Relational Normalization Theory

Chapter 6

1

Limitations of E-R Designs

- Provides a set of guidelines, does not result in a unique database schema
- Does not provide a way of evaluating alternative schemas
- Normalization theory provides a mechanism for analyzing and refining the schema produced by an E-R design

2

Redundancy

- Dependencies between attributes cause redundancy
  - Ex. All addresses in the same town have the same zip code

SSN	*Name*	*Town*	*Zip*
1234	Joe	Stony Brook	11790
4321	Mary	Stony Brook	11790
5454	Tom	Stony Brook	11790
	…………………..		

Redundancy

3

Redundancy and Other Problems

- Set valued attributes in the E-R diagram result in multiple rows in corresponding table
- Example: Person (*SSN*, *Name*, *Address*, *Hobbies*)
  - A person entity with multiple hobbies yields multiple rows in table Person
    - Hence, the association between *Name* and *Address* for the same person is stored redundantly
  - *SSN* is key of entity set, but (*SSN*, *Hobby*) is key of corresponding relation
    - The relation Person can't describe people without hobbies

4

Example

ER Model

SSN	*Name*	*Address*	*Hobby*
1111	Joe	123 Main	{biking, hiking}

Relational Model

SSN	*Name*	*Address*	*Hobby*
1111	Joe	123 Main	biking
1111	Joe	123 Main	hiking

…………

Redundancy

5

Anomalies

- Redundancy leads to anomalies:
  - **Update anomaly**: A change in *Address* must be made in several places
  - **Deletion anomaly**: Suppose a person gives up all hobbies. Do we:
    - Set Hobby attribute to null? No, since *Hobby* is part of key
    - Delete the entire row? No, since we lose other information in the row
  - **Insertion anomaly**: *Hobby* value must be supplied for any inserted row since *Hobby* is part of key

6

Decomposition

- **Solution**: use two relations to store Person information
  - Person1 (*SSN, Name, Address*)
  - Hobbies (*SSN*, *Hobby*)
- The decomposition is more general: people with hobbies can now be described
- No update anomalies:
  - Name and address stored once
  - A hobby can be separately supplied or deleted

7

Normalization Theory

- Result of E-R analysis need further refinement
- Appropriate decomposition can solve problems
- The underlying theory is referred to as *normalization theory* and is based on *functional dependencies* (and other kinds, like *multivalued dependencies*)

8

Functional Dependencies

- **Definition:** A *functional dependency* (FD) on a relation schema **R** is a constraint $\boldsymbol{X} \rightarrow \boldsymbol{Y}$, where X and Y are subsets of attributes of **R.**
- **Definition**: An FD $\boldsymbol{X} \rightarrow \boldsymbol{Y}$ is *satisfied* in an instance **r** of **R** if for every pair of tuples, *t* and s: if *t* and *s* agree on all attributes in *X* then they must agree on all attributes in *Y*
  - Key constraint is a special kind of functional dependency: all attributes of relation occur on the right-hand side of the FD:
    - *SSN* → *SSN, Name, Address*

9

Functional Dependencies

- *Address* → *ZipCode*
  - Stony Brook's ZIP is 11733
- *ArtistName* → *BirthYear*
  - Picasso was born in 1881
- *Autobrand* → *Manufacturer*, *Engine type*
  - Pontiac is built by General Motors with gasoline engine
- *Author, Title* → *PublDate*
  - Shakespeare's Hamlet published in 1600

10

Functional Dependency - Example

- Consider a brokerage firm that allows multiple clients to share an account, but each account is managed from a single office and a client can have no more than one account in an office
  - HasAccount (*AcctNum, ClientId, OfficeId*)
    - keys are (*ClientId, OfficeId*), (*AcctNum, ClientId*)
  - *Client, OfficeId* $\rightarrow$ *AcctNum*
  - *AcctNum* $\rightarrow$ *OfficeId*
    - Thus, attribute values need not depend only on key values

11

Entailment, Closure, Equivalence

- **Definition**: If $\boldsymbol{F}$ is a set of FDs on schema **R** and f is another FD on **R**, then $\boldsymbol{F}$ *entails* f if every instance **r** of **R** that satisfies every FD in $\boldsymbol{F}$ also satisfies f
  - Ex: $\boldsymbol{F} = \{A \rightarrow B, B \rightarrow C\}$ and f is $A \rightarrow C$
    - If *Town* $\rightarrow$ *Zip* and *Zip* $\rightarrow$ *AreaCode* then *Town* $\rightarrow$ *AreaCode*
- **Definition**: The *closure* of $\boldsymbol{F}$, denoted $\boldsymbol{F}^+$, is the set of all FDs entailed by $\boldsymbol{F}$
- **Definition**: $\boldsymbol{F}$ and $\boldsymbol{G}$ are *equivalent* if $\boldsymbol{F}$ entails $\boldsymbol{G}$ and $\boldsymbol{G}$ entails $\boldsymbol{F}$

12

Entailment (cont'd)

