Logic Languages

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Languages

- Paradigms of Programming Languages:
  - Imperative = Turing machines
  - Functional Programming = lambda calculus
  - Logical Programming = first-order predicate calculus
- Prolog and its variants make up the most commonly used Logical programming languages.
  - One variant is XSB Prolog (developed here at Stony Brook)
  - Other Prolog systems: SWI Prolog, Sicstus, Yap Prolog, Ciao Prolog, GNU Prolog, etc.
    - ISO Prolog standard.

Relations/Predicates

- Predicates are building-blocks in predicate calculus: **p(a$_1$,a$_2$,...,a$_k$)**
  - **parent(X, Y)**: X is a parent of Y.

```
parent(pam, bob). parent(bob, ann).
parent(tom, bob). parent(bob, pat).
parent(tom, liz). parent(pat, jim).
```

  - **male(X)**: X is a male.

```
male(tom).
male(bob).
male(jim).
```

We attach meaning to them, but within the logical system they are simply structural building blocks, with no meaning beyond that provided by explicitly-stated interrelationships

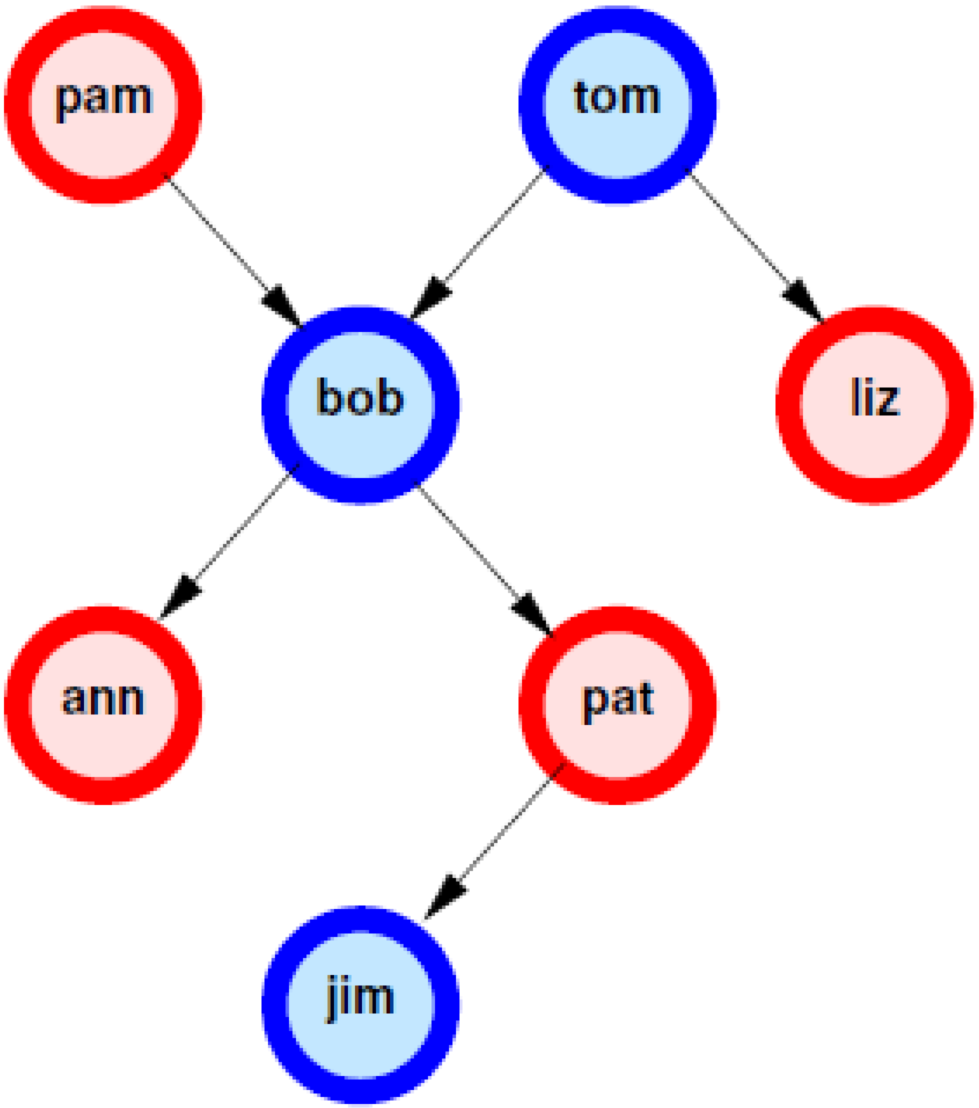
Relations

- **`female(X)`** : X is a female.

```
female(pam).
female(pat).
female(ann).
female(liz).
```

Relations

```
parent(pam, bob).
parent(tom, bob).
parent(tom, liz).
parent(bob, ann).
parent(bob, pat).
parent(pat, jim).
female(pam).
female(pat).
female(ann).
female(liz).
male(tom).
male(bob).
male(jim).
```



Relations

- Rules:
  - `mother(X, Y)`: X is the mother of Y.

    -In First Order Logic (FOL or predicate calculus):

$$\forall X,Y\ (parent(X,Y) \wedge female(X) \Rightarrow mother(X,Y))$$

    -In Prolog:

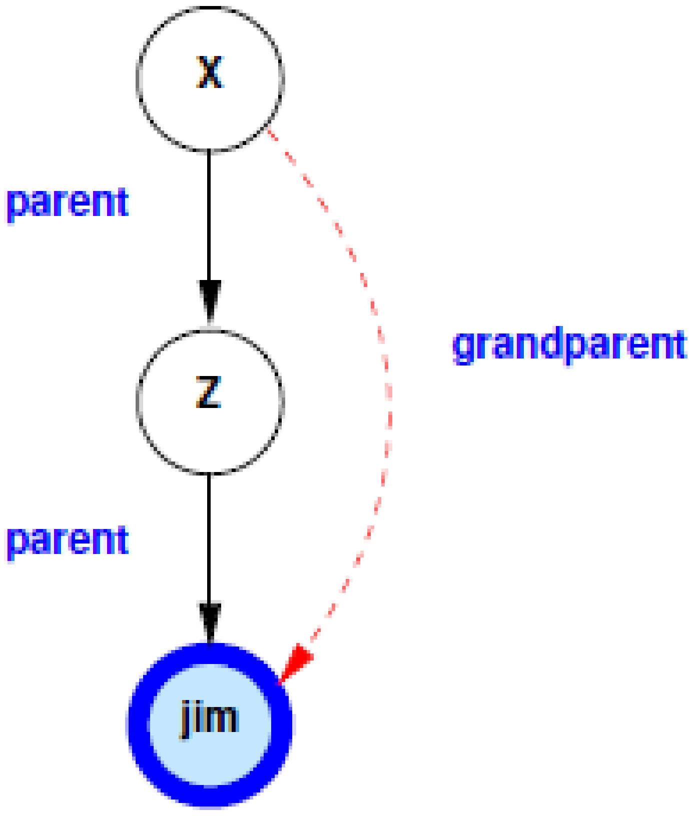
```
mother(X,Y) :-
      parent(X,Y),
      female(X).
```

  - all variables are universally quantified outside the rule
  - "," means *and* (conjunction), ":-" means *if* (implication) and ";" means *or* (disjunction).

Relations

- More Relations:

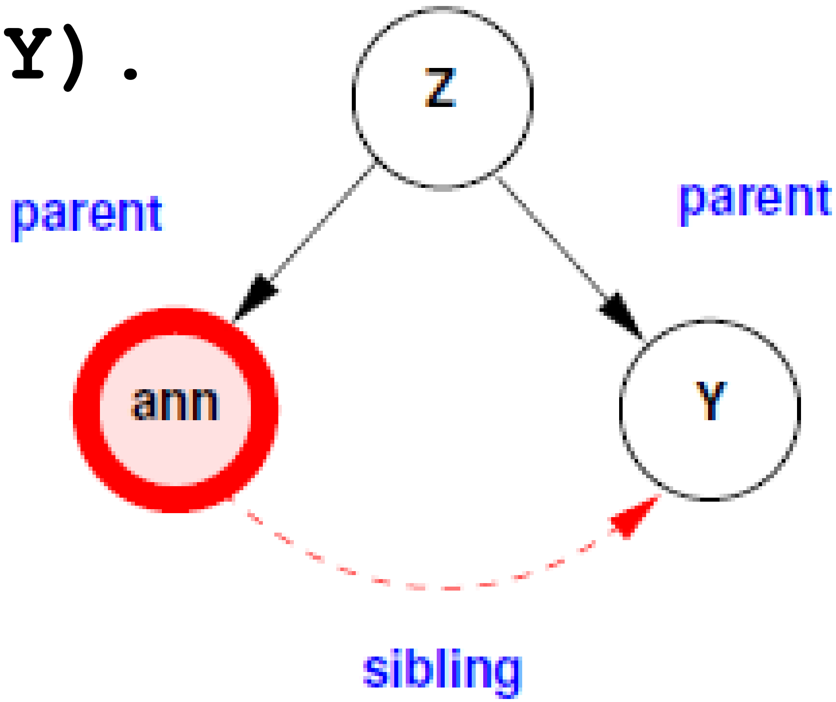
```
grandparent(X,Y) :-
    parent(X,Z),
    parent(Z,Y).
```



Relations

```
sibling(X,Y) :- parent(Z,X),
    parent(Z,Y), X \= Y.

?- sibling(ann,Y).
```



Relations

- More Relations:

```
cousin(X,Y) :- …
greatgrandparent(X,Y) :- …
greatgreatgrandparent(X,Y) :- …
```

