optimization problems: first with the objective of maximizing the overall demand (*Max-Demand*) satisfied among the various base stations and the second with the objective of minimizing the overall interference in the network (*Min-Interference*) when all the demands of the base stations are satisfied. We show that the optimization problems are NP-hard and design efficient algorithms to solve them. Our simulation results on sample network topologies show that our algorithms scale very well for large network sizes.**

## I. Introduction

The existing command-and-control spectrum management that relies on static allocation of spectrum and rigid specification of spectrum usage parameters (such as technology, power, geographical scope etc.) has rendered spectrum *access limited* rather than throughput limited [1]. Recent trends in radio front end, antenna and signal processing technologies show feasibility of replacing this antiquated spectrum management with a more dynamic one wherein networks and end user devices can access varying amounts of spectrum on a spatio-temporal scale.

The most sophisticated form of dynamic spectrum access (DSA), such as the one explored in the DARPA XG program [2], envisions individual (network or end user) nodes operating over very wide radio bands (e.g.: usable 0-3 GHz). Here, each node performs rapid spectrum sensing to detect spectrum holes and distributed coordination to opportunistically use them. Even though such operation may yield the best spectrum access and utilization, the complexity of sensing and coordination may be prohibitively large. The efforts to understand and address this complexity, often captured by an increasingly popular term of "cognitive radio", have spawned extensive new research. However, this most general form of DSA may be suitable for ad hoc on-demand networks but is unsuitable for infrastructure based networks, most prominent of which are the commercial cellular networks used by millions of end users worldwide.

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Measurement studies have shown that the cellular spectrum (e.g.: In U.S.A., 824-849 MHz, 869-894 MHz, 1.850-1.910 GHz, 1.930-1.990 GHz) is highly utilized but the spectrum utilization varies dramatically over space and time [3–5]. For example, metro areas with dense populations experience significant utilization in peak periods but the utilization during off-peak periods and in sparse areas is often low. As the high bandwidth wireless data applications are widely adopted, cellular networks will continue to evolve to higher access speeds and therefore, will require larger amount of spectrum. However, releasing more spectrum using current long-term command-and-control model of spectrum licensing is a flawed approach (Section IV.A, IV.B and IV.C in FCC Spectrum Policy Task Force Report [1]). First, existing licenses cover large spatial region and provide exclusive multi-year rights of usage to a single service provider. This often leads to a capital intensive process – a "big player syndrome" where only large providers can license spectrum as evidenced in multi-billion dollar auctions worldwide for 3G spectrum. Also, it precludes fine grained secondary usage of spectrum [1], often results in underutilization of spectrum for protracted periods of time in areas of low demand [4, 5] and leads to slow network and service deployments [6].

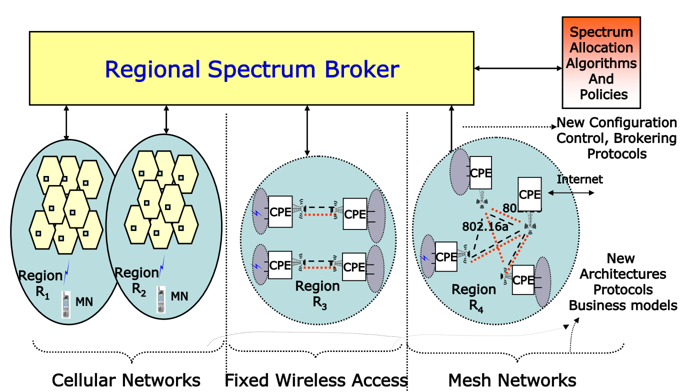



Fig. 1. Coordinated DSA model.

Buddhikot et al. proposed a concept of Coordinated Dynamic Spectrum Access (CDSA) for cellular networks to alleviate these limitations and enable capacity-on-demand services [5, 7, 8]. In their CDSA model (Figure 1), a centralized spectrum broker coordinates access to spectrum in a given region and assigns short term spectrum leases to competing radio infrastructure providers. In contrast to existing cellular spectrum licenses, the spectrum broker can grant spectrum leases that are for small geographical regions (e.g.: per base station) and valid for short durations (e.g.: tens of minutes) [8]. Such a spectrum lease gives the lessee exclusive rights to use the spectrum in the designated region for the duration of the lease without exceeding the maximum power limit. The technology and purpose for which the spectrum is used is not specified as part of the lease and the lease (token) holder can choose the technology and service that it deems fit for its use. However, the broker may require that the spectrum demand request specify the technology that may be used if the request is granted to ensure feasibility of spectrum allocation.

Variants of the CDSA model have been explored in projects such as E2R project [9] and OverDrive Project [10, 11]. Centralized coordination, aggregation of spectrum demands and restricting only network nodes (instead of end-user devices) to participate in spectrum access, make the CDSA model practically realizable. However, the design of scalable, large spectrum brokers is necessary for successful realization of this model. One of the main challenges in building such brokers is the design of fast spectrum allocation algorithms. In this paper, we address this challenge.

### *A.* Research Contributions

In this work, we formulate the spectrum allocation problem as two optimization problems: first with the objective of maximizing the overall spectrum demands (*Max-Demand DSA*) satisfied among various base stations such that no two interfering base stations that belong to different radio infrastructure providers are assigned the same channels and the second with the objective of minimizing the overall interference (*Min-Interference DSA*) in the network when all the demands of the base stations are satisfied. We show that both the optimization problems are NP-Hard[1] and design efficient algorithms to solve them.

We propose a graph construct called *interference graph* that captures conflict relationships between transmitters (base stations) of various radio infrastructure providers that co-exist in a region. We develop a constant factor approximation algorithm for the *Max-Demand DSA* problem, when the interference graph of the network is modeled as a $\delta$-degree bounded graph and the minimum demand of each node is 0. We also design an algorithm with a constant factor approximation that depends on the number of channels available for the *Min-Interference DSA* problem. We report simulation results on sample network topologies to show that our algorithms scale very well for large network sizes.

[1]In simple terms, no algorithm exists for NP-hard problems that can find an optimal solution with a running time as a polynomial function of the input size $n$. As such, running time of any algorithm to find an optimal solution for NP-Hard problems increases exponentially with the problem size.

### *B.* Outline

The rest of the paper is organized as follows: Section II describes the reference system architecture for which the problem of spectrum allocation needs to be solved. In Section III, we present our network model and formulate our spectrum allocation problem as two optimization problems. In Section IV, we design efficient algorithms for the spectrum allocation problem. In Section V, we provide detailed performance evaluation of our algorithms. We discuss related work in Section VI. Section VII concludes the paper and outlines our future work.