- Satisfaction, entailment, and equivalence are *semantic* concepts – defined in terms of the actual relations in the "real world."
  - They define *what these notions are*, **not** how to compute them
- How to check if ***F*** entails f or if ***F*** and ***G*** are equivalent?
  - Apply the respective definitions for all possible relations?
    - *Bad idea*: might be infinite number for infinite domains
    - Even for finite domains, we have to look at relations of *all* arities
  - **Solution**: find algorithmic, *syntactic* ways to compute these notions
    - *Important*: The syntactic solution must be "correct" with respect to the semantic definitions
    - Correctness has two aspects: *soundness* and *completeness* – see later

13

Armstrong's Axioms for FDs

- This is the *syntactic* way of computing/testing the various properties of FDs

- **Reflexivity**: If $Y \subseteq X$ then $X \rightarrow Y$ (trivial FD)
  - *Name, Address* $\rightarrow$ *Name*
- **Augmentation**: If $X \rightarrow Y$ then $X Z \rightarrow YZ$
  - If *Town* $\rightarrow$ *Zip* then *Town, Name* $\rightarrow$ *Zip, Name*
- **Transitivity**: If $X \rightarrow Y$ and $Y \rightarrow Z$ then $X \rightarrow Z$

14

Soundness

- Axioms are *sound*: If an FD $f: X \rightarrow Y$ can be derived from a set of FDs $\boldsymbol{F}$ using the axioms, then f holds in every relation that satisfies every FD in $\boldsymbol{F}$.
- Example: Given $X \rightarrow Y$ and $X \rightarrow Z$ then

$$\begin{array}{ll} X \rightarrow XY & \textit{Augmentation by } X \\ YX \rightarrow YZ & \textit{Augmentation by } Y \\ X \rightarrow YZ & \textit{Transitivity} \end{array}$$

  - Thus, $X \rightarrow Y Z$ is satisfied in every relation where both $X \rightarrow Y$ and $X \rightarrow Z$ are satisfied
    - Therefore, we have derived the *union rule* for FDs: we can take the union of the RHSs of FDs that have the same LHS

15

Completeness

- Axioms are *complete:* If $\boldsymbol{F}$ entails f, then f can be derived from $\boldsymbol{F}$ using the axioms
- A consequence of completeness is the following (naïve) algorithm to determining if $\boldsymbol{F}$ entails f:
  - Algorithm: Use the axioms in all possible ways to generate $\boldsymbol{F}^+$ (the set of possible FD's is finite so this can be done) and see if f is in $\boldsymbol{F}^+$

16

Correctness

- The notions of *soundness* and *completeness* link the syntax (Armstrong's axioms) with semantics (the definitions in terms of relational instances)
- This is a precise way of saying that the algorithm for entailment based on the axioms is "correct" with respect to the definitions

17

Generating F^+

$\underline{\boldsymbol{F}}$

$$\begin{tikzcd}
AB \to C \arrow[rrd, dash, dotted] & & & & \\
A \to D \arrow[r, dash, dotted, "aug"] & AB \to BD \arrow[r, dash, dotted, "union"] & AB \to BCD \arrow[r, dash, dotted, "trans"] & AB \to BCDE \arrow[r, dash, dotted, "decomp"] & AB \to CDE \\
D \to E \arrow[r, dash, dotted, "aug"] & BCD \to BCDE \arrow[rru, dash, dotted] & & &
\end{tikzcd}$$

Thus, $AB \to BD$, $AB \to BCD$, $AB \to BCDE$, and $AB \to CDE$ are all elements of $\boldsymbol{F}^+$

18

Attribute Closure

- Calculating *attribute closure* leads to a more efficient way of checking entailment
- The *attribute closure* of a set of attributes, X, with respect to a set of functional dependencies, $\boldsymbol{F}$, (denoted $X^{+}{}_{\boldsymbol{F}}$) is the set of all attributes, A, such that $X \rightarrow A$
  - $X^{+}{}_{\boldsymbol{F1}}$ is not necessarily the same as $X^{+}{}_{\boldsymbol{F2}}$ if $\boldsymbol{F1} \neq \boldsymbol{F2}$
- *Attribute closure and entailment*:
  - Algorithm: Given a set of FDs, $\boldsymbol{F}$, then $X \rightarrow Y$ if and only if $X^{+}{}_{\boldsymbol{F}} \supseteq Y$

19

Example - Computing Attribute Closure

$\boldsymbol{F}$: $AB \rightarrow C$
$A \rightarrow D$
$D \rightarrow E$
$AC \rightarrow B$

X	$X_{\boldsymbol{F}}^{+}$
A	$\{A, D, E\}$
AB	$\{A, B, C, D, E\}$ (Hence AB is a key)
B	$\{B\}$
D	$\{D, E\}$

Is $AB \rightarrow E$ entailed by $\boldsymbol{F}$? *Yes*
Is $D \rightarrow C$ entailed by $\boldsymbol{F}$? *No*

Result: $X_{\boldsymbol{F}}^{+}$ allows us to determine FDs of the form $X \rightarrow Y$ entailed by $\boldsymbol{F}$

20

Computation of Attribute Closure $X^+_{\boldsymbol{F}}$

closure := *X*; // *since* $X \subseteq X^+_{\boldsymbol{F}}$
repeat
old := *closure*;
if there is an FD $Z \rightarrow V$ in $\boldsymbol{F}$ such that
$Z \subseteq$ *closure* **and** $V \not\subseteq$ *closure*
then *closure* := *closure* $\cup$ *V*
until *old* = *closure*