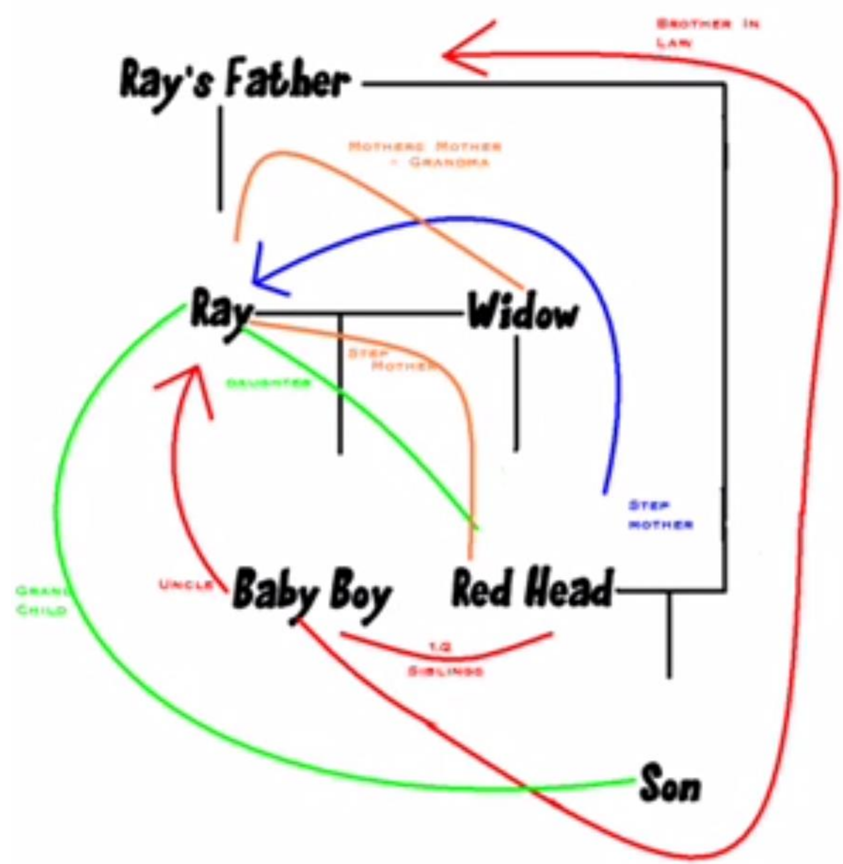
Recursion

```
ancestor(X,Y) :-
   parent(X,Y).
ancestor(X,Y) :-
   parent(X,Z),
   ancestor(Z,Y).
?- ancestor(X,jim).
?- ancestor(pam,X).
?- ancestor(X,Y).
```

Relations

- How to implement "I'm My Own Grandpa"?

https://www.youtube.com/watch?v=eYlJH81dSiw



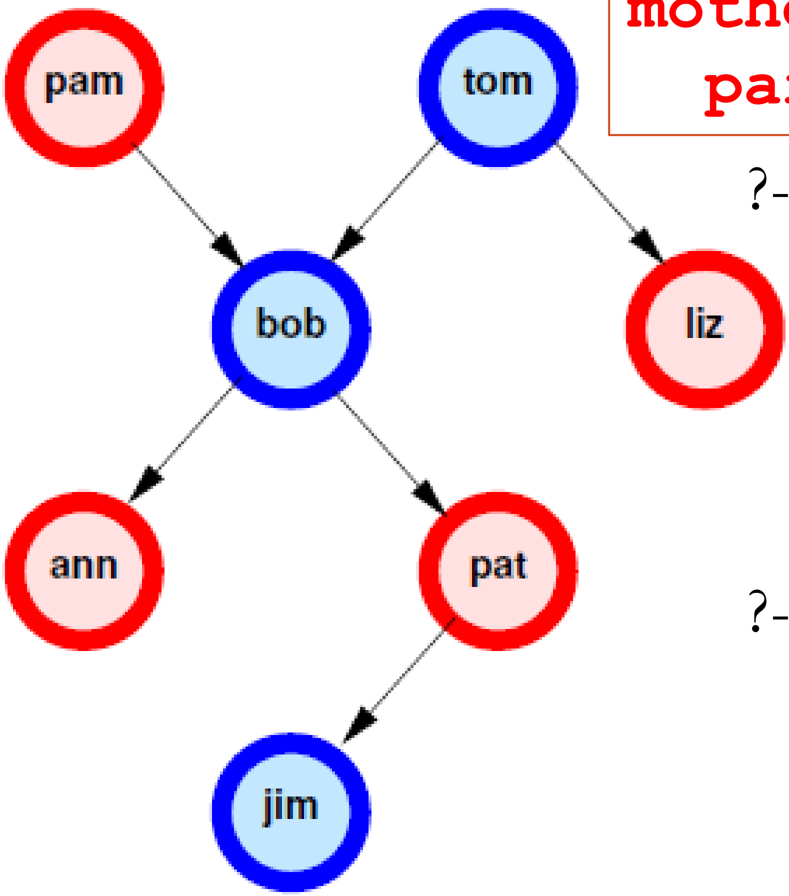
Recursion

- What about:

```
ancestor(X,Y) :-
    ancestor(X,Z),
    parent(Z,Y).
ancestor(X,Y) :-
    parent(X,Y).
?- ancestor(X,Y).
```

INFINITE LOOP

Computations in Prolog




```
mother(X,Y):-
   parent(X,Y),female(X).
```

?- mother(M, bob).

?- parent(M, bob), female(M).

?- M=pam, female(pam).

M = pam   true

?- father(M, bob).

?- parent(M, bob), male(M)

(i) ?- M=pam, male(pam).

fail

(ii) ?- M=tom, male(tom).

M = tom   true

Prolog Execution

- Call: Call a predicate (invocation)
- Exit: Return an answer to the caller
- Fail: Return to caller with no answer
- Redo: Try next path to find an answer

The XSB Prolog System

- http://xsb.sourceforge.net
  - Developed at Stony Brook by David Warren and many contributors
- Overview of Installation:
  - Unzip/untar; this will create a subdirectory XSB
  - Windows: you are done
  - Linux:

```
cd  XSB/build
./configure
./makexsb
```

    That's it!
  - Cygwin under Windows: same as in Linux

Use of XSB

- Put your ruleset *and* data in a file with extension .P (or .pl)

```
p(X) :- q(X,_).
q(1,a).
q(2,a).
q(b,c).
```

- Don't forget: all rules and facts end with a period (.)
- Comments: /*…*/ or %…. (% acts like // in Java/C++)
- Type

  …**/XSB/bin/xsb** (Linux/Cygwin)

  …**\XSB\config\x86-pc-windows\bin\xsb** (Windows)

  where … is the path to the directory where you downloaded XSB
- You will see a prompt

```
| ?-
```

and are now ready to type queries

Use of XSB

- Loading your program, myprog.P or myprog.pl

```
?- [myprog].
```

  XSB will compile myprog.P (if necessary) and load it.

  Now you can type further queries, e.g.

```
?- p(X).
?- p(1).
```

- Some Useful Built-ins:
  - **write(X)** – write whatever X is bound to
  - **writeln(X)** – write then put newline
  - **nl** – output newline
  - Equality: **=**
  - Inequality: **\=**

  http://xsb.sourceforge.net/manual1/index.html (Volume 1)
  http://xsb.sourceforge.net/manual2/index.html (Volume 2)

Use of XSB

- Some Useful Tricks:
  - XSB returns only the first answer to the query
  - To get the next, type **;** <Return>. For instance:

```
| ?- q(X).
X = 2;
X = 4
yes
```

  - Usually, typing the **;**'s is tedious. To do this programmatically, use this idiom:

```
| ?- (q(_X), write('X='), writeln(_X), fail ; true).
```

_X here tells XSB to not print its own answers, since we are printing them by ourselves. (XSB won't print answers for variables that are prefixed with a _.)

Logic Programming Concepts

- In logic, most statements can be written many ways.
  - That's great for people but a nuisance for computers.
  - It turns out that if you make certain **restrictions** on the **format of statements** you can prove theorems mechanically.
    - Most common restriction is to have a single conclusion implied by a conjunction of premises (i.e., ***Horn clauses***)
      - Horn clauses are named for the logician Alfred Horn, who first pointed out their significance in 1951
    - That's what logic programming systems do!

Syntax of Prolog Programs

- A ***Prolog*** *program* is a sequence of clauses
- Each *clause* (sometimes called a *rule* or *Horn rule*) is of the form:

```
Head :- Body.
```

  - **`Head`** is one *term*
  - **`Body`** is a comma-separated list of terms
  - A clause with an empty body is called a ***fact***

Logic Programming Concepts

- Operators:
  - conjunction, disjunction, negation, implication
- Universal and existential quantifiers
- Statements
  - sometimes true, sometimes false, sometimes unknown
  - axioms - assumed true
  - theorems - provably true
  - goals - things we'd like to prove true

Logic Programming Concepts

- A ***term*** can be a ***constant***, ***variable***, or ***structure*** (consisting of a ***functor*** and a parenthesized list of arguments)
- A ***constant*** is either an ***atom*** or a ***number***
  - An ***atom*** is either what looks like an identifier beginning with a lowercase letter, or a single quoted string
  - A ***number*** looks like an integer or real from some more ordinary language
- A ***variable*** looks like an identifier beginning with an upper-case letter
- There are NO declarations (vars, terms, or predicates)
  - All types are discovered implicitly

Logic Programming Concepts

- The Prolog interpreter has a collection of facts and rules in its DATABASE
  - Facts (i.e., clauses with empty bodies):

```
raining(ny).          raining(seattle).
```

  - *Facts* are axioms (things the interpreter assumes to be true)
    - Prolog provides an automatic way to deduce true results from facts and rules
  - A rule (i.e., a clause with both sides):

```
wet(X) :- raining(X).
```

  - The meaning of a *rule* is that **the conjunction of the structures in the body implies the head**.