## II. Reference Architecture for the Spectrum Allocation Problem

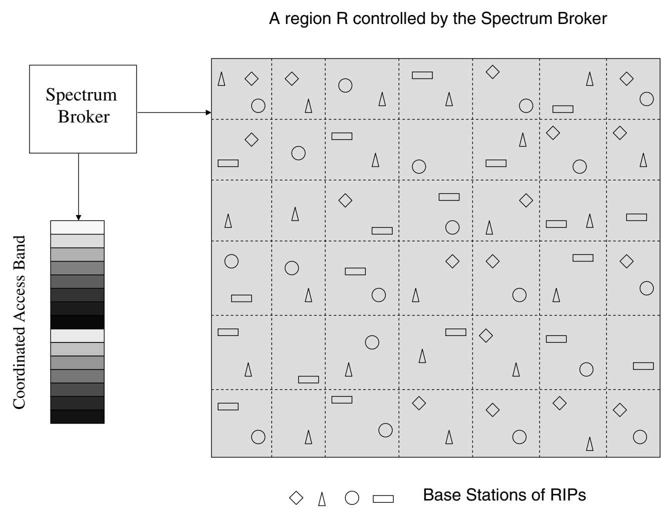



Fig. 2. System Architecture: A representative example of a region controlled by the spectrum broker along with the coordinated access band managed.

In this section, we describe the reference system architecture for our spectrum allocation problem and discuss the constraints that need to be satisfied by the spectrum allocation algorithms. The discussion here closely follows the model outlined in [8]. In this model, a part of the spectrum, designated as the Coordinated Access Band (CAB), is meant to be dynamically shared under the control of a spectrum broker. Regulatory authorities such as FCC can conduct a one-time auction to license CAB and the winner of such an auction then owns and operates a *spectrum broker* [7].

Each region R, which is under the control of a spectrum broker can have a number of base stations owned by several Radio Infrastructure Providers (RIPs). The Wireless Service Providers (WSPs) who offer wireless services such as voice, data etc. to the end users are customers of these RIPs and may use different RIPs in different regions and at different times. The network elements such as the Radio Network Controllers (RNCs) that control the base stations aggregate the end user demands and generate a spectrum demand request to the spectrum broker. The aggregation of end user demands can be done either by predicting the expected end user traffic or the end users themselves signal their bandwidth requirement to their respective base stations using a two way control channel.

This model differs from existing vertical integration model for cellular networks where each service provider licenses and owns spectrum in a region, operates a radio infrastructure and also, offers services to end-users. Our model represents a horizontal model where at the top-level, spectrum access is managed by spectrum provider (the spectrum broker owner), spectrum is used by another level of providers – the RIPs and the end-user services are offered by customer facing WSPs.

The portions of the spectrum that are highly underutilized or unused in spatial or temporal dimension qualify as prime candidates to be used as CAB. Examples of such spectrum bands are Specialized Mobile Radio (SMR) (851-854/806-809 MHz, 861-866/816-821 MHz), public safety bands (764-776, 794-806 MHz), and unused broadcast UHF TV channels (450-470 MHz, 470-512 MHz (channels 14-20), 512-698 MHz (channels 21-51), 698-806 MHz (channels 52-69)).

One can conceivably designate existing cellular bands in 450 MHz, 800 MHz, and 1.9 GHz range also as CAB bands. However, we believe that this move, though technically feasible, may not serve the short-term interest of the incumbent wireless service providers who have spent billions of dollars licensing and deploying their networks and services As such we advocate a *hybrid* model, wherein the existing cellular bands serve as guaranteed capacity or baseline allocation for the incumbent cellular providers and no new providers can avail this spectrum as guaranteed today by the license regime. The CAB band spectrum, however, is guaranteed time bound dynamic access shared among competing providers. This enables existing providers to use CAB spectrum to add capacity to their networks for alleviating traffic hot spots. On the other hand, it also enables new, potentially regional metro scale radio infrastructure providers to compete without requiring large investment in long term licenses of today.

This model satisfies several goals advocated by FCC Spectrum Policy Task Force Report [1]. It improves spectrum access in spatio-temporal scale by promoting time-bound access. The spectrum broker can employ market based mechanisms (e.g: auctions or peak load pricing or hybrids [12]) to price spectrum access. Also, as the broker is cognizant of spectrum demands over time and space, it can better optimize allocation and improve spectrum utilization which is in contrast to state-of-the-art, where a license holder's spectrum may be underutilized in time and space. Our model provides a practical way to protect incumbents and introduce a graceful DSA mechanism in cellular networks.

We designate the smallest amount of contiguous spectrum that can be requested via CDSA as a *channel* of $C$ units. If the broker manages a spectrum band of $B$ units, it can dynamically allocate $K = B/C$ channels.

**Demand model:** Each spectrum demand request for a base station consists of the amount of spectrum required specified as a range between $d_{min}$ and $d_{max}$ channels. This means that this base station requires at least $d_{min}$ channels and at most $d_{max}$ channels to support its end users. This minimum demand ensures that each base station gets at least some number of channels so that they can meet the minimal end user traffic and serve them continuously. Optionally, each demand request may also have an associated scope parameter which captures how many of the neighboring base stations should be preferably given the same channels as in case of CDMA networks or strictly different channels as in TDMA networks.

We assume a batched spectrum request processing model in which the spectrum demands received in a time window of $\tau$ units are grouped and processed together. The allocated spectrum is used in subsequent time windows. This is in contrast to an on-line spectrum request model where spectrum request come at any time and if allocated, can be used immediately by the base stations.

Also, note that the problem of spectrum allocation arising from such a model is different than the spectrum/channel allocation problem that has been widely researched in existing cellular networks. There the problem is solved as a one-time provisioning of a network of a single RIP (and WSP) using a single technology.

**Cellular Infrastructure model:** We assume a cellular infrastructure model similar to [8] with three possibilities: (1) a single RIP per region, (2) base stations of different RIPs with collocated antennas at a common site, (3) non-collocated base stations of different RIPs with non-collocated antennas. Of these, the cases 2, 3 are the most interesting and challenging for the design of spectrum allocation algorithms.

We assume that the spectrum broker knows the terrain propagation model in the given region, the exact location of the transmitters (base stations) and their other characteristics (e.g: frequency range of operations, maximum power level, number of transmitters, waveforms supported etc.) and can estimate the interference level between any two base stations given their location.

**Interference Constraints:** When the spectrum broker assigns channels to the base stations of different RIPs, the following constraints must be satisfied.