– If $T \subseteq$ *closure* then $X \rightarrow T$ is entailed by $\boldsymbol{F}$

21

Example: Computation of Attribute Closure

Problem: Compute the attribute closure of *AB* with respect to the set of FDs :

$AB \rightarrow C$ (a)
$A \rightarrow D$ (b)
$D \rightarrow E$ (c)
$AC \rightarrow B$ (d)

Solution:

Initially *closure* = {*AB*}
Using (a) *closure* = {*ABC*}
Using (b) *closure* = {*ABCD*}
Using (c) *closure* = {*ABCDE*}

22

Normal Forms

- Each normal form is a set of conditions on a schema that guarantees certain properties (relating to redundancy and update anomalies)
- First normal form (1NF) is the same as the definition of relational model (relations = sets of tuples; each tuple = sequence of atomic values)
- Second normal form (2NF) – a research lab accident; has no practical or theoretical value – won't discuss
- The two commonly used normal forms are *third normal form* (3NF) and *Boyce-Codd normal form* (BCNF)

23

BCNF

- **Definition**: A relation schema **R** is in BCNF if for every FD $X \rightarrow Y$ associated with **R** either
  - $Y \subseteq X$ (i.e., the FD is trivial) or
  - X is a superkey of **R**
- **Example**: Person1(*SSN*, *Name*, *Address*)
  - The only FD is $SSN \rightarrow Name, Address$
  - Since *SSN* is a key, Person1 is in BCNF

24

(non) BCNF Examples

- Person (*SSN*, *Name*, *Address*, *Hobby*)
  - The FD *SSN* $\rightarrow$ *Name*, *Address* does not satisfy requirements of BCNF
    - since the key is (*SSN*, *Hobby*)
- HasAccount (*AcctNum, ClientId, OfficeId*)
  - The FD *AcctNum*$\rightarrow$ *OfficeId* does not satisfy BCNF requirements
    - since keys are (*ClientId, OfficeId*) and (*AcctNum, ClientId*); not *AcctNum*.

25

Redundancy

- Suppose **R** has a FD $A \rightarrow B$, *and A is not a superkey*. If an instance has 2 rows with same value in *A*, they *must* also have same value in *B* (=> redundancy, if the A-*value* repeats twice)

SSN $\rightarrow$ *Name, Address*

redundancy

SSN	*Name*	*Address*	*Hobby*
1111	Joe	123 Main	stamps
1111	Joe	123 Main	coins

- If *A* is a superkey, there cannot be two rows with same value of *A*
  - Hence, BCNF eliminates redundancy

26

Third Normal Form

- A relational schema **R** is in 3NF if for every FD $X \rightarrow Y$ associated with **R** either:
  - $Y \subseteq X$ (i.e., the FD is trivial); or
  - X is a superkey of **R;** or
  - Every $A \in Y$ is part of some key of **R**
- 3NF is weaker than BCNF (every schema that is in BCNF is also in 3NF)

BCNF conditions

27

3NF Example

- HasAccount (*AcctNum, ClientId, OfficeId*)
  - *ClientId, OfficeId* $\rightarrow$ *AcctNum*
    - OK since LHS contains a key
  - *AcctNum* $\rightarrow$ *OfficeId*
    - OK since RHS is part of a key
- HasAccount is in 3NF but it might still contain redundant information due to *AcctNum* → *OfficeId* (which is not allowed by BCNF)

28

3NF (Non) Example

- Person (*SSN*, *Name*, *Address*, *Hobby*)
  - (*SSN*, *Hobby*) is the only key.
  - $SSN \rightarrow Name$ violates 3NF conditions since *Name* is not part of a key and *SSN* is not a superkey

29

Decompositions

- **Goal**: Eliminate redundancy by decomposing a relation into several relations in a higher normal form
- Decomposition must be *lossless*: it must be possible to reconstruct the original relation from the relations in the decomposition
  - We will see why

30

Decomposition

- Schema **R** = (R, ***F***)
  - R is set a of attributes
  - ***F*** is a set of functional dependencies over R
    - Each key is described by a FD
- The *decomposition of schema* **R** is a collection of schemas $\mathbf{R}_i = (R_i, \boldsymbol{F}_i)$ where
  - $R = \cup_i R_i$ for all i (*no new attributes*)
  - $\boldsymbol{F}_i$ is a set of functional dependences involving only attributes of R_i
  - ***F*** entails $\boldsymbol{F}_i$ for all i (*no new FDs*)
- The *decomposition of an instance*, **r**, of **R** is a set of relations $\mathbf{r}_i = \pi_{R_i}(\mathbf{r})$ for all i