Note: Single-assignment variables: X must have the same value on both sides.

  - *Query* or *goal* (i.e., a clause with an empty head):

```
?- wet(X).
```

Logic Programming Concepts

- So, rules are theorems that allow the interpreter to infer things
- To be interesting, <u>rules generally contain variables</u>

```
employed(X) :- employs(Y,X).
```

can be read as:

"*for all* **X**, **X** *is employed* ***if*** *there exists a* **Y** *such that* **Y** *employs* **X**"

  - **<u>Note the direction of the implication</u>**
  - **<u>Also, the example does NOT say that X is employed ONLY IF there is a Y that employs X</u>**
    - **<u>there can be other ways for people to be employed</u>**
      - **<u>like, we know that someone is employed, but we don't know who is the employer or maybe they are self employed:</u>**

```
employed(bill).
```

Logic Programming Concepts

- The scope of a variable is the clause in which it appears:
  - Variables whose first appearance is on the left hand side of the clause (i.e., the head) have implicit **universal** quantifiers
    - For example, we infer for all possible **`X`** that they are **`employed`**

```
employed(X) :- employs(Y,X).
```

  - Variables whose first appearance is in the body of the clause have implicit **existential** quantifiers **in that body**
    - For example, there exists some **`Y`** that **`employs X`**
    - Note that these variables are also universally quantified outside the rule (by logical equivalences)

Logic Programming Concepts

```
grandmother(A, C) :-
      mother(A, B),
      mother(B, C).
```

can be read as:

"for all A, C [A is the grandmother of C if there exists a B such that A is the mother of B and B is the mother of C]"

- We probably want another rule that says:

```
grandmother(A, C) :-
      mother(A, B),
      father(B, C).
```

Recursion

- Transitive closure:
  - Example: a graph declared with facts (true statements)

```
edge(1,2).
edge(2,3).
edge(2,4).
```

1) if there's an **edge** from **X** to **Y**, we can **reach** **Y** from **X**:

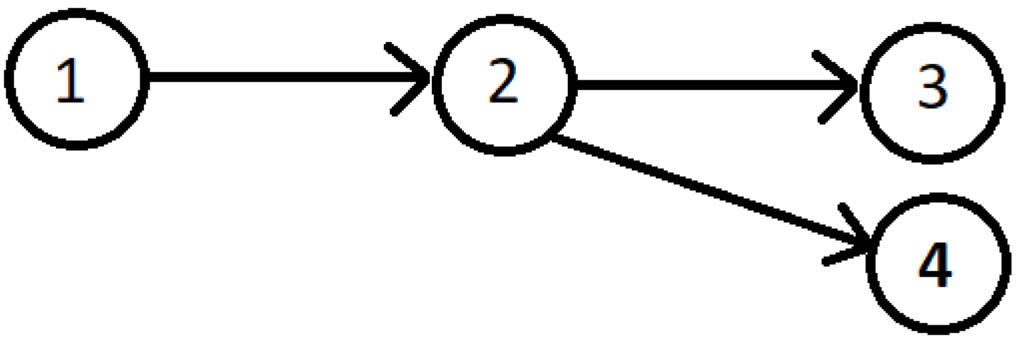
```
reach(X,Y)  :-  edge(X,Y).
```

2) if there's an **edge** from **X** to **Z**, and we can **reach** **Y** from **Z**, then we can **reach** **Y** from **X**:

```
reach(X,Y)  :-
        edge(X,Z),
        reach(Z, Y).
```

```
?- reach(X,Y).
  X = 1
  Y = 2;
  X = 2
  Y = 3;
  X = 2
  Y = 4;
  X = 1
  Y = 3;
  X = 1
  Y = 4;
  no
```

← Type a semi-colon repeatedly for more answers




```
reach(X,Y) :- edge(X,Y).
reach(X,Y) :-
        edge(X,Z),
        reach(Z, Y).
```

Prolog Programs

- We will now explore Prolog programs in more detail:
  - Syntax of Prolog Programs
    - ***Terms*** can be:
      - Atomic data
      - Variables
      - Structures

Atomic Data

- ***Numeric constants***: integers, floating point numbers (e.g. **`1024`**, **`-42`**, **`3.1415`**, **`6.023e23`**,...)
- ***Atoms***:
  - Identifiers: sequence of letters, digits, underscore, beginning with a lower case letter (e.g. **`paul`**, **`r2d2`**, **`one_element`**).
  - Strings of characters enclosed in single quotes (e.g. **`'Stony Brook'`**)

Variables

- Variables are denoted by identifiers beginning with an *Uppercase letter* or *underscore* (e.g. **`X`**, **`Index`**, **`_param`**).
- *These are Single-Assignment Logical variables:*
  - Variables can be assigned only once
  - Different occurrences of the same variable in a clause denote the same data
- Variables are implicitly declared upon first use
  - Variables are not typed
    - All types are discovered implicitly (no declarations in LP)
  - If the variable does not start with underscore, it is assumed that it appears multiple times in the rule.
    - If is does not appear multiple times, then a warning is produced: "*Singleton variable*"
    - You can use variables preceded with underscore to eliminate this warning

Variables

- ***Anonymous variables*** (also called *Don't care variables*): variables **beginning with "_"**
  - Underscore, by itself (i.e., **_**), represents a variable
    - Each occurrence of **_** corresponds to a different variable; even within a clause, **_** does not stand for one and the same object.
  - A variable with a name beginning with "**_**", but has more characters. E.g.: **`_radius`**, **`_Size`**
    - we want to give it a descriptive name
    - sometimes it is used to create relationships within a clause (and must therefore be used more than once): a warning is produced: "*Singleton-marked variable appears more than once*"

Variables

- Warnings are used to identify bugs (most because of copy-paste errors)
  - Instead of declarations and type checking
  - Fix all the warnings in a program, so you know that you don't miss any logical error

Variables

- Variables can be assigned only once, but that value can be further refined:

```
?- X=f(Y),
       Y=g(Z),
       Z=2.
```

Therefore, `X=f(g(2)), Y=g(2), Z=2`

- The order also does not matter:

```
?- Z=2,
       X=f(Y),
       Y=g(Z).
X = f(g(2)), Y=g(2), Z=2
```

- Even infinite structures:

```
?- X=f(X).
X=f(f(f(f(f(f(f(f(f(f(...))
```

Logic Programming Queries

- To run a Prolog program, one asks the interpreter a question
  - This is done by asking a query which the interpreter tries to prove:
    - If it can, it says **yes**
    - If it can't, it says **no**
    - If your query contained variables, the interpreter prints the values it had to give them to make the query true

```
?- wet(ny).      ?- reach(a, d).  ?- reach(d, a).
Yes              Yes              No
?- wet(X).       ?- reach(X, d).  ?- reach(X, Y).
X = ny;          X=a              X=a, Y=d
X = seattle;     ?- reach(a, X).
no               X=d
```

Meaning of Logic Programs

- **Declarative Meaning:** What are the ***logical consequences*** of a program?
- **Procedural Meaning:** For what values of the variables in the query can I ***prove*** the query?
  - The user gives the system a goal:
    - The system attempts to find axioms + inference rules to **prove** that goal
      - If goal contains variables, then also gives the values for those variables for which the goal is proven

Declarative Meaning

```
brown(bear).              big(bear).
gray(elephant).           big(elephant).
black(cat).               small(cat).
dark(Z) :- black(Z).
dark(Z) :- brown(Z).
dangerous(X) :- dark(X), big(X).
```

- The *logical consequences* of *a program* L is the smallest set such that
  - All facts of the program are in L,
  - If **H :- B_1, B_2, …, B_n.** is an instance of a clause in the program such that **B_1, B_2, …, B_n** are all in L, then **H** is also in L.
  - For the above program we get **dark(cat)** and **dark(bear)** and consequently **dangerous(bear)** in addition to the original facts.

Procedural Meaning of Prolog

```
brown(bear).          big(bear).
gray(elephant).       big(elephant).
black(cat).           small(cat).
dark(Z) :- black(Z).
dark(Z) :- brown(Z).
dangerous(X) :- dark(X), big(X).
```

- A *query* is, in general, a conjunction of goals: $G_1, G_2, \ldots, G_n$
- To *prove* $G_1, G_2, \ldots, G_n$:
  - Find a clause H `:-` $B_1, B_2, \ldots, B_k$ such that G_1 and H match.
  - Under the substitution for variables, prove $B_1, B_2, \ldots, B_k, G_2, \ldots, G_n$

  If nothing is left to prove then the proof succeeds!

  If there are no more clauses to match, the proof fails!