1) *Collocated cross-provider constraint:* Two collocated base stations belonging to two different RIPs that share the antenna infrastructure should not be assigned same channels. Assigning same channels to two collocated base stations cause a high level of interference (modeled using the penalty function $p$ defined later) due to their close proximity.
2) *Remote cross-provider constraint:* Two remote base stations belonging to two different RIPs should not be assigned the same channel if they can interfere with each other. The level of interference (modeled using the penalty function $p$) depends on the location of the base stations in the region, the terrain-propagation model and the frequency range under consideration.
3) *Soft Handoff Constraint:* This constraint applies only to CDMA networks where neighboring base stations

must use the same CDMA carrier channels allowing multipath propagation to be constructively exploited to support soft handoff. The discussion of our algorithms does not specifically account for the above due to space constraints; however, they can be easily extended to account for the same.

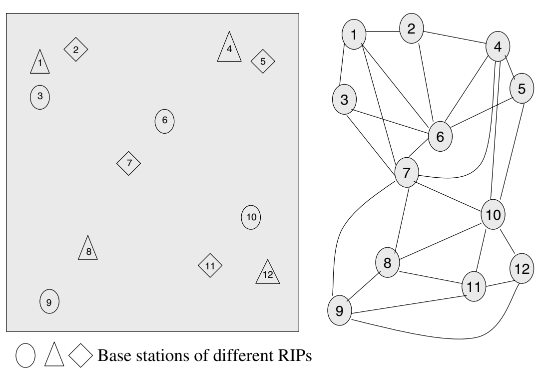
## III. Problem Formulation

In this section, we first describe the graph construct that captures our system model and formulate the spectrum allocation problem as two optimization problems.

### *A.* Interference Graph

The networks of various RIPs in the region $R$ controlled by the spectrum broker are collectively modeled as a weighted undirected graph called the *interference graph* $G = (V, E)$ where each base station in the region is represented by a node in the graph.

There is an edge $(i, j) \in E$ between nodes $i$ and $j$, if the base stations represented by them belong to different RIPs and interfere with each other when they transmit in the same channel. The edges in the interference graph capture the constraints defined in Section II that the spectrum allocation algorithm must satisfy.




a) Region R controlled by the Spectrum Broker    b) Interference Graph

Fig. 3. The region R controlled by the spectrum broker and the corresponding Interference Graph.

We illustrate the concept of interference graph in Figure 3. The region $R$ controlled by the spectrum broker containing the base stations of the various providers is shown in Figure 3.a and the corresponding interference graph is shown in Figure 3.b. Note that nodes 1, 4, 8, and 12 belong to the same RIP and do not have an edge between them. Nodes 1, 2, and 3 which belong to different RIPs interfere with each other. Nodes 1 and 10 which belong to different RIPs do not have an edge as they do not interfere according to the terrain propagation model and the frequency range under consideration (as predicted by the spectrum broker). Each edge $(i, j) \in E$ has a weight $p_{ij}$ associated with it which is the penalty when nodes $i$ and $j$ are assigned the same channels. The value of the penalty function depends on the location of nodes $i$ and $j$ and the terrain propagation model. The penalty function differentiates between the different kind of constraints described in the section II. The penalty is very high for the edges representing the collocated cross provider constraint like the edge between node 1,3 compared to the edge between node 1,6 which represents a remote cross provider constraint.

We assume that the CAB band managed by the spectrum broker is divided into $K$ different channels. For clarity of presentation, we assume that the $K$ channels are non-interfering, which means any two interfering nodes can transmit in two different channels simultaneously. However, our techniques are applicable even when the channels are partially overlapping.

TABLE I
NOTATIONS USED

| Notation | Explanation |
|---|---|
| $G(V, E)$ | Interference graph of the network |
| $N = \lvert V \rvert$ | Number of nodes in the graph |
| $\kappa = \{1, 2, ..., K\}$ | Set of $K$ available channels |
| $d_{min}(i)$ | Minimum demand of node $i \in V$ |
| $d_{max}(i)$ | Maximum demand of node $i \in V$ |
| $F : V \rightarrow \wp^{\kappa}$ | Spectrum allocation function such that $d_{min}(i) \leq \lvert F(i) \rvert \leq d_{max}(i)$ for each node $i \in V$. Here $\wp^{\kappa}$ is the power set of $\kappa$. |
| $p_{ij}$ | Penalty when nodes $i$ and $j \in V$ are assigned the same channels |

Table I summarizes the notations we use. In addition, we use the variables $i$,$j$ to represent the nodes in the interference graph $G$ and $k$ to refer to a channel throughout this paper. Since assigning channels to the base stations can be thought of as assigning colors to the vertices in the interference graph, we use the terms channels and colors interchangeably in the rest of the paper. We use the terms nodes and base stations interchangeably.

Given the network of base stations, the demands of each base station and a fixed number of channels, it is not always possible to allocate as many channels demanded by each base station such that no two interfering base stations belonging to different providers are assigned the same channels. So it is desirable to formulate the spectrum allocation problem as an optimization problem to optimize certain objectives over the set of feasible solutions. In our work, we focus on two optimization problems, one aimed at maximizing the demands satisfied and the other aimed at minimizing total interference in the network. For each problem, we show its relation to a well known computationally hard (NP-Hard) problem and argue need for practical approximate solutions.

### *B.* Maximum Demands Serviced Dynamic Spectrum Access (Max-Demand DSA)

In this version of the spectrum allocation problem, the objective is to maximize the overall demand serviced among the different base stations such that no two base stations belonging to different service providers that interfere with each other (represented by the collocated and remote cross provider constraints) are assigned same channels. Informally, given the network of base stations, the minimum and maximum demands of each base station and the fixed number of available channels, we first see if the minimum demand of each base station can be serviced with the available number of channels. If we can service the minimum demands, then we try to maximize the overall number of demands that can be serviced for all the base stations. Now, we define the problem formally.

**Problem Definition:** Given an unweighted (without the penalties)[2] interference graph $G = (V, E)$, the demand $\{d_{min}(i), d_{max}(i)\}$ for each node $i \in V$ and the available number of channels $K$,

- Is it possible to find an allocation function $F$ such that $\forall i \in V, |F(i)| = d_{min}(i)$, and $\forall (i,j) \in E, F(i) \cap F(j)$ is empty ?
- If *yes*, then find an allocation function $F$ so as to

$$maximize \quad \sum_{i \in V} (|F(i)| - d_{min}(i))$$

such that $\forall i \in V,\ d_{min}(i) \leq |F(i)| \leq d_{max}(i)$, and $\forall (i,j) \in E, F(i) \cap F(j)$ is empty.