31

Example Decomposition

Schema (R, F) where

R = {SSN, $Name$, $Address$, $Hobby$}

F = {$SSN \rightarrow Name, Address$}

can be decomposed into

R_1 = {SSN, $Name$, $Address$}

$\boldsymbol{F}_1$ = {$SSN \rightarrow Name, Address$}

and

R_2 = {SSN, $Hobby$}

$\boldsymbol{F}_2$ = { }

32

Lossless Schema Decomposition

- A decomposition should not lose information
- A decomposition ($\mathbf{R}_1$,.., $\mathbf{R}_n$) of a schema, $\mathbf{R}$, is *lossless* if every valid instance, $\mathbf{r}$, of $\mathbf{R}$ can be reconstructed from its components:

$$\mathbf{r} = \mathbf{r}_1 \bowtie \mathbf{r}_2 \bowtie \ldots\ldots \bowtie \mathbf{r}_n$$

- where each $\mathbf{r}_i = \pi_{\mathbf{R}i}(\mathbf{r})$

33

Lossy Decomposition

The following is always the case (Think why?):

$$\mathbf{r} \subseteq \mathbf{r}_1 \bowtie \mathbf{r}_2 \bowtie \ldots \bowtie \mathbf{r}_n$$

But the following is not always true:

$$\mathbf{r} \supseteq \mathbf{r}_1 \bowtie \mathbf{r}_2 \bowtie \ldots \bowtie \mathbf{r}_n$$

Example: $\mathbf{r} \not\supseteq \mathbf{r}_1 \bowtie \mathbf{r}_2$

SSN	*Name*	*Address*
1111	Joe	1 Pine
2222	Alice	2 Oak
3333	Alice	3 Pine

SSN	*Name*
1111	Joe
2222	Alice
3333	Alice

Name	*Address*
Joe	1 Pine
Alice	2 Oak
Alice	3 Pine

The tuples (2222, Alice, 3 Pine) and (3333, Alice, 2 Oak) *are in the join, but not in the original*

34

Lossy Decompositions: *What is Actually Lost*?

- In the previous example, the tuples (2222, Alice, 3 Pine) and (3333, Alice, 2 Oak) were *gained*, not lost!
  - Why do we say that the decomposition was lossy?

- What was lost is *information*:
  - That 2222 lives at 2 Oak: *In the decomposition, 2222 can live at either 2 Oak or 3 Pine*
  - That 3333 lives at 3 Pine: *In the decomposition, 3333 can live at either 2 Oak or 3 Pine*

35

Testing for Losslessness

- A (binary) decomposition of **R** = (R, $\boldsymbol{F}$) into $\mathbf{R}_1$ = (R_1, $\boldsymbol{F}_1$) and $\mathbf{R}_2$ = (R_2, $\boldsymbol{F}_2$) is lossless *if and only if* :
  - either the FD
    - $(R_1 \cap R_2) \rightarrow R_1$ is in $\boldsymbol{F}^+$
  - or the FD
    - $(R_1 \cap R_2) \rightarrow R_2$ is in $\boldsymbol{F}^+$

36

Example

Schema (R, F) where

$R = \{SSN, Name, Address, Hobby\}$

$\boldsymbol{F} = \{SSN \rightarrow Name, Address\}$

can be decomposed into

$R_1 = \{SSN, Name, Address\}$

$\boldsymbol{F}_1 = \{SSN \rightarrow Name, Address\}$

and

$R_2 = \{SSN, Hobby\}$

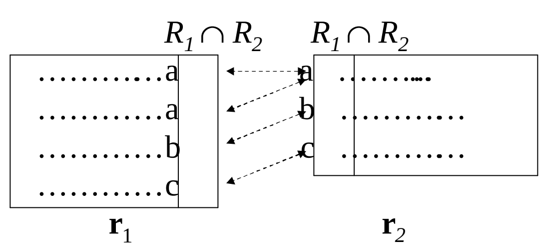
$\boldsymbol{F}_2 = \{\ \}$

Since $R_1 \cap R_2 = SSN$ and $SSN \rightarrow R_1$ the decomposition is lossless

37

Intuition Behind the Test for Losslessness

- Suppose $R_1 \cap R_2 \rightarrow R_2$. Then a row of $\mathbf{r}_1$ can combine with <u>exactly</u> one row of $\mathbf{r}_2$ in the natural join (since in $\mathbf{r}_2$ a particular set of values for the attributes in $R_1 \cap R_2$ defines a unique row)



38

Proof of Lossless Condition

- $\mathbf{r} \subseteq \mathbf{r}_1 \bowtie \mathbf{r}_2$ – *this is true for any decomposition*
- $\mathbf{r} \supseteq \mathbf{r}_1 \bowtie \mathbf{r}_2$

  If $R_1 \cap R_2 \rightarrow R_2$ then

  $card\ (\mathbf{r}_1 \bowtie \mathrm{r}_2) = card\ (\mathbf{r}_1)$

  (since each row of r_1 joins with exactly one row of r_2)

  But $card\ (\mathbf{r}) \geq card\ (\mathbf{r}_1)$ (*since* $\mathbf{r}_1$ *is a projection of* $\mathbf{r}$)
  and therefore $card\ (\mathbf{r}) \geq card\ (\mathbf{r}_1 \bowtie \mathbf{r}_2)$