Procedural Meaning of Prolog

```
brown(bear).              big(bear).
gray(elephant).           big(elephant).
black(cat).               small(cat).
dark(Z) :- black(Z).
dark(Z) :- brown(Z).
dangerous(X) :- dark(X), big(X).
```

- To prove: `?- dangerous(Q).`
  1. Select `dangerous(X):-dark(X),big(X)` and prove `dark(Q),big(Q).`
  2. To prove `dark(Q)` select the first clause of dark, i.e. `dark(Z):-black(Z)`, and prove `black(Q),big(Q).`
  3. Now select the fact `black(cat)` and prove `big(cat).` This proof fails!
  4. Go back to step 2, and select the second clause of dark, i.e. `dark(Z):-brown(Z)`, and prove `brown(Q),big(Q).`

Procedural Meaning of Prolog

```
brown(bear).            big(bear).
gray(elephant).         big(elephant).
black(cat).             small(cat).
dark(Z) :- black(Z).
dark(Z) :- brown(Z).
dangerous(X) :- dark(X), big(X).
```

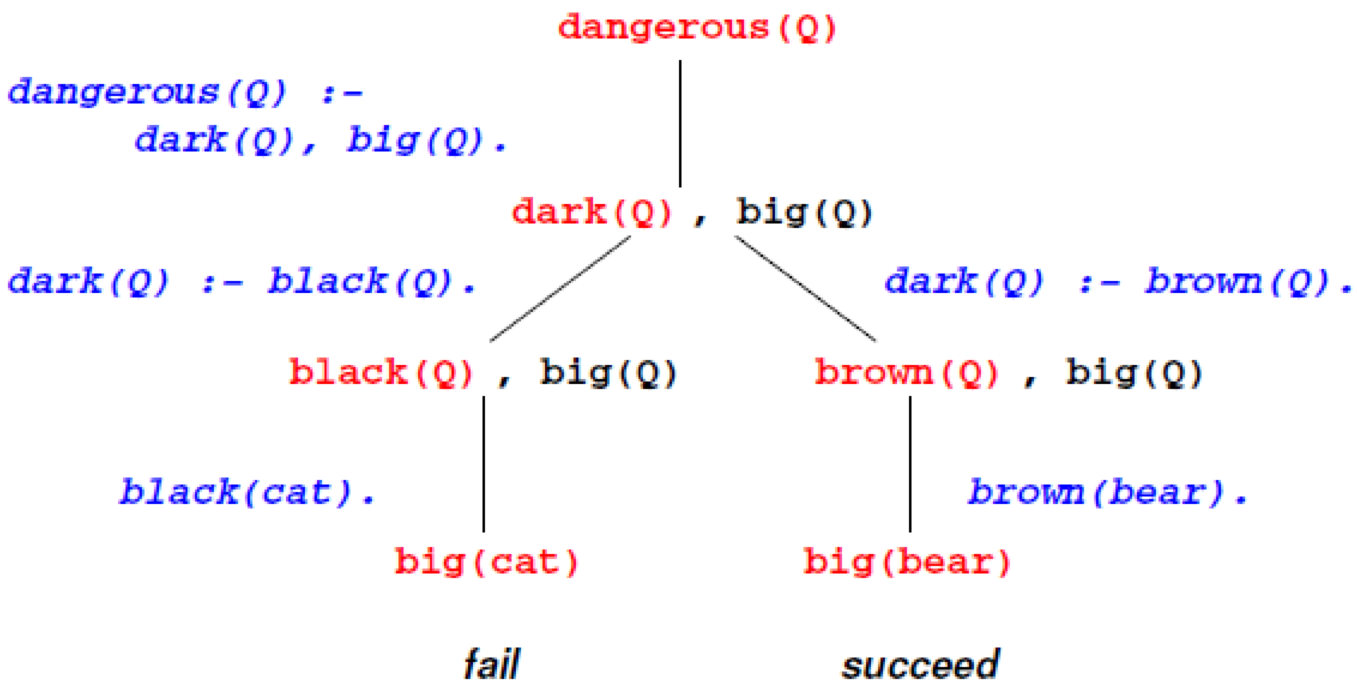
- To prove: `?- dangerous(Q).`

5. Now select **`brown(bear)`** and prove **`big(bear)`**.
6. Select the fact **`big(bear)`**.

There is nothing left to prove, so the proof succeeds

Procedural Meaning of Prolog

```
brown(bear).          big(bear).
gray(elephant).       big(elephant).
black(cat).           small(cat).
dark(Z) :- black(Z).
dark(Z) :- brown(Z).
dangerous(X) :- dark(X), big(X).
```



Procedural Meaning of Prolog

- The Prolog interpreter works by what is called ***BACKWARD CHAINING*** (also called ***top-down***, ***goal directed***)
  - It begins with the thing it is trying to prove and works backwards looking for things that would imply it, until it gets to facts.
- It is also possible to work forward from the facts trying to see if any of the things you can prove from them are what you were looking for
  - This methodology is called ***bottom-up resolution***
  - It can be very time-consuming
  - Example: Answer set programming, DLV, Potassco (the Potsdam Answer Set Solving Collection), OntoBroker
- Fancier logic languages use both kinds of chaining, with special smarts or hints from the user to bound the searches

Procedural Meaning of Prolog

- When it attempts resolution, the Prolog interpreter pushes the current goal onto a stack, makes the first term in the body the current goal, and goes back to the beginning of the database and starts looking again.
- If it gets through the first goal of a body successfully, the interpreter continues with the next one.
- If it gets all the way through the body, the goal is satisfied and it backs up a level and proceeds.

Procedural Meaning of Prolog

- The Prolog interpreter starts at the beginning of your database (**this ordering is part of Prolog**, NOT of logic programming in general) and looks for something with which to unify the current goal
  - If it finds a fact, great; it succeeds,
  - If it finds a rule, it attempts to satisfy the terms in the body of the rule depth first.
  - This process is motivated by the *RESOLUTION PRINCIPLE*, due to Robinson, 1965:
    - It says that if C1 and C2 are Horn clauses, where C2 represents a true statement and the head of C2 unifies with one of the terms in the body of C1, then we can replace the term in C1 with the body of C2 to obtain another statement that is true if and only if C1 is true

Procedural Meaning of Prolog

- If it fails to satisfy the terms in the body of a rule, the interpreter **undoes** the unification of the left hand side (BACKTRACKING) (this includes un-instantiating any variables that were given values as a result of the unification) and keeps looking through the database for something else with which to unify
- If the interpreter gets to the end of database without succeeding, it **backs** out a level (that's how it might **fail** to satisfy something in a body) and continues from there.

Procedural Meaning of Prolog

- PROLOG IS NOT PURELY DECLARATIVE:
  - The ordering of the database and the left-to-right pursuit of sub-goals gives a deterministic imperative semantics to searching and backtracking
  - Changing the order of statements in the database can give you different results:
    - It can lead to infinite loops
    - It can result in inefficiency

Procedural Meaning of Prolog

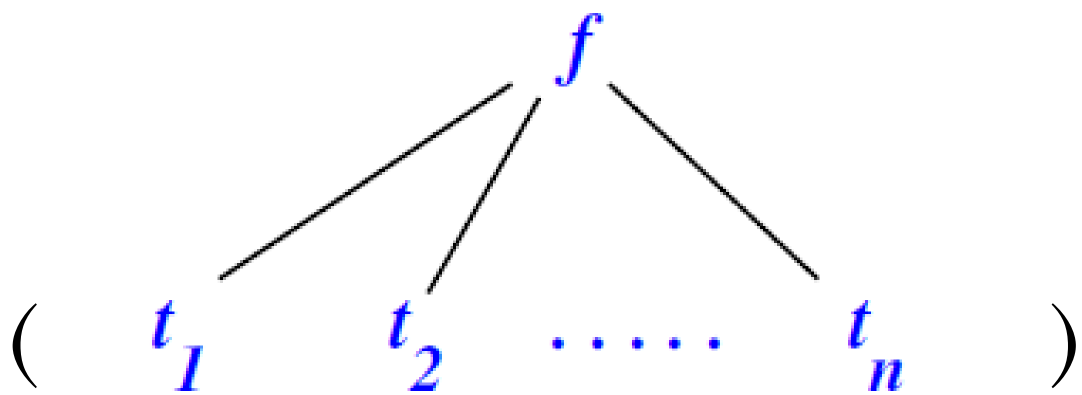
- Transitive closure with ***left recursion*** in Prolog will run into an infinite loop:

```
reach(X,Y) :-
      reach(X,Z),
      edge(Z, Y).
reach(X,Y) :-
      edge(X,Y).

 ?- reach(A,B).
  Infinite loop
```

Structures

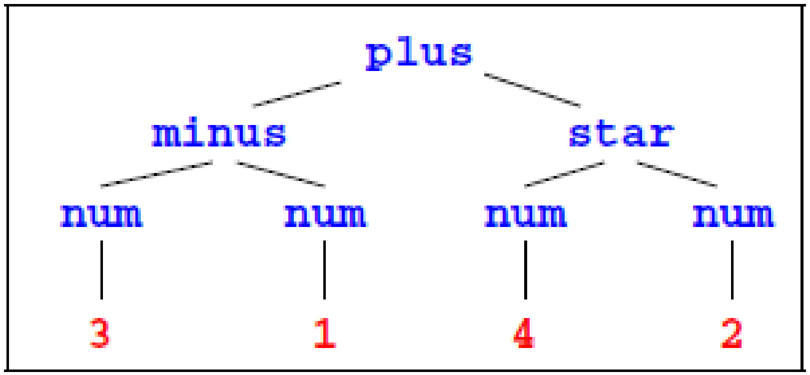
- If **f** is an identifier and t_1, t_2, …, t_n are terms, then **f**(t_1, t_2, …, t_n) is a term



- In the above, **f** is called a ***functor*** and t_is are called ***arguments***
- Structures are used to group related data items together (in some ways similar to struct in C and objects in Java)
  - Structures are used to construct trees (and, as a special case of trees, **lists**)