**Relationship with Maximum K-Colorable Induced Subgraph (Max K-CIS) Problem:**
We show that there is a close connection between our *Max-Demand DSA* problem and the $K$-colorability problem in graph theory. We formally define the $K$-colorability problem, present some standard results and show how they relate to our problem. This serves as a motivation for our algorithm design for the *Max-Demand DSA* problem.

*Definition 1:* Given a graph $G = (V, E)$ and an integer $K$, the maximum $K$-colorable induced subgraph (*Max K-CIS*) problem is to find a $K$-colorable subgraph of $G$ with the maximum number of vertices. A graph is said to be $K$-*colorable* if it is possible to color the nodes using only $K$ colors in such a way that no two adjacent vertices are colored with the same color.

Now in our *Max-Demand DSA* problem, let us assume the minimum demands of each node is 0. Given the interference graph $G(V, E)$, we create a new graph $G_{max} = (V_{max}, E_{max})$ such that for each node $i \in V$, we create $d_{max}(i)$ copies of it in $V_{max}$ and form a clique among those nodes. For each edge $(i, j) \in E$, we add to $E_{max}$ an edge from each copy of node $i$ to each copy of node $j$. Thus each edge $(i, j) \in E$ translates to $d_{max}(i) \times d_{max}(j)$ edges in $E_{max}$. It is easy to see that, our *Max-Demand DSA* problem is to find a $K$-colorable induced subgraph of $G_{max}$ with the maximum number of vertices since coloring a node in $G_{max}$ means we are servicing one demand of a base station.

It is well known that deciding if a graph is $K$-colorable is NP-complete [13]. This means that it is unlikely that we can find an optimal solution to the *Max-Demand DSA* problem in polynomial time even when the minimum demand of all the nodes is 0. It is even hard to approximate an optimal solution for the *Max K-CIS* problem. This is true as it can be shown that approximating the *Max K-CIS* problem is as hard as approximating the maximum independent set problem (see below) for any fixed value of K [14].

*Definition 2:* The maximum independent set problem is to find a set of vertices of maximum cardinality such that no two vertices have an edge between them.

It is well known that the problem of approximating the maximum independent set problem to within a factor better than $\Omega(n^{1-\epsilon})$ is NP-complete for general graphs [15]. However the problem becomes easier in the case of unit-disk graphs.[3] Here the maximum independent set problem can be approximated within a constant factor of 3 [16] and the *Max-K-CIS* problem can be approximated within a factor of 1.582 [17]. Our solution is motivated by this relationship between *Max-Demand DSA* and *MaX K-CIS* problems. In Section IV-A, we design an efficient algorithm for Max-Demand DSA based on known techniques to solve the *Max K-CIS* problem.

### *C.* Minimum Interference Dynamic Spectrum Access (Min-Interference DSA)

In this section, we formulate our spectrum allocation problem with the objective to minimize the overall interference in the network (incurred when either the collocated or remote cross provider constraints are violated) when all the demands ($d_{max}$) of the base stations are serviced. Informally, we need to find an assignment of $d_{max}(i)$ channels to each node $i$ in the network, such that the penalty incurred by assigning common channels to nodes that interfere with each other is minimized.

If $F$ is the spectrum allocation function, such that $|F(i)| = d_{max}(i), \forall i \in V$, then the interference in the network is defined as follows:

$$I(F) = \sum_{(i,j) \in E} p_{ij} \times |F(i) \cap F(j)| \quad (1)$$

Interference in the network is the sum of the penalties of the edges for each common channel assigned to both of its endpoints. In the following, $I(F)$ is also refereed to as the cost of the spectrum allocation function $F$. Now we define the problem formally.

**Problem Definition:** Given the weighted interference graph $G = (V, E)$, the demand $\{d_{min}(i), d_{max}(i)\}$ for each node $i \in V$, and the available number of channels $K$, find an spectrum allocation function $F$ such that $|F(i)| = d_{max}(i), \forall i \in V$, so as to minimize $I(F)$, i.e.,

$$minimize \quad \sum_{(i,j) \in E} p_{ij} \times |F(i) \cap F(j)|,$$

[2]For this problem, we implicitly assume the penalties to be very high, i.e., we enforce the constraint that any two nodes connected by an edge *must* be assigned different channels.

[3]A graph is said to be a *unit disk graph* if and only if its vertices can be put in one to one correspondence with equisized circles in a plane in such a way that two vertices are joined by an edge if and only if the corresponding circles intersect. Unit disk graphs have been used to model wireless broadcast networks [16] [17] with nodes having equal transmission range and a communication link exist between two nodes if they are within the transmission range of each other.

where $p_{ij}$ is the weight (penalty due to interference) on the edge $(i, j)$.

**Relationship with Max-K-Cut Problem:**

We show that there is a close connection between our *Min-Interference DSA* problem and the *Max-K-Cut* problem [18] in graph theory and derive insights from the techniques to solve the *Max-K-Cut* problem to design our algorithm.

*Definition 3:* Given a graph $G = (V, E)$, the *Max-K-Cut* problem [18] is to find a $K$-partitioning of the vertex set $V$, such that the number of edges that have their endpoints in different partitions (which form part of the cut) is maximized. The *weighted Max-K-Cut* problem is to partition the vertex set such that the sum of the weights of the edges whose endpoints are in different partitions is maximized.

In our *Min-Interference DSA* problem, if we assume the maximum demand of each node to be 1, then our problem boils down to assigning one of the $K$ colors to each node in the graph such that the sum of the weights ($p_{ij}$'s) of the *monochromatic* edges (i.e., with endpoints assigned the same color) is minimized. The above is tantamount to maximizing the sum of the weights of the non-monochromatic edges (which is the objective of the *Max-K-Cut* problem). In essence, we can use the *Max-K-Cut* solution to derive a solution for our *Min-Interference DSA* problem, and vice-versa. Since *Max-K-Cut* is NP-hard [18, 19], the above shows that our *Min-Interference DSA* is also NP-hard even when the maximum demand at each node is 1.

When the demand $d_{max}(i)$ of each node $i \in V$ is more than 1, we can create a new graph $G_{max} = (V_{max}, E_{max})$ with $d_{max}(i)$ copies of each node $i \in V$ and form a clique between them. For any edge $(i, j) \in E$ in the original graph, we add an edge to $E_{max}$ between each copy of node $i$ to each copy of node $j$ similar to the way we did for the *Max Demand DSA* problem. However, a *Max-K-Cut* on this new graph $G_{max}$ may not give a solution for our *Min-Interference DSA* problem, since it is possible that, for some node $i$ in the original graph $G$, more than one copy of $i$ has been assigned to the same partition. This will gives rise to an infeasible solution for our problem since $|F(i)|$ will then be less than $d_{max}(i)$.