  Hence $\mathbf{r} = \mathbf{r}_1 \bowtie \mathbf{r}_2$

39

Dependency Preservation

- Consider a decomposition of $\mathbf{R} = (R, \boldsymbol{F})$ into $\mathbf{R}_1 = (R_1, \boldsymbol{F}_1)$ and $\mathbf{R}_2 = (R_2, \boldsymbol{F}_2)$
  - An FD $X \rightarrow Y$ of $\boldsymbol{F}^+$ is in $\boldsymbol{F}_i$ iff $X \cup Y \subseteq R_i$
  - An FD, $f \in \boldsymbol{F}^+$ may be in neither $\boldsymbol{F}_1$, nor $\boldsymbol{F}_2$, nor even $(\boldsymbol{F}_1 \cup \boldsymbol{F}_2)^+$
    - Checking that f is true in $\mathbf{r}_1$ or $\mathbf{r}_2$ is (relatively) easy
    - Checking f in $\mathbf{r}_1 \bowtie \mathbf{r}_2$ is harder – requires a join
    - *Ideally*: want to check FDs locally, in $\mathbf{r}_1$ and $\mathbf{r}_2$, and have a guarantee that every $f \in F$ holds in $\mathbf{r}_1 \bowtie \mathbf{r}_2$
- The decomposition is *dependency preserving* iff the sets $\boldsymbol{F}$ and $\boldsymbol{F}_1 \cup \boldsymbol{F}_2$ are equivalent: $\boldsymbol{F}^+ = (\boldsymbol{F}_1 \cup \boldsymbol{F}_2)^+$
  - Then checking all FDs in $\boldsymbol{F}$, as $\mathbf{r}_1$ and $\mathbf{r}_2$ are updated, can be done by checking $\boldsymbol{F}_1$ in $\mathbf{r}_1$ and F_2 in $\mathbf{r}_2$

40

Dependency Preservation

- If f is an FD in $\boldsymbol{F}$, but f is not in $\boldsymbol{F}_1 \cup \boldsymbol{F}_2$, there are two possibilities:
  - $f \in (\boldsymbol{F}_1 \cup \boldsymbol{F}_2)^+$
    - If the constraints in $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ are maintained, f will be maintained automatically.
  - $f \notin (\boldsymbol{F}_1 \cup \boldsymbol{F}_2)^+$
    - f can be checked only by first taking the join of $\mathbf{r}_1$ and $\mathbf{r}_2$. This is costly.

41

Example

Schema $(R, \boldsymbol{F})$ where

$R = \{SSN, Name, Address, Hobby\}$

$\boldsymbol{F} = \{SSN \rightarrow Name, Address\}$

can be decomposed into

$R_1 = \{SSN, Name, Address\}$

$\boldsymbol{F}_1 = \{SSN \rightarrow Name, Address\}$

and

$R_2 = \{SSN, Hobby\}$

$\boldsymbol{F}_2 = \{ \}$

Since $\boldsymbol{F} = \boldsymbol{F}_1 \cup \boldsymbol{F}_2$ the decomposition is dependency preserving

42

Example

- Schema: $(ABC;\ F)$, $\boldsymbol{F} = \{A \rightarrow B, B \rightarrow C, C \rightarrow B\}$
- Decomposition:
  - $(AC, \boldsymbol{F}_1)$, $\boldsymbol{F}_1 = \{A \rightarrow C\}$
    - Note: $A \rightarrow C \notin \boldsymbol{F}$, but in $\boldsymbol{F}^+$
  - $(BC, \boldsymbol{F}_2)$, $\boldsymbol{F}_2 = \{B \rightarrow C, C \rightarrow B\}$

- $A \rightarrow B \notin (\boldsymbol{F}_1 \cup \boldsymbol{F}_2)$, **but** $A \rightarrow B \in (\boldsymbol{F}_1 \cup \boldsymbol{F}_2)^+$.
  - So $\boldsymbol{F}^+ = (\boldsymbol{F}_1 \cup \boldsymbol{F}_2)^+$ and thus the decompositions is still dependency preserving

43

Example

- HasAccount (*AcctNum, ClientId, OfficeId*)
  - f_1: *AcctNum* $\rightarrow$ *OfficeId*
  - f_2: *ClientId, OfficeId* $\rightarrow$ *AcctNum*
- Decomposition:
  - R_1 = (*AcctNum, OfficeId*; {*AcctNum* $\rightarrow$ *OfficeId*})
  - R_2 = (*AcctNum, ClientId*; {})
- Decomposition is lossless:
  - $R_1 \cap R_2$ = {*AcctNum*} and *AcctNum* $\rightarrow$ *OfficeId*
- In BCNF
- Not dependency preserving: $f_2 \notin (\boldsymbol{F}_1 \cup \boldsymbol{F}_2)^+$
- HasAccount *does not* have BCNF decompositions that are both lossless and dependency preserving! (Check, eg, by enumeration)
- Hence: BCNF+lossless+dependency preserving decompositions are not always achievable!