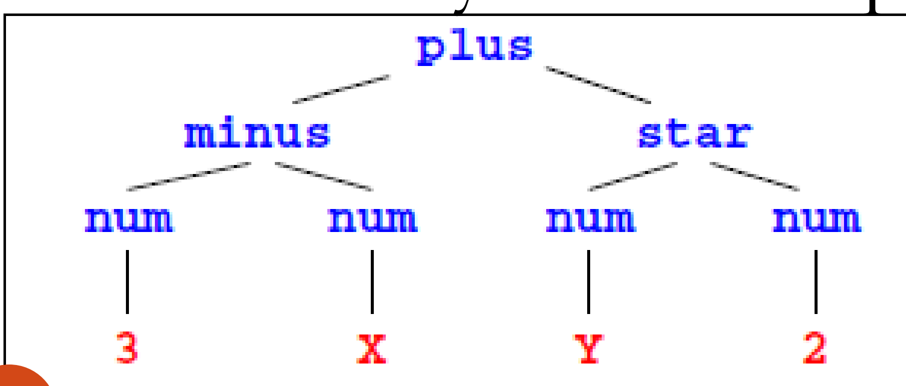
Trees

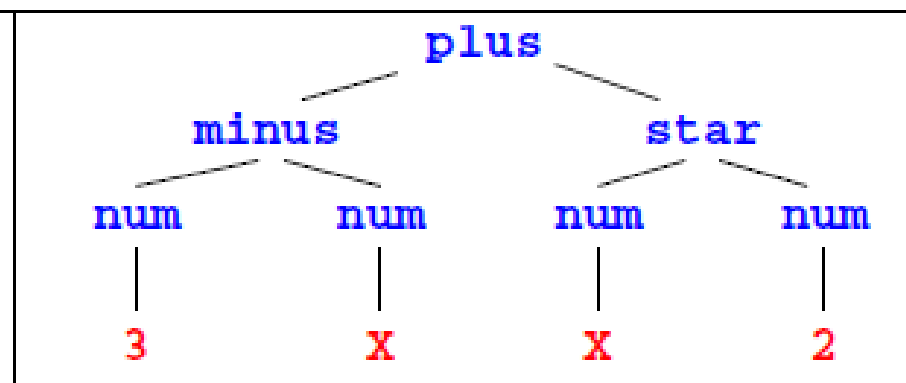
- Example: expression trees:




```
plus(minus(num(3),num(1)),star(num(4),num(2)))
```

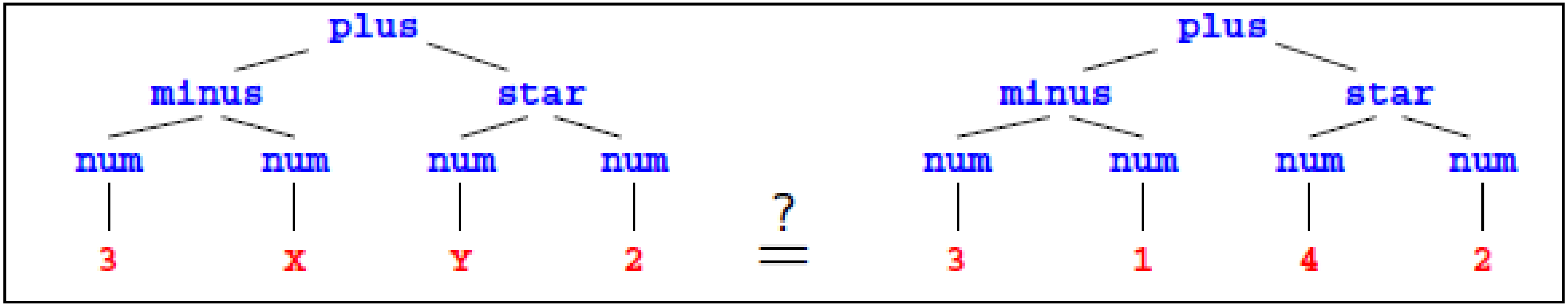
- Data structures may have variables AND the same variable may occur multiple times in a data structure





Matching

- **`t1 = t2`**: finds substitutions for variables in **`t1`** and **`t2`** that make the two terms identical
  - (We'll later introduce *unification*, a related operation that has logical semantics)



Yes, with $X = 1$, $Y = 4$.

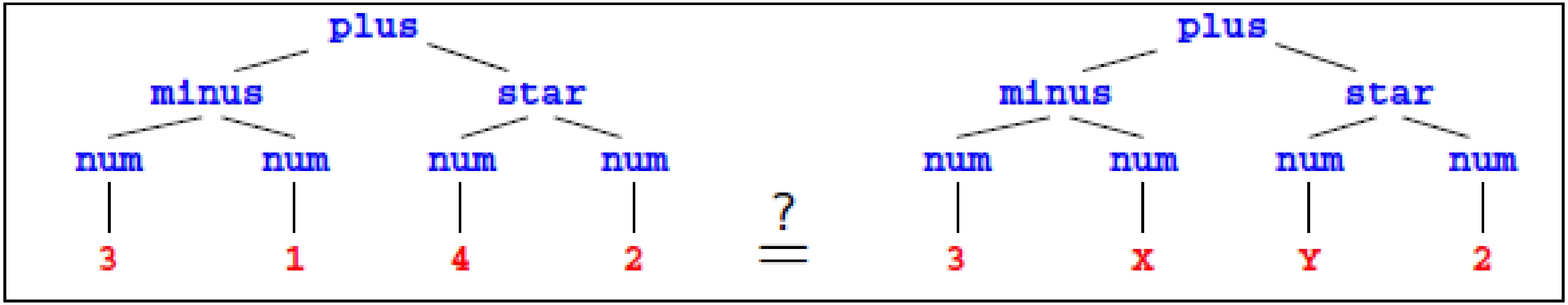
Matching

- Matching: given two terms, we can ask if they "*match*" each other
  - A constant matches with itself: **`42`** unifies with **`42`**
  - A variable matches with anything:
    - if it matches with something other than a variable, then it instantiates,
    - if it matches with a variable, then the two variables become associated.
      - **`A=35, A=B`** ➔ **`B`** becomes **`35`**
      - **`A=B, A=35`** ➔ **`B`** becomes **`35`**
  - Two structures match if they:
    - Have the same functor,
    - Have the same arity, and
    - Match recursively
      - **`foo(g(42),37)`** matches with **`foo(A,37)`**, **`foo(g(A),B)`**, etc.

Matching

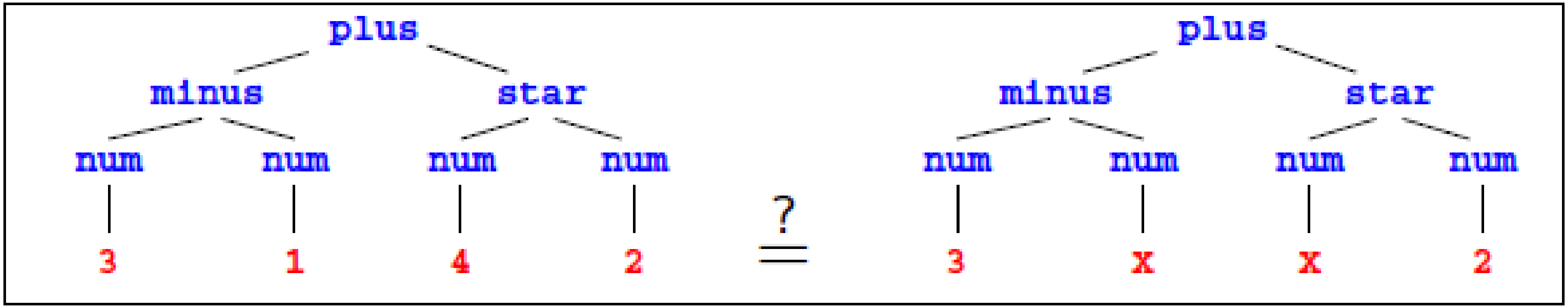
- The general Rules to decide whether two terms **`S`** and **`T`** *match* are as follows:
  - If **`S`** and **`T`** are constants, **`S=T`** if both are same object
  - If **`S`** is a variable and **`T`** is anything, **`T=S`**
  - If **`T`** is variable and **`S`** is anything, **`S=T`**
  - If **`S`** and **`T`** are structures, **`S=T`** if
    - **`S`** and **`T`** have same functor, same arity, and
    - All their corresponding arguments components have to match

Matching



Yes, with $X = 1$, $Y = 4$.

Matching



No! X cannot be 1 and 4 at the same time.

Matching

- Which of these match?
  - `A`
  - `100`
  - `func(B)`
  - `func(100)`
  - `func(C, D)`
  - `func(+(99, 1))`

Matching

- Which of these match?
  - **`A`**
  - **`100`**
  - **`func(B)`**
  - **`func(100)`**
  - **`func(C, D)`**
  - **`func(+(99, 1))`**
- **`A`** matches with **`100`**, **`func(B)`**, **`func(100)`**, **`func(C,D)`**, **`func(+(99, 1))`**.
- **`100`** matches only with **`A`**.
- **`func(B)`** matches with **`A`**, **`func(100)`**, **`func(+(99, 1))`**
- **`func(C, D)`** matches with **`A`**.
- **`func(+(99, 1))`** matches with **`A`** and **`func(B)`**.