In order to overcome this, we define a new problem known as the *Multi-color Max-K-Cut* problem in which, we assign each node to multiple different partitions (equal to the node's demand) such that the sum of the weights of edges crossing the partitions is maximized. We develop simple and efficient heuristics to solve the *Multi-color Max-K-Cut* problem in turn solving our *Min-Interference DSA* problem in Section IV-B.

## IV. New Algorithms

In this section, we develop efficient algorithms for the spectrum allocation problems formulated in Section III.

### *A.* Maximum Demands Serviced DSA (Max-Demand DSA)

As discussed in section III, the *Max-K-CIS* problem is hard to approximate in general graphs. So it is unlikely that we can get a good solution for our *Max-Demand DSA* problem when the interference graph $G$ is modeled as a general graph. If we consider a unit-disk graph model for the interference graph, the *Max-K-CIS* problem can be approximated within a constant factor from the optimal [17]. But in a unit-disk graph model, each node should have the same transmission range, which is not well suited to capture the characteristics of a realistic cellular network as each base station belonging to different providers can use different transmit powers.

In this section, we use a *$\delta$-degree bounded* graph to model the interference graph representing the networks of base stations of different RIPs in a region.

*Definition 4:* A graph $G = (V, E)$ is said to be *$\delta$-degree bounded*, if the maximum node degree of any node in $G$ is less than or equal to $\delta$.

This model does not require the base stations to have the same transmission range. It also captures the locality of interference in the cellular network. Any base station can interfere only with base stations within its interference range which depends upon its transmit power and the terrain propagation model. Considering the sparse nature of deployment of base stations in macro or micro-cellular networks in a region, a $\delta$-degree bounded graph capture the characteristics of a realistic cellular network quite well.

Next, we describe an algorithm to find the maximum independent sets *(Max-IS)* in a *$\delta$-degree bounded* graph and use it as a building block for our *Max-Demand DSA* algorithm. The pseudo-code to find the maximum independent set is shown in Algorithm 1. Step 1 of the algorithm 1 can be done quickly as $\delta$ is very small compared to $|V|$. Also step 1 and 2 have to be repeated at most $|V|$ times.

*Theorem 1:* Given the *$\delta$-degree bounded* graph $G = (V, E)$, let IS and OPT denote the independent set produced by the *Max-IS* algorithm and an optimal algorithm, then $|IS| \geq \frac{|OPT|}{\delta}$

*Proof:* It is easy to see that for every vertex $i$ added to IS by the Algorithm 1, the maximum number of vertices that can be added to the optimal solution is at most $\delta$ as the size of the maximum independent set in the induced subgraph in the neighborhood of node $i$ can be at most $\delta$. So, it follows that $|IS| \geq \frac{|OPT|}{\delta}$. ■

A solution to the *Max-K-CIS* problem can be obtained by repeating the Max-IS algorithm $K$ times, removing the nodes in independent set formed from the graph in every iteration.

**Algorithm Max-Demand DSA:** Now we describe the algorithm for our *Max-Demand DSA* problem. Our algorithm runs in two phases. In phase I, we check if the minimum demands of all nodes in the network can be serviced by using the $K$ available channels. If we can service the minimum demands, then in Phase II we try to maximize the number of demands that can be serviced beyond the minimum demands of each node.

**Phase I:** Given the interference graph $G = (V, E)$, we create a new graph $G_{min} = (V_{min}, E_{min})$ such that for each node $i \in V$, we create $d_{min}(i)$ copies of it in $V_{min}$ and form a clique among those nodes. For each edge $(i, j) \in E$, we add an

**Algorithm 1**. Algorithm to find maximum independent set in a $\delta$-degree bounded graph *(Max-IS)*.

**Input** : The $\delta$*-degree bounded* graph $G(V, E)$.

**Output**: The maximum independent set IS

1) Pick a node $i \in V$ such that the maximum independent set in the induced subgraph in the neighborhood of $i$ is minimum among all nodes.
2) Add $i$ to the solution IS and remove $i$ and all its neighbors from $V$
3) Repeat step 1 and 2 until all vertices in $V$ are removed from the graph.

edge from each copy of node $i$ to each copy of node $j$ to $E_{min}$. Thus each edge $(i, j) \in E$ translates to $d_{min}(i) \times d_{min}(j)$ edges in $E_{min}$. Now we try to color the nodes of graph $G_{min}$ using $K$ colors by solving the *Max-K-CIS* problem in graph $G_{min}$. As mentioned before, the *Max-K-CIS* problem can be solved by repeating Algorithm 1 $K$ times. If all the nodes in $G_{min}$ can be colored using the $K$ colors (using the above method), then the minimum demands of all the nodes can be serviced such that no two interfering nodes are assigned the same channels.

**Phase II:** Now we add extra copies $(d_{max}(i) - d_{min}(i))$ of each node $i \in V$ to the already colored graph $G_{min}$ to form the new graph $G_{max}$. We then again solve the *Max-K-CIS* problem in $G_{max}$ to color as many extra vertices as possible using the $K$ colors. Note that the Phase II of our algorithm is same as finding $K$ disjoint independent sets (or a $K$-colorable induced subgraph) with maximum number of nodes among the nodes that are newly added to form $G_{max}$. Below, we show that repeated application of Algorithm 1 delivers a solution ($K$ disjoint independent sets) of total size at least a constant fraction of the optimal solution. The below theorem is similar to the result of *Max-K-CIS* in unit-disk graphs [17]; we present the proof below, as [17] does not give one.

*Theorem 2:* Phase II of the *Max-Demand DSA* achieves a $(1 - \frac{1}{e^{1/\delta}})$ approximation ratio.