BCNF Decomposition Algorithm

Input**:** **R** = (R; $\boldsymbol{F}$)

Decomp := **R**
while there is **S** = (S; $\boldsymbol{F}'$) $\in$ *Decomp* and **S** not in BCNF **do**
Find $X \rightarrow Y \in \boldsymbol{F}'$ that violates BCNF // *X isn't a superkey in* **S**
Replace **S** in *Decomp* with $\mathbf{S_1} = (XY;\ \boldsymbol{F}_1)$, $\mathbf{S_2} = (S - (Y - X);\ \boldsymbol{F}_2)$
// $\boldsymbol{F}_1$ = *all FDs of* $\boldsymbol{F}'$ *involving only attributes of* XY
// $\boldsymbol{F}_2$ = *all FDs of* $\boldsymbol{F}'$ *involving only attributes of* $S - (Y - X)$
end
return *Decomp*

45

Simple Example

- HasAccount :

(ClientId, OfficeId, AcctNum)

ClientId,OfficeId $\rightarrow$ *AcctNum*
AcctNum $\rightarrow$ *OfficeId*

- Decompose using *AcctNum* $\rightarrow$ *OfficeId* :

(OfficeId, AcctNum)

BCNF: *AcctNum* is key
FD: *AcctNum* $\rightarrow$ *OfficeId*

(ClientId , AcctNum)

BCNF (only trivial FDs)

46

A Larger Example

Given: **R** = (R; $\boldsymbol{F}$) where $R = ABCDEGHK$ and

$\boldsymbol{F} = \{ABH \rightarrow C, A \rightarrow DE, BGH \rightarrow K, K \rightarrow ADH, BH \rightarrow GE\}$

step 1: Find a FD that violates BCNF

Not $ABH \rightarrow C$ since $(ABH)^+$ includes all attributes

(BH is a key)

$A \rightarrow DE$ violates BCNF since A is not a superkey ($A^+ = ADE$)

step 2: Split **R** into:

$\mathbf{R_1}$ **=** (ADE, $\boldsymbol{F}_1 = \{A \rightarrow DE\}$)

$\mathbf{R_2}$ **=** ($ABCGHK$; $\boldsymbol{F}_1 = \{ABH \rightarrow C, BGH \rightarrow K, K \rightarrow AH, BH \rightarrow G\}$)

Note 1: $\mathbf{R_1}$ is in BCNF

Note 2: Decomposition <u>is</u> *lossless* since A is a key of $\mathbf{R_1}$.

Note 3: FDs $K \rightarrow D$ and $BH \rightarrow E$ are not in $\boldsymbol{F}_1$ or $\boldsymbol{F}_2$. But both can be derived from $\boldsymbol{F}_1 \cup \boldsymbol{F}_2$

(*E.g.*, $K \rightarrow A$ and $A \rightarrow D$ implies $K \rightarrow D$)

Hence, decomposition <u>*is*</u> *dependency preserving*.

47

Example (con't)

Given: $\mathbf{R_2}$ **=** ($ABCGHK$; $\{ABH \rightarrow C, BGH \rightarrow K, K \rightarrow AH, BH \rightarrow G\}$)

step 1: Find a FD that violates BCNF.

Not $ABH \rightarrow C$ or $BGH \rightarrow K$, since BH is a key of $\mathbf{R_2}$

$K \rightarrow AH$ violates BCNF since K is not a superkey ($K^+ = AH$)

step 2: Split $\mathbf{R_2}$ into:

$\mathbf{R_{21}}$ **=** (KAH, $\boldsymbol{F}_{21} = \{K \rightarrow AH\}$)

$\mathbf{R_{22}}$ **=** ($BCGK$; $\boldsymbol{F}_{22} = \{\}$)

Note 1: Both $\mathbf{R_{21}}$ and $\mathbf{R_{22}}$ are in BCNF.

Note 2: The decomposition is *lossless* (since K is a key of $\mathbf{R_{21}}$)

Note 3: FDs $ABH \rightarrow C, BGH \rightarrow K, BH \rightarrow G$ are not in $\boldsymbol{F}_{21}$ or $\boldsymbol{F}_{22}$, and they can't be derived from $\boldsymbol{F}_1 \cup \boldsymbol{F}_{21} \cup \boldsymbol{F}_{22}$.
Hence the decomposition is *not* dependency-preserving

48

Properties of BCNF Decomposition Algorithm

Let $X \rightarrow Y$ violate BCNF in $\mathbf{R} = (R,\boldsymbol{F})$ and $\mathbf{R_1} = (R_1,\boldsymbol{F}_1)$, $\mathbf{R_2} = (R_2,\boldsymbol{F}_2)$ is the resulting decomposition. Then:

- There are *fewer violations* of BCNF in $\mathbf{R_1}$ and $\mathbf{R_2}$ than there were in $\mathbf{R}$
  - $X \rightarrow Y$ implies X is a key of $\mathbf{R_1}$
  - Hence $X \rightarrow Y \in \boldsymbol{F}_1$ does not violate BCNF in $\mathbf{R_1}$ and, since $X \rightarrow Y \notin \boldsymbol{F}_2$, does not violate BCNF in $\mathbf{R_2}$ either
  - Suppose f is $X' \rightarrow Y'$ and $f \in \boldsymbol{F}$ doesn't violate BCNF in $\mathbf{R}$. If $f \in \boldsymbol{F}_1$ or $\boldsymbol{F}_2$ it does not violate BCNF in $\mathbf{R_1}$ or $\mathbf{R_2}$ either since X' is a superkey of $\mathbf{R}$ and hence also of $\mathbf{R_1}$ and $\mathbf{R_2}$.