Accessing arguments of a structure

- Matching is the predominant means for accessing structures arguments
  - Let `date('Sep', 1, 2020)` be a structure used to represent dates, with the month, day and year as the three arguments (**in that order!**)

then `date(M,D,Y) = date('Sep',1,2020).`

makes

```
M = 'Sep',   D = 1,   Y = 2020
```

  - If we want to get only the day, we can write

```
date(_, D, _) = date('Sep', 1, 2020).
```

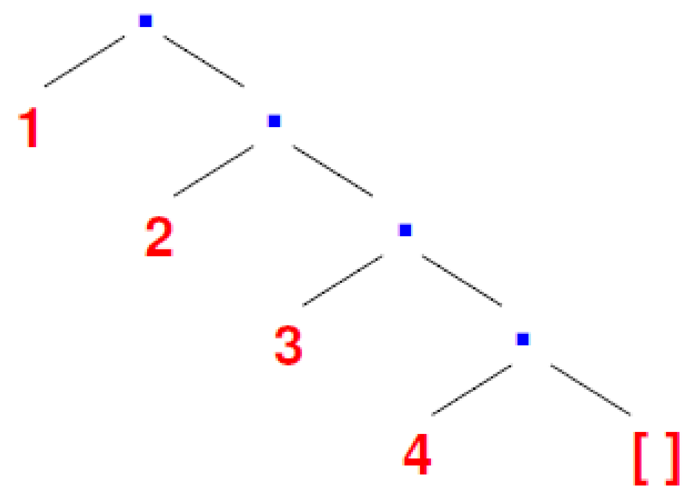
Then we only get: `D = 1`

Lists

- Prolog uses a special syntax to represent and manipulate lists:
  - **`[1,2,3,4]`**: represents the list with **`1`**, **`2`**, **`3`** and **`4`**, respectively.
  - This can also be written as **`[1|[2,3,4]]`**: a list with **`1`** as the ***head*** (i.e., first element) and **`[2,3,4]`** as its ***tail*** (i.e., the list of remaining elements).
    - If **`X = 1`** and **`Y = [2,3,4]`** then **`[X|Y]`** is same as **`[1,2,3,4]`**.
  - The empty list is represented by **`[]`** or **`nil`**
- The symbol "**`|`**" (*pipe*) and is used to separate the beginning elements of a list from its tail
  - For example: **`[1,2|[3,4]] = [1,2,3|[4]] = [1,2,3,4] = [1,2,3,4|[]] = [1|[2,3,4]] = [1|[2|[3,4]]] = [1|[2|[3|[4|[]]]]]`**

Lists

- Lists are special cases of trees (syntactic sugar, i.e., internally, they use structures)
  - For instance, the list **`[1,2,3,4]`** is represented by the following structure:



  - where the function symbol **`'.'/2`** is the list constructor:

`[1,2,3,4]` is same as **`'.'(1,'.'(2,'.'(3,'.'(4,[]))))`**

- **In XSB:**

```
?- L = '.'(1,[2]).
   L = [1,2]
```

Lists

- ***Strings***: are sequences of characters surrounded by double quotes **`"abc"`**, **`"John Smith"`**, **`"to be, or not to be"`**.
  - A string is <u>equivalent to a list of the (numeric) character codes</u>
    - <u>**`In XSB:`**</u>

```
?- X="abc".
X = [97,98,99]
```

Programming with Lists

- `member`/2 finds if a given element occurs in a list:
  - The program:

```
member(X, [X|_]).
member(X, [_|Ys]) :-
            member(X,Ys).
```

  - Example queries:

```
?- member(2,[1,2,3]).
?- member(X,[l,i,s,t]).
?- member(f(X),[f(1),g(2),f(3),h(4)]).
?- member(1,L).
```

Programming with Lists

- **append/3** concatenate two lists to form the third list:
  - The program:
    - Empty list **append L** is **L**:

```
append([], L, L).
```

    - Otherwise, break the first list up into the head **X**, and the tail **L**: if **L** append **M** is **N**, then **[X|L]** append **M** is **[X|N]** :

```
append([X|L], M, [X|N]) :-
          append(L, M, N).
```

  - Example queries:

```
?- append([1,2],[3,4],X).
?- append(X, Y, [1,2,3,4]).
?- append(X, [3,4], [1,2,3,4]).
?- append([1,2], Y, [1,2,3,4]).
```

Programming with Lists

- Is the predicate a function?
  - **No.** We are not applying arguments to get a result. Instead, we are proving that a theorem holds. Therefore, we can leave any variables unbound.

```
?- append(L, [2, 3], [1, 2, 3]).
   L = [ 1 ]
?- append([ 1 ], L, [1, 2, 3]).
   L = [2, 3]
?- append(L1, L2, [1, 2, 3]).
 L1 = [],         L2 = [1, 2, 3];
 L1 = [1],        L2 = [2, 3];
 L1 = [1, 2],     L2 = [3] ;
 L1 = [1, 2, 3],  L2 = [];
 no
```

Append example trace

```
append([],L,L).
append([X|L], M, [X|N]) :- append(L,M,N).
```

```
append([1,2],[3,4],X)?
```

Append example trace

```
append([],L,L).
append([X|L],M,[X|N]) :- append(L,M,N).
```

`append([1,2],[3,4],A)?`	`X=1,L=[2],M=[3,4],A=[X|N]`

Append example trace

```
append([],L,L).
append([X|L],M,[X|N]) :- append(L,M,N).
```

append([2],[3,4],N)?	
append([1,2],[3,4],A)?	X=1,L=[2],M=[3,4],A=[X|N]

Append example trace

```
append([],L,L).
append([X|L],M,[X|N']) :- append(L,M,N').
```

append([2],[3,4],N)?	X=2,L=[],M=[3,4],N=[2|N']
append([1,2],[3,4],A)?	X=1,L=[2],M=[3,4],A=[1|N]

Append example trace

```
append([],L,L).
append([X|L],M,[X|N']) :- append(L,M,N').
```

append([],[3,4],N')?	
append([2],[3,4],N)?	X=2,L=[],M=[3,4],N=[2|N']
append([1,2],[3,4],A)?	X=1,L=[2],M=[3,4],A=[1|N]

Append example trace

```
append([],L,L).
append([X|L],M,[X|N']) :- append(L,M,N').
```

append([],[3,4],N')?	L = [3,4], N' = L
append([2],[3,4],N)?	X=2,L=[],M=[3,4],N=[2|N']
append([1,2],[3,4],A)?	X=1,L=[2],M=[3,4],A=[1|N]

Append example trace

```
append([],L,L).
append([X|L],M,[X|N′]) :- append(L,M,N′).
```

```
A = [1|N]
N = [2|N′]
N′= L
L = [3,4]
```

Answer: **A = [1,2,3,4]**

`append([],[3,4],N′)?`	`L = [3,4], N′ = L`
`append([2],[3,4],N)?`	`X=2,L=[],M=[3,4],N=[2|N′]`
`append([1,2],[3,4],A)?`	`X=1,L=[2],M=[3,4],A=[1|N]`

Programming with Lists

- **`len`**/2 to find the length of a list (the first argument):
  - The program:

```
len([], 0).
len([_|Xs], N+1) :-
      len(Xs, N).
```

  - Example queries:

```
?- len([], X).
      X = 0
?- len([l,i,s,t], 4).
      false
?- len([l,i,s,t], X).
      X = 0+1+1+1+1
```

Arithmetic

```
?- 1+2 = 3.
     false
```

- In Predicate logic, the basis for Prolog, the only symbols that have a meaning are the predicates themselves
  - In particular, function symbols are uninterpreted: have no special meaning and can only be used to construct data structures

Arithmetic

- Meaning for arithmetic expressions is given by the built-in predicate "**`is`**":

```
?- X is 1 + 2.
        succeeds, binding X = 3.
?- 3 is 1 + 2.
        succeeds.
```

- General form: **`R is E`** where **`E`** is an expression to be evaluated and **`R`** is matched with the expression's value
- **`Y is X + 1`**, where **`X`** is a free variable, will give an error because **`X`** does not (yet) have a value, so, **`X + 1`** cannot be evaluated

The list length example revisited

- **`length`**/2 **finds** the length of a list (first argument):
  - The program:

```
length([], 0).
length([_|Xs], M):-
            length(Xs, N),
            M is N+1.
```

  - Example queries:

```
?- length([], X).
?- length([l,i,s,t], 4).
?- length([l,i,s,t], X).
      X = 4
?- length(List, 4).
      List = [_1, _2, _3, _4]
```

Conditional Evaluation

- Conditional operator: the if-then-else construct in Prolog:
  - *if A then B else C* is written as **`( A -> B ; C)`**
    - To Prolog this means: try **`A`**. If you can prove it, go on to prove **`B`** and ignore **`C`**. If **`A`** fails, however, go on to prove **`C`** ignoring **`B`**.

```
max(X,Y,Z) :-
     (  X =< Y
     -> Z = Y
     ;  Z = X
     ).
```

```
?- max(1,2,X).
X = 2.
```

Conditional Evaluation

- Consider the computation of **`n!`** (i.e. the factorial of **`n`**)

```
% factorial(+N, -F)
factorial(N, F) :- ...
```

  - **`N`** is the input parameter and **`F`** is the output parameter!
  - The body of the rule species how the output is related to the input
    - For factorial, there are two cases: **`N <= 0`** and **`N > 0`**
      - if **`N <= 0`**, then **`F = 1`**
      - if **`N > 0`**, then **`F = N * factorial(N - 1)`**

```
factorial(N, F) :-
   (N > 0
    -> N1 is N-1,
          factorial(N1, F1),
          F is N*F1
    ; F = 1
   ).
```

```
?- factorial(12,X).
X = 479001600
```

Imperative features

- Other imperative features: we can think of prolog rules as imperative programs w/ backtracking

```
program :-
        member(X, [1, 2, 3, 4]),
        write(X),
        nl,
        fail;
        true.
?- program. % prints all solutions
```

- **fail**: always fails, causes backtracking
- **!** is the cut operator: prevents other rules from matching (we will see it later)

Arithmetic Operators

- Integer/Floating Point operators: +, -, *, /
  - Automatic detection of Integer/Floating Point
- Integer operators: mod, // (integer division)
- Comparison operators: <, >, =<, >=,

Expr1 =:= *Expr2* (succeeds if expression *Expr1* evaluates to a number equal to *Expr2*),