*Proof:* Let us assume that $OPT$ is the total size of the optimal $K$ disjoint independent sets, and let $N_1, N_2, \ldots, N_K$ be the sizes of the $K$ independent sets obtained by iterative application of our *Max-IS* algorithm. We can show that at the $i^{th}$ iteration (when $N_1, N_2, \ldots, N_{i-1}$ independent sets have already been selected), there exists an independent set of size $\frac{OPT - \sum_{j=1}^{i-1} N_j}{\delta K}$ in the remaining graph (i.e., original graph minus the $N_1, N_2, \ldots, N_{i-1}$ independent sets). Since Algorithm 1 guarantees an approximation ratio of $1/\delta$, we have

$$N_i \geq \frac{OPT - \sum_{j=1}^{i-1} N_j}{\delta K}, \qquad \text{for all} \quad 1 \leq i \leq K$$

Manipulation of the above equation gives

$$\begin{aligned} OPT - \sum_{j=1}^{i} N_j &\leq (1 - \frac{1}{K\delta})(OPT - \sum_{j=1}^{i-1} N_j) \\ OPT - \sum_{j=1}^{K} N_j &\leq (OPT)(1 - \frac{1}{K\delta})^K \end{aligned}$$

Since $(1 - 1/k)^k \leq 1/e$ for all $k$, where e = 2.718, we get:

$$\frac{\sum_{j=1}^{K} N_j}{OPT} \geq 1 - (1 - \frac{1}{K\delta})^K \geq (1 - \frac{1}{e^{1/\delta}}).$$

■

## *B.* Minimum Interference DSA (Min-Interference DSA)

In this section, first we present a simple random heuristic to solve the *Multi-Color Max-K-Cut* problem and prove its performance ratio. Next, we start with the solution from the random heuristic and find a better solution using a Tabu search [20] based heuristic for the *Min-Interference DSA* problem.

**Multi-Color Max-K-Cut Problem, and $R_k$ Algorithm:** Given the weighted interference graph $G = (V, E)$ with demands $d_{max}(i)$ for each node $i \in V$ and the total number of colors $K$, the *Multi-Color Max-K-Cut* problem is to assign $d_{max}(i)$ different colors to each node $i$ such that the sum of the weights of the non-monochromatic edges is maximized, i.e.,

$$\sum_{(i,j)\in E} p_{ij}(d_{max}(i)d_{max}(j) - |F(i) \cap F(j)|)$$

is maximized, where $F(i)$ is the set of colors assigned to $i$ and $F(j)$ is the set of colors assigned to $j$.

The $R_k$ (random) algorithm is as follows. For each node $i$, we randomly pick $d_{max}(i)$ different colors from the available $K$ colors and assign them to node $i$. Below, we show that $R_k$ algorithm is expected to give a good solution.

*Theorem 3:* Algorithm $R_k$ achieves a $\left(1 - \frac{1}{K}\right)$ approximation ratio.

*Proof:* We can see that $R_k$ gives a feasible solution as each node $i \in V$ in the graph gets $d_{max}(i)$ different colors assigned.

Let us now consider an edge $(i, j) \in E$. Since we choose $F(i)$ (the set of colors assigned to $i$) to be a random set of size $d_{max}(i)$ from the set of available $K$ colors, each color is expected to be in $F(i)$ with a probability of $d_{max}(i)/K$. Similarly, for $F(j)$. Since, $F(i)$ and $F(j)$ are chosen independently, the probability of any particular color being in $F(i)$ as well as $F(j)$ is $d_{max}(i)d_{max}(j)/K^2$. Thus, the expected value of $|F(i) \cap F(j)|$ is $d_{max}(i)d_{max}(j)/K$, and hence, the expected value of the $R_k$ solution is:

$$\sum_{(i,j)\in E} p_{ij} d_{max}(i)d_{max}(j)(1 - 1/K),$$

which is $(1-1/K)$ of the maximum possible (optimal) value. ■

**Tabu Search Algorithm for Min-Interference DSA:** The random heuristic $R_k$ gives a solution to the *Multi-Color Max-K-Cut* problem with a good approximation guarantee. In this section, we present a Tabu search [20] based heuristic that starts with the random solution obtained by algorithm $R_k$ and improves the solution to get a better solution for the *Min-Interference DSA* problem. Tabu search is a popular local search algorithm used for graph coloring that searches through the solution space guided by the value of the objective function. The pseudo code of our tabu search algorithm is presented in Algorithm 2.

We start with a random initial solution $F_0$ wherein each node $i \in V$ is assigned to $d_{max}(i)$ different random colors in $\kappa$. Starting from such a random solution $F_0$, we create a sequence of solutions $F_0, F_1, F_2, \ldots, F_l, \ldots$, in an attempt to reach a solution with minimum network interference. In the $l^{th}$ iteration ($l \geq 0$), we create the next solution $F_{l+1}$ in the sequence (from $F_l$) as follows.

**The $l^{th}$ Iteration.** First, we generate a certain number (say, $r$) of random neighboring solutions of $F_l$. A random neighboring solution of $F_l$ is generated by picking a random vertex $i \in V$ and a color in $F_l(i)$ and changing it to a random color in $(\mathcal{K} - \{F_l(i)\})$. Thus, a neighboring solution of $F_l$ differs from $F_l$ in the color assignment of only one vertex by a single color. Among the set of such randomly generated neighboring solutions of $F_l$, we pick the neighboring solution with the lowest network interference as the next solution $F_{l+1}$. Note that we do not require $I(F_{l+1})$ to be less than $I(F_l)$, so as to allow escaping from local minima.

**Tabu List:** To achieve fast convergence, we avoid reassigning the same color to a vertex more than once by maintaining a *tabu list* $\tau$ of limited size. In particular, if $F_{l+1}$ was created from $F_l$ by changing a color $c \in F_l(i)$ of a vertex $i$ to a new color $k$, then we add $(i, c)$ to the tabu list $\tau$. Now, when generating random neighboring solutions, we ignore neighboring solutions that assign the color $c$ to $i$ if $(i, c)$ is in $\tau$.

**Termination.** We keep track of the best (i.e., with lowest interference) solution $F_{best}$ seen so far by the algorithm. We terminate the algorithm when the maximum number (say, $count_{max}$) of allowed iterations have passed without any improvement in $I(F_{best})$.

Note that the number of neighbor solutions and the value of $count_{max}$ affects the running time of the algorithm. Higher those values, we search more in the solution space and get better solutions if any, but at the cost of increased running time of the algorithm. There is a trade off between running time and solution quality. Based on our simulation experience, we set $count_{max}$ to the number of nodes in the network and generated 100 neighboring solutions in each iteration.

## V. Performance Evaluation

In this section, we present a detailed performance evaluation of our algorithms using graph based simulations. We generate different interference graphs by randomly placing nodes in a fixed region of space and assign them randomly to different service providers. We add an edge between two nodes, if they belong to different service providers and are within the interference range.