49

Properties of BCNF Decomposition Algorithm

- A BCNF decomposition is *not necessarily* dependency preserving
- But *always* lossless:

  since $R_1 \cap R_2 = X, \quad X \rightarrow Y,$ and $\quad R_1 = XY$
- BCNF+lossless+dependency preserving is sometimes unachievable (recall HasAccount)

50

Third Normal Form

- Compromise – Not all redundancy removed, but dependency preserving decompositions are <u>always</u> possible (and, of course, lossless)
- 3NF decomposition is based on a *minimal cover*

51

Minimal Cover

- A *minimal cover* of a set of dependencies, ***F***, is a set of dependencies, ***U***, such that:
  - U is equivalent to ***F*** ($F^+ = U^+$)
  - All FDs in ***U*** have the form $X \rightarrow A$ where A is a single attribute
  - It is not possible to make ***U*** smaller (while preserving equivalence) by
    - Deleting an FD
    - Deleting an attribute from an FD (either from LHS or RHS)
  - FDs and attributes that can be deleted in this way are called *redundant*

52

Computing Minimal Cover

- **Example**: $\boldsymbol{F} = \{ABH \rightarrow CK, A \rightarrow D, C \rightarrow E,$
  $BGH \rightarrow L, L \rightarrow AD, E \rightarrow L, BH \rightarrow E\}$
- **step 1**: Make RHS of each FD into a single attribute
  - *Algorithm*: Use the decomposition inference rule for FDs
  - Example: $L \rightarrow AD$ replaced by $L \rightarrow A, L \rightarrow D$; $ABH \rightarrow CK$ by $ABH \rightarrow C$, $ABH \rightarrow K$
- **step 2**: Eliminate redundant attributes from LHS.
  - *Algorithm*: If FD $XB \rightarrow A \in \boldsymbol{F}$ (where B is a single attribute) and $X \rightarrow A$ is entailed by $\boldsymbol{F}$, then B was unnecessary
  - Example: Can an attribute be deleted from $ABH \rightarrow C$?
    - Compute $AB^+_{\boldsymbol{F}}$, $AH^+_{\boldsymbol{F}}$, $BH^+_{\boldsymbol{F}}$.
    - Since $C \in (BH)^+_{\boldsymbol{F}}$, $BH \rightarrow C$ is entailed by $\boldsymbol{F}$ and A is redundant in $ABH \rightarrow C$.

53

Computing Minimal Cover (con't)

- **step 3**: Delete redundant FDs from $\boldsymbol{F}$
  - *Algorithm*: If $\boldsymbol{F} - \{f\}$ entails f, then f is redundant
    - If f is $X \rightarrow A$ then check if $A \in X^+_{\boldsymbol{F}-\{f\}}$
  - Example: $BGH \rightarrow L$ is entailed by $E \rightarrow L$, $BH \rightarrow E$, so it is redundant

- *Note*: The order of steps 2 and 3 cannot be interchanged!! See the textbook for a counterexample

54

Synthesizing a 3NF Schema

Starting with a schema **R** = (R, $\boldsymbol{F}$)

- **step 1**: Compute a minimal cover, $\boldsymbol{U}$, of $\boldsymbol{F}$. The decomposition is based on $\boldsymbol{U}$, but since $\boldsymbol{U}^+ = \boldsymbol{F}^+$ the same functional dependencies will hold
  - A minimal cover for
    $\boldsymbol{F}=\{ABH \to CK, A \to D, C \to E, BGH \to L, L \to AD, E \to L, BH \to E\}$
    is
    $U=\{BH \to C, BH \to K, A \to D, C \to E, L \to A, E \to L\}$

55

Synthesizing a 3NF schema (con't)

- **step 2**: Partition $\boldsymbol{U}$ into sets $\boldsymbol{U}_1$, $\boldsymbol{U}_2$, … $\boldsymbol{U}_n$ such that the LHS of all elements of $\boldsymbol{U}_i$ are the same
  - $\boldsymbol{U}_1 = \{BH \to C, BH \to K\}$, $U_2 = \{A \to D\}$,
    $\boldsymbol{U}_3 = \{C \to E\}$, $U_4 = \{L \to A\}$, $U_5 = \{E \to L\}$

56

Synthesizing a 3NF schema (con't)

- **step 3**: For each $\boldsymbol{U}_i$ form schema $\mathbf{R_i} = (R_i, \boldsymbol{U}_i)$, where R_i is the set of all attributes mentioned in $\boldsymbol{U}_i$
  - Each FD of $\boldsymbol{U}$ will be in some $\mathbf{R_i}$. Hence the decomposition is *dependency preserving*
  - $\mathbf{R_1} = (BHCK;\ BH \rightarrow C, BH \rightarrow K)$, $\mathbf{R_2} = (AD;\ A \rightarrow D)$, $\mathbf{R_3} = (CE;\ C \rightarrow E)$, $\mathbf{R_4} = (AL;\ L \rightarrow A)$, $\mathbf{R_5} = (EL;\ E \rightarrow L)$

57

Synthesizing a 3NF schema (con't)