Expr1 =\= *Expr2* (succeeds if expression *Expr1* evaluates to a number non-equal to *Expr2*)

Programming with Lists

- Quicksort:

```
quicksort([], []).
quicksort([X0|Xs], Ys) :-
  partition(X0, Xs, Ls, Gs),
  quicksort(Ls, Ys1),
  quicksort(Gs, Ys2),
  append(Ys1, [X0|Ys2], Ys).
partition(Pivot,[],[],[]).
partition(Pivot,[X|Xs],[X|Ys],Zs) :-
   X =< Pivot,
   partition(Pivot,Xs,Ys,Zs).
partition(Pivot,[X|Xs],Ys,[X|Zs]) :-
   X > Pivot,
   partition(Pivot,Xs,Ys,Zs).
```

Programming with Lists

- Quicksort:

```
quicksort([], []).
quicksort([X0|Xs], Ys) :-
  partition(X0, Xs, Ls, Gs),
  quicksort(Ls, Ys1),
  quicksort(Gs, Ys2),
  append(Ys1, [X0|Ys2], Ys).
partition(Pivot,[],[],[]).
partition(Pivot,[X|Xs],[X|Ys],Zs) :-
   X =< Pivot,
   !, % cut
   partition(Pivot,Xs,Ys,Zs).
partition(Pivot,[X|Xs],Ys,[X|Zs]) :-
   partition(Pivot,Xs,Ys,Zs).
```

Programming with Lists

- We want to define **delete**/3, to remove an element from a list:
  - **delete(2,[1,2,3],X)** should succeed with **X=[1,3]**
  - **delete(2, [2,1,2], X)** should succeed with **X=[1,2]; X =[2,1]; fail**
  - **delete(X,[1,2,3],[1,3])** should succeed with **X=2**
  - **delete(2, X, [1,3])** should succeed with **X=[2,1,3]; X =[1,2,3]; X=[1,3,2]; fail**

Programming with Lists

- **Algorithm:**
  - When **`X`** is selected from **`[X|Ys]`**, **`Ys`** results as the rest of the list
  - When **`X`** is selected from the tail of **`[H|Ys]`**, **`[H|Zs]`** results, where **`Zs`** is the result of taking **`X`** out of **`Ys`**

Programming with Lists

- The program:

```
delete(X,[],_) :- fail.% not needed
delete(X, [X|Ys], Ys).
delete(X, [Y|Ys], [Y|Zs]) :-
      delete(X, Ys, Zs).
```

- Example queries:

```
?- delete(s, [l,i,s,t], Z).
X = [l, i, t]
?- delete(X, [l,i,s,t], Z).
?- delete(s, Y, [l,i,t]).
?- delete(X, Y, [l,i,s,t]).
```

Permutations

- Define **`permute`**/2, to find a permutation of a given list.
  - E.g. **`permute([1,2,3], X)`** should return **`X=[1,2,3]`** and upon backtracking, **`X=[1,3,2]`**, **`X=[2,1,3]`**, **`X=[2,3,1]`**, **`X=[3,1,2]`**, and **`X=[3,2,1]`**.
  - Hint: What is the relationship between the permutations of **`[1,2,3]`** and the permutations of **`[2,3]`**?

permute([2,3],Y)	permute([1,2,3],Y)
[2,3]	[1,2,3]
	[2,1,3]
	[2,3,1]
[3,2]	[1,3,2]
	[3,1,2]
	[3,2,1]

Programming with Lists

- The program:

```
permute([], []).
permute([X|Xs], Ys) :-
      permute(Xs, Zs),
      delete(X, Ys, Zs).
```

- Example query:

```
?- permute([1,2,3], X).
X = [1,2,3];
X = [2,1,3];
X = [2,3,1];
X = [1,3,2] …
```

The Issue of Efficiency

- Define a predicate, **`rev`**/2 that finds the **reverse** of a given list
  - E.g. **`rev([1,2,3],X)`** should succeed with **`X=[3,2,1]`**
  - Hint: what is the relationship between the reverse of **`[1,2,3]`** and the reverse of **`[2,3]`**? Answer: **`append([3,2],[1],[3,2,1])`**

```
rev([], []).
rev([X|Xs], Ys) :- rev(Xs, Zs),
    append(Zs, [X], Ys).
```

- How long does it take to evaluate **`rev([1,2,…,n],X)`**?
  - **`T(n) = T(n - 1)+`** time to add **`1`** element to the end of an **`n - 1`** element list **`= T(n - 1) + n - 1 = T(n - 2) + n - 2 + n - 1 = ...`**

→ $T(n) = O(n^2)$ (quadratic)

Making rev/2 faster

- Keep an accumulator: stack all elements seen so far
  - i.e. a list, with elements seen so far in **reverse** order
- The program:

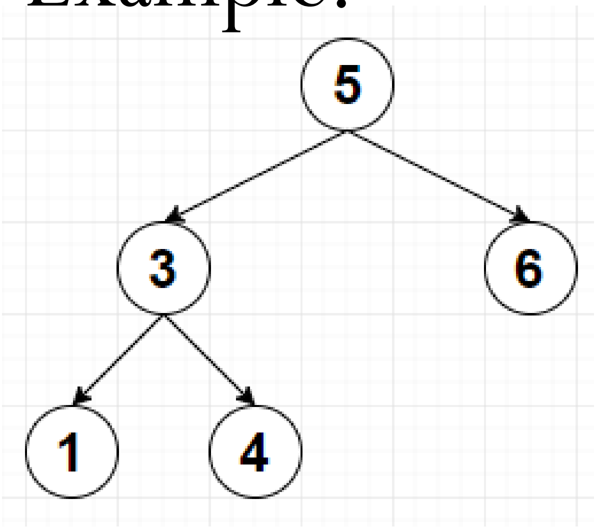
```
rev(L1, L2) :- rev_h(L1, [], L2).
rev_h([X|Xs], AccBefore, Out):-
     rev_h(Xs, [X|AccBefore], Out).
rev_h([], Acc, Acc). % Base case
```

- Example query:

```
   ?- rev([1,2,3], X).
         will call rev_h([1,2,3], [], X)
         which calls rev_h([2,3], [1], X)
         which calls rev_h([3], [2,1], X)
         which calls rev_h([], [3,2,1], X)
         which returns X = [3,2,1]
```

Tree Traversal

- Assume you have a binary tree, represented by
  - **`node/3`** facts for internal nodes: **`node(a,b,c)`** means that **`a`** has **`b`** and **`c`** as children
  - **`leaf/1`** facts: for leaves: **`leaf(a)`** means that **`a`** is a leaf
  - Example:

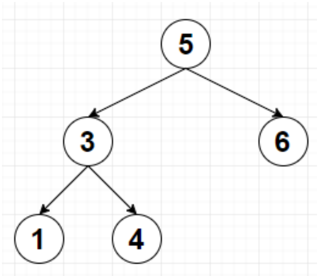



```
node(5, 3, 6).
node(3, 1, 4).
leaf(1).
leaf(4).
leaf(6).
```

Tree Traversal

- Write a predicate **preorder**/2 that traverses the tree (starting from a given node) and returns the list of nodes in pre-order

```
preorder(Root, [Root]) :-
    leaf(Root).
preorder(Root, [Root|L]) :-
    node(Root, Child1, Child2),
    preorder(Child1, L1),
    preorder(Child2, L2),
    append(L1, L2, L).
?- preorder(5, L).
L = [5, 3, 1, 4, 6]
```



- The program takes **O(n²)** time to traverse a tree with **n** nodes. **How to append 2 lists in shorter time?**

Difference Lists

- The lists in Prolog are singly-linked; hence we can access the first element in constant time, **but** need to scan the entire list to get the **last** element
- However, unlike functional languages like Lisp or SML, we can use **variables** in data structures:
  - We can exploit this to make lists "***open tailed***" (also called ***difference lists*** in Prolog): end the list with a variable tail and pass that variable, so we can add elements at the end of the list

Difference Lists

- When `X=[1,2,3|Y]`, `X` is a list with `1`, `2`, `3` as its first three elements, followed by `Y`
  - Now if `Y=[4|Z]` then `X=[1,2,3,4|Z]`
    - We can now think of `Z` as "pointing to" the end of `X`
  - **We can now add an element to the end of `X` in constant time!!**
    - And continue adding more elements, e.g. `Z=[5|W]`

Difference Lists: Conventions

- A ***difference list*** is represented by two variables: one referring to the entire list, and another to its (uninstantiated) tail
  - e.g. **`X = [1,2,3|Z], Z`**
- Most Prolog programmers use the notation **`List-Tail`** to denote a list **`List`** with tail **`Tail`**.
  - e.g. **`X-Z`**
  - Note that "**`-`**" is used as a <u>data structure infix symbol</u> (not used for arithmetic here)

Difference Lists

- Append 2 open ended lists:

```
dappend(X,T, Y,T2, L,T3) :-
    T = Y,
    T2 = T3,
    L = X.
?- dappend([1,2,3|T],T, [4,5,6|T2],T2, L,T3).
L = [1,2,3,4,5,6|T3]
```

- Simplified version:

```
dappend(X,T, T,T2, X,T2).
```

- More simplified **notation** (with "-"):

```
dappend(X-T, T-T2, X-T2).
?- dappend([1,2,3|T]-T, [4,5,6|T2]-T2, L-T3).
L = [1,2,3,4,5,6|T2]
```

Difference Lists

- Add an element at the end of a list:

```
add(L-T, X, L2-T2) :-
    T = [X|T2],
    L = L2.
?- add([1,2,3|T]-T, 4, L-T2).
L = [1,2,3,4|T2]
```