**Graph Parameters:** In all our experiments, we used 1000 nodes in the network randomly assigned to 10 service providers. Each node has a transmission range of 150m. Two nodes have an edge between them, if they belong to different service providers and are within 300m from each other. We generated graphs of different densities by randomly placing the 1000 nodes in a fixed area of size as shown in following table.

| Area ($m^2$) | Max node degree | Avg. node degree |
|---|---|---|
| 3000x3000 | 40 | 25 |
| 3600x3600 | 30 | 20 |
| 5400x5400 | 20 | 10 |
| 7200x7200 | 10 | 5 |

In the USA, there are 5 VHF-LO (Channel 2-6, 55.25-83.25 MHz), 7 VHF-HI (Channel 7-13, 175.25-211.25 MHz), and 56 UHF (Channel 14-69, 472.25-801.25 MHz) on-the-air broadcast TV channels. Of these total 68 channels, channels 7,4, 21, 37, 47, 52, 63, 66-69 are unavailable or already allocated, leaving approximately 58 channels [21]. Since, each channel is 6 MHz wide (to accommodate NTSC analog signal), a total of 348 MHz of prime spectrum is potential candidate for DSA. The current CDMA systems use $\approx 1.25$ MHz of spectrum per carrier (channel). Therefore, we can accommodate $\approx 278$ CDMA channels in this spectrum. In our simulations, we varied the number of channels from 40 to 240 in steps of 40. We assume that the channels are orthogonal and each edge in the graph has equal weight.

In the following, first we show the performance of our *Max-Demand DSA* algorithm for various network settings and then show the performance of our *Min-Interference DSA* algorithm.

**Max-Demand DSA:** We evaluate the performance of our *Max-Demand DSA* algorithm using the metric *"percentage of demands serviced beyond the minimum demand*". Given the minimum and maximum demands of each node in the graph, we first see if the minimum demands can be serviced for each node using the available number of channels and then maximize the demands for each node as much as possible. In our simulations, we used four sets of demands. In the first set, the minimum demand of each node was randomly picked from 1 to 10 and the maximum demand for each node was randomly picked from 10 to 20. Similarly we used the values (20,40),(30,60) and (40,80) for the other three sets of demands. We show the performance of our *Max-Demand DSA* algorithm in Figure 4 and 5. Each value in the plots is an average of 50 different runs.

Figure 4 shows the percentage of demands serviced beyond the minimum demand using different number of channels for a network with maximum node degree 10. The four bars for each value of the channels show the percentage of demands serviced

**Algorithm 2**. Tabu-search Algorithm for the Min-Interference DSA problem.

**Input** : Interference Graph $G(V, E)$
Set of channels $\kappa$
The demands $d_{max}(i)$ for each node $i \in V$.
**Output**: Spectrum Allocation Function $F_{best} : V \rightarrow \wp^{\kappa}$

// The main idea is to start with a random coloring and iteratively improve it.
Start with a random assignment function $F_0$ obtained using $R_k$;
$I_{best} = I(F_0)$; $F_{best} = F = F_0$;
$\tau = null$; $count = 0$;
**while** $I(F) > 0$ and $count \leq count_{max}$ **do**
Generate a certain number of neighbors of $F$;
Each neighbor is generated by randomly picking a node $i \in V_c$ and a color $c \in F(i)$ and changing it to a random $k \in (\mathcal{K} - \{F(i)\})$ such that $((i, k) \notin \tau)$.
Let $F^{'}$ be the neighbor with lowest cost.
Add $(i, c)$ to $\tau$, where $c \in F(i) - F^{'}(i)$.
If $\tau$ is full, delete its oldest entry;
**if** $(I(F^{'}) < I_{best})$ **then**
$I_{best} = I(F^{'})$; $F_{best} = F^{'}$; $count = 0$;
**else** $count = count + 1$;
**endif**;
$F = F^{'}$;
**end while**
**RETURN** $F_{best}$;

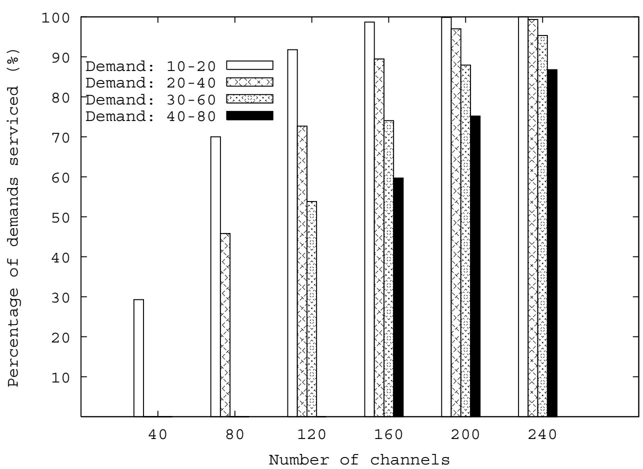



Fig. 4. Percentage of demands serviced beyond the minimum demand using different number of channels for networks with maximum node degree 10.

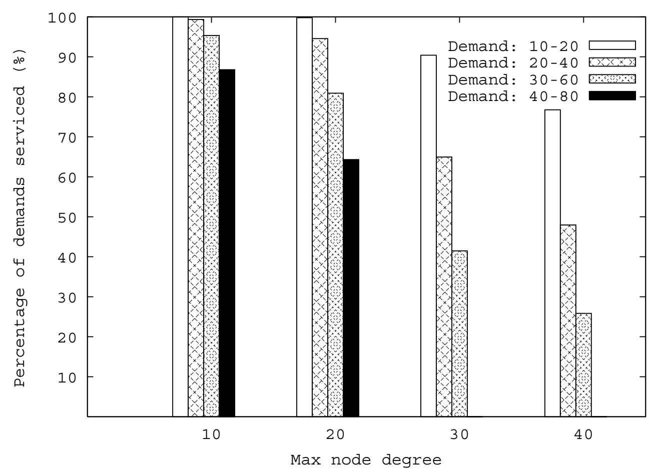



Fig. 5. Percentage of demands serviced beyond the minimum demand in networks of different densities with 240 channels.

beyond the minimum demand for the four different demand sets. Using 40 channels, we are able to service the minimum demand of all the nodes only when they are 10 or less. In other cases, 40 channels are not sufficient to serve the minimum demands using the *Max-Demand DSA* algorithm. We can see that it is not always possible to serve the minimum demands of all the nodes when the number of channels available is less. We also see an increasing trend in the performance as the number of channels increase.

In Figure 5, we show the percentage of the demands for different network densities when the number of channels is set to 240. Note that as the density of the network increase, the percentage of demands served using the same number of channels decreases. The percentage of demands served is also decreasing for the same network density and same number of channels when the demands increase.