- **step 4**: If no R_i is a superkey of **R,** add schema $\mathbf{R_0} = (R_0, \{\})$ where R_0 is a key of **R**.
  - $\mathbf{R_0} = (BGH, \{\})$
    - $\mathbf{R_0}$ might be needed when not all attributes are necessarily contained in $R_1 \cup R_2 \ldots \cup R_n$
      - A missing attribute, *A*, must be part of all keys (since it's not in any FD of *U*, deriving a key constraint from *U* involves the augmentation axiom)
    - $\mathbf{R_0}$ might be needed even if all attributes are accounted for in $R_1 \cup R_2 \ldots \cup R_n$
      - Example: $(ABCD; \{A \rightarrow B, C \rightarrow D\})$.
        Step 3 decomposition: $R_1 = (AB; \{A \rightarrow B\})$, $R_2 = (CD; \{C \rightarrow D\})$.
        Lossy! Need to add (AC; { }), for losslessness
  - Step 4 guarantees lossless decomposition.

58

BCNF Design Strategy

- The resulting decomposition, $\mathbf{R_0, R_1, \ldots R_n}$, is
  - Dependency preserving (since every FD in *U* is a FD of some schema)
  - Lossless (although this is not obvious)
  - In 3NF (although this is not obvious)
- Strategy for decomposing a relation
  - Use 3NF decomposition first to get lossless, dependency preserving decomposition
  - If any resulting schema is not in BCNF, split it using the BCNF algorithm (but this may yield a non-dependency preserving result)

59

Normalization Drawbacks

- By limiting redundancy, normalization helps maintain consistency and saves space
- But performance of querying can suffer because related information that was stored in a single relation is now distributed among several
- **Example**: A join is required to get the names and grades of all students taking CS305 in S2002.

```
SELECT  S.Name, T.Grade
FROM  Student S, Transcript T
WHERE  S.Id = T.StudId  AND
         T.CrsCode = 'CS305'  AND  T.Semester = 'S2002'
```

60

Denormalization

- **Tradeoff**: *Judiciously* introduce redundancy to improve performance of certain queries
- **Example**: Add attribute *Name* to Transcript

```
SELECT  T.Name, T.Grade
FROM    Transcript'  T
WHERE   T.CrsCode = 'CS305'  AND  T.Semester = 'S2002'
```

  – Join is avoided
  – If queries are asked more frequently than Transcript is modified, added redundancy might improve average performance
  – But, Transcript' is no longer in BCNF since key is (*StudId, CrsCode, Semester*) and *StudId* $\rightarrow$ *Name*

61

Fourth Normal Form

redundancy

SSN	*PhoneN*	*ChildSSN*
111111	123-4444	222222
111111	123-4444	333333
111111	321-5555	222222
111111	321-5555	333333
222222	987-6666	444444
222222	777-7777	444444
222222	987-6666	555555
222222	777-7777	555555

Person

- Relation has redundant data
- Yet it is in BCNF (since there are no non-trivial FDs)
- Redundancy is due to set valued attributes (in the E-R sense), not because of the FDs

62

Multi-Valued Dependency

- **Problem**: multi-valued (or binary join) dependency
  - **Definition**: If every instance of schema **R** can be (losslessly) decomposed using attribute sets (X, Y) such that:

$$\mathbf{r} = \pi_X(\mathbf{r}) \bowtie \pi_Y(\mathbf{r})$$

then a *multi-valued dependency*

$$\mathbf{R} = \pi_X(\mathbf{R}) \bowtie \pi_Y(\mathbf{R})$$

holds in **r**

Ex: $\text{Person} = \pi_{SSN,PhoneN}(\text{Person}) \bowtie \pi_{SSN,ChildSSN}(\text{Person})$

63

Fourth Normal Form (4NF)

- A schema is in *fourth normal form* (4NF) if for every multi-valued dependency

$$R = X \bowtie Y$$

in that schema, either:

- $X \subseteq Y$ or $Y \subseteq X$ (trivial case); or
- $X \cap Y$ is a superkey of R (*i.e.*, $X \cap Y \rightarrow R$)

64

Fourth Normal Form (Cont'd)

- *Intuition*: if $X \cap Y \rightarrow R$, there is a unique row in relation **r** for each value of $X \cap Y$ (hence no redundancy)
  - Ex: *SSN* does not uniquely determine *PhoneN* or *ChildSSN*, thus Person is not in 4NF.
- *Solution*: Decompose R into X and Y
  - Decomposition is lossless – but not necessarily dependency preserving (since 4NF implies BCNF – next)

65

4NF Implies BCNF

- Suppose R is in 4NF and $X \rightarrow Y$ is an FD.
  - $R1 = XY$, $R2 = R - Y$ is a lossless decomposition of R
  - Thus R has the multi-valued dependency:

$$R = R_1 \bowtie R_2$$

  - Since R is in 4NF, one of the following must hold :
    - $XY \subseteq R - Y$ (an impossibility)
    - $R - Y \subseteq XY$ (i.e., $R = XY$ and X is a superkey)
    - $XY \cap R - Y$ (= X) is a superkey
  - Hence $X \rightarrow Y$ satisfies BCNF condition

66