**Min-Interference DSA:** We evaluate the performance of our *Min-Interference DSA* algorithm using the metric *"percentage of interference remaining"* in the network after the spectrum allocation. We show the performance of the random and tabu-search algorithm as discussed in Section IV-B. We used four sets of demands for this set of experiments also. In the first set of demands, each node picks a value randomly from 1 to 10 for $d_{max}$. Similarly we used the values 20,30,40 for the other three sets of demands. The performance of our random and tabu-search heuristics are shown in Figure 6 and 7. Each value in the plots is an average of 50 different runs. The y-axis is in log scale for clarity of presentation in both the plots.

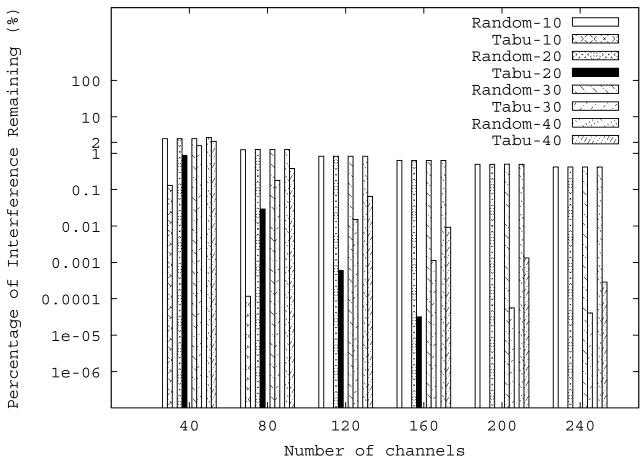



Fig. 6. Percentage of interference remaining in the network after spectrum allocation using different number of channels for networks with maximum node degree 10.

Figure 6 shows the percentage of interference remaining in the network after allocating as many channels as demanded for each node using the random and tabu-search heuristics in a network with maximum node degree 10 while varying the number of channels from 40 to 240. For each channel, the first two bars correspond to the percentage of interference remaining in the network after spectrum allocation using the random and tabu-search heuristic when the node demands are between 1 and 10 (first set of demands). Similarly the next three sets of bars correspond to performance for the next three sets of demands.

Note that the random algorithm, has the same performance irrespective of the number of demands as discussed in section IV-B. It depends only on the value of $k$ as we can see for k=40, the percentage of interference remaining in the network is around 2.5%. The performance of the tabu-search algorithm is extremely good compared to the random algorithm and is always better than the random algorithm. In most cases the percentage of interference remaining in the network using the tabu-search algorithm is less than 0.1%.

In Figure 7, we show the percentage of interference remaining in the network for networks of different densities using 240 channels. Note here the performance of the random algorithm is around 0.417% irrespective of the network density and number of demands. Our tabu-search algorithm starts using this solution and performs extremely well achieving 0 interference in most case. This shows that, we can achieve extremely good assignments with the available number of

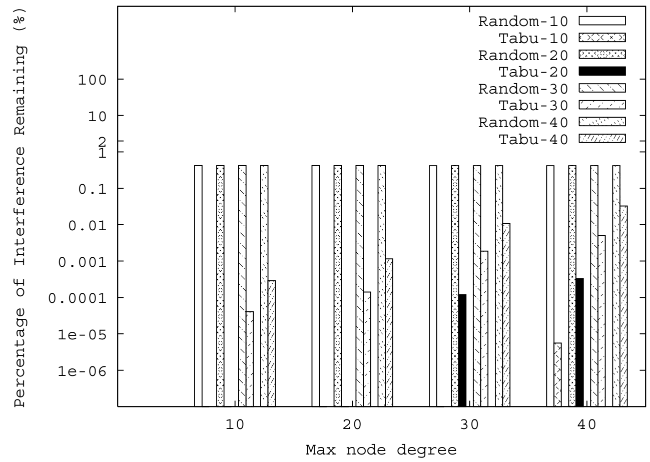



Fig. 7. Percentage of interference remaining in the network after spectrum allocation in networks of different densities with 240 channels.

channels reducing the interference to less than $\frac{1}{k}$ of the initial interference using our tabu-search heuristic.

Our simulation results show that our algorithms for both the *Max-demand DSA* and *Min-Interference DSA* problems scale very well for large network sizes and provide very good spectrum allocation solutions.

## VI. Related Work

Frequency assignment in cellular networks is a well studied problem. Several papers have come up with different kinds of formulations for the frequency assignment problem. [22] and [23] give a comprehensive survey on frequency assignment problems in cellular networks.

Many heuristic algorithms have been proposed to solve various flavors of frequency assignment problem in cellular networks using simulated annealing [24], tabu search [25], evolutionary algorithms [26], neural networks [27] and graph coloring algorithms [22]. [28] provides a good survey of models and algorithms for frequency assignment in homogeneous single provider TDMA cellular networks. Also, [29] provides a good overview of application of graph coloring to frequency assignment problems in such cellular networks. The authors in [30] find lower bound for the fixed spectrum frequency assignment problem using linear programming techniques which is related to our *Min Interference DSA* problem. All these proposal differ in the nature of algorithms and vary in the formulation of the objective function. Our problem formulations presented in this paper differ from the other proposed schemes mainly due of the presence of the minimum and maximum demands for each node and a fixed number of channels to serve these demands.

In the world of wireless networks, graph coloring algorithms have been used to develop efficient channel assignment protocols in wireless LANs [17] [31] and multihop wireless mesh networks [32] [33].

## VII. **Conclusions and Future Work**

The concept of Coordinated Dynamic Spectrum Access (CDSA), where a centralized spectrum broker controls spatio-temporal dynamic access to a reserved spectrum, is one of the most promising approaches in the context of infrastructure networks such as cellular networks. However, the success of this approach depends on design of a scalable spectrum broker which in turn requires the design of fast spectrum allocation algorithms. In this paper, we addressed this challenge.

Specifically, we reported two formulations of the spectrum allocation problem as two optimization problems: first with the objective of maximizing the overall number of demands (*Max-Demand*) satisfied among the various base stations and the second with the objective of minimizing the overall interference in the network (*Min-Interference*) when all the demands of the base stations are satisfied. We showed that the optimization problems are NP-hard and designed efficient algorithms to solve them. We also presented simulation results on sample network topologies showing that our algorithms scale very well for large network sizes.

In our future work, we plan to obtain realistic network topologies of existing service providers and generate super-imposed network infrastructure maps which will be used to generate interference graphs. We also plan to build an experimental spectrum broker simulator that accounts for notions for demand stickiness[8], demand scope, advance reservations, and fairness across providers [8].

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