- We can simplify it as:

```
add(L-T, X, L-T2) :-
    T = [X|T2].
```

- This can be simplified more like:

```
add(L-[X|T2], X, L-T2).
```

- Alternative using **dappend**:

```
add(L-T, X, L-T2) :-
    dappend(L-T,[X|T2]-T2,L-T2).
```

Difference Lists

- Check if a list is a palindrome:

```
palindrome(X) :-
     palindromeHelp(X-[]).
palindromeHelp(A-A). % an empty list
palindromeHelp([_|A]-A).%1-element list
palindromeHelp([C|A]-D) :-
     palindromeHelp(A-B),
     B=[C|D].
?- palindrome([1,2,2,1]).
yes
?- palindrome([1,2,3,2,1]).
yes
?- palindrome([1,2,3,4,5]).
no
```

Tree Traversal, Revisited

```
preorder1(Node, List, Tail) :-
   node(Node, Child1, Child2),
   List = [Node|List1],
   preorder1(Child1, List1, Tail1),
   preorder1(Child2, Tail1, Tail).
preorder1(Node, [Node|Tail], Tail) :-
   leaf(Node).
preorder(Node, List) :-
    preorder1(Node, List, []).
```

- The program takes **`O(n)`** time to traverse a tree with **`n`** nodes

Difference Lists: Conventions

- The preorder traversal program may be rewritten as:

```
preorder1(Node, [Node|L]-T) :-
   node(Node, Child1, Child2),
   preorder1(Child1, L-T1),
   preorder1(Child2, T1-T).
preorder1(Node, [Node|T]-T).
```

Difference Lists: Conventions

- The inorder traversal program:

```
inorder1(Node, L-T) :-
   node(Node, Child1, Child2),
   inorder1(Child1, L-T1),
   T1 = [Node|T2],
   inorder1(Child2, T2-T).
inorder1(Node, [Node|T]-T).
inorder(Node,L):-
   inorder1(Node, L-[]).
```

Difference Lists: Conventions

- The postorder traversal program:

```
postorder1(Node, L-T) :-
    node(Node, Child1, Child2),
    postorder1(Child1, L-T1),
    postorder1(Child2, T1-T2),
    T2 = [Node|T].
postorder1(Node, [Node|T]-T).
postorder(Node,L):-
    postorder1(Node, L-[]).
```

Graphs in Prolog

- There are several ways to represent graphs in Prolog:
  - represent each edge separately as one clause (fact):

    ```
    edge(a,b).
    edge(b,c).
    ```

    - isolated nodes cannot be represented, unless we have also **`node`**/1 facts
  - the whole graph as one data object: as a pair of two sets (nodes and edges): **`graph([a,b,c,d,f,g], [e(a,b), e(b,c),e(b,f)])`**
    - list of arcs: **`[a-b, b-c, b-f]`**
  - adjacency-list: **`[n(a,[b]), n(b,[c,f]), n(d,[])]`**

Graphs in Prolog

- Path from one node to another one:
  - A predicate **`path(+G,+A,+B,-P)`** to find an acyclic path **`P`** from node **`A`** to node **`B`** in the graph **`G`**
  - The predicate should return **all paths via backtracking**
  - We will solve it using the graph as a data object, like in **`graph([a,b,c,d,f,g], [e(a,b), e(b,c),e(b,f)]`**

Graphs in Prolog

- **adjacent** for directed edges:

```
adjacent(X,Y,graph(_,Es)) :-
    member(e(X,Y),Es).
```

- **adjacent** for undirected edges (ie. no distinction between the two vertices associated with each edge):

```
adjacent(X,Y,graph(_,Es)) :-
    member(e(X,Y),Es).
adjacent(X,Y,graph(_,Es)) :-
    member(e(Y,X),Es).
```

Graphs in Prolog

- Path from one node to another one:

```
path(G,A,B,P) :-
    pathHelper(G,A,[B],P).
% Base case
pathHelper(_,A,[A|P1],[A|P1]).
pathHelper(G,A,[Y|P1],P) :-
    adjacent(X,Y,G),
    \+ member(X,[Y|P1]),
    pathHelper(G,A,[X,Y|P1],P).
```

Graphs in Prolog

- Cycle from a given node in a directed graph:
  - a predicate **`cycle(G,A,Cycle)`** to find a closed path (cycle) **`Cycle`** starting at a given node **`A`** in the graph **`G`**
  - The predicate should return all cycles via backtracking

```
cycle(G,A,Cycle) :-
    adjacent(A,B,G),
    path(G,B,A,P1),
    Cycle = [A|P1].
```

Complete program in XSB

```
:- import member/2 from basics.
adjacent(X,Y,graph(_,Es)) :-
      member(e(X,Y),Es).
path(G,A,B,P) :-
      pathHelper(G,A,[B],P).
pathHelper(_,A,[A|P1],[A|P1]).
pathHelper(G,A,[Y|P1],P) :-
      adjacent(X,Y,G),
      \+ member(X,[Y|P1]),
      pathHelper(G,A,[X,Y|P1],P).
cycle(G,A,Cycle) :-
      adjacent(A,B,G),
      path(G,B,A,P),
      Cycle = [A|P].
?- Graph = graph([a,b,c,d,f,g],
              [e(a,b), e(b,c),e(c,a),e(a,e),e(e,a)]),
      cycle(Graph,a,Cycle),
      writeln(Cycle),
      fail; true.
```

Aggregates in XSB

- **setof(Template,Goal,Set)**: **Set** is the set of all instances of **Template** such that **Goal** is provable
- **findall(Template,Goal,List)** is similar to predicate **bagof**/3, except that variables in **Goal** that do not occur in **Template** are treated as existential, and alternative lists are not returned for different bindings of such variables
- **bagof(Template,Goal,Bag)** has the same semantics as **setof**/3 except that the third argument returns an unsorted list that may contain duplicates. **X^Goal** will not bind **X**
- **tfindall(Template,Goal,List)** is similar to predicate **findall**/3, but the **Goal** must be a call to a single tabled predicate

Aggregates in XSB

```
p(1,1).
p(1,2).
p(2,1).
?- setof(Y,p(X,Y),L).
L=[1,2]
?- findall(Y,p(X,Y),L).
L=[1,2,1]
?- bagof(Y,p(X,Y),L).
X=1, L=[1,2] ;
X=2, L=[1] ;
fail
```

XSB Prolog

- Negation: **`\+`** is negation-as-failure
- Another negation called **`tnot`** *(TABLING = memoization)*
  - Use: … `:- …, tnot(foobar(X)).`
  - All variables under the scope of **`tnot`** must also occur to the left of that scope in the body of the rule in other positive relations:
  - Ok: `…:-p(X,Y),tnot(foobar(X,Y)),…`
  - Not ok: `…:-p(X,Z),tnot(foobar(X,Y)), …`
- XSB also supports Datalog:

```
:- auto_table.
```

at the top of the program file

XSB Prolog

- Read/write from and to files:
  - Edinburgh style:

```
?- tell('a.txt'),
   write('Hello, World!'), told.

?- see('a.txt'), read(X), seen.
```

XSB Prolog

- Read/write from and to files:
  - ISO style:

```
?- open('a.txt', write, X),
   write(X,'Hello, World!'),
   close(X).
```

Cut (logic programming)

- Cut (`!` in Prolog) is a goal which always succeeds, but **cannot be backtracked past**:

```
max(X,Y,Y)  :-   X =< Y,  !.
max(X,_,X).
```

  - **cut says "stop looking for alternatives"**
  - **no check is needed in the second rule anymore because if we got there, then `X =< Y` must have failed, so `X > Y` must be true.**
  - **Red cut: if someone deletes !, then the rule is incorrect - above**
  - **Green cut: if someone deletes !, then the rule is correct**

```
max(X,Y,Y)  :-   X =< Y,  !.
max(X,Y,X)  :-   X > Y.
```

    - by explicitly writing `X > Y`, it guarantees that the second rule will always work even if the first one is removed by accident or changed (cut is deleted)

Cut (logic programming)

- No backtracking pass the guard, but ok after:

```
p(a). p(b).
q(a). q(b). q(c).

?- p(X),!.
X=a ;
no

?- p(X),!,q(Y).
X=a, Y=a ;
X=a, Y=b ;
X=a, Y=c ;
no
```

Testing types

- **`atom(X)`**

  Tests whether **X** is bound to a symbolic atom

  ```
  ?- atom(a).
  yes
  ?- atom(3).
  no
  ```

- **`integer(X)`**

  Tests whether **X** is bound to an integer

- **`real(X)`**

  Tests whether **X** is bound to a real number

Testing for variables

- **`is_list(L)`**
  Tests whether **`L`** is bound to a list
- **`ground(G)`**
  Tests whether **`G`** has unbound logical variables
- **`var(X)`**
  Tests whether **`X`** is bound to a Prolog variable

Control / Meta-predicates

- **`call(P)`**

  Force **`P`** to be a goal; succeed if **`P`** does, else fail

- **`clause(H,B)`**

  Retrieves clauses from memory whose head matches **`H`** and body matches **`B`**. **`H`** must be sufficiently instantiated to determine the main predicate of the head

- **`copy_term(P,NewP)`**

  Creates a new copy of the first parameter (with new variables)

  - It is used in iteration through non-ground clauses, so that the original calls are not bound to values

Control / Meta-predicates

- Write a Prolog relation

`map(BinaryRelation,InputList, OutputList)`

which applies a binary relation on each of the elements of the list **`InputList`** as the first argument and collects the second argument in the result list.

  - Example:

`?- map(inc1(X,Y),[5,6],R).` returns **`R=[6,7]`**

where **`inc1(X,Y)`** was defined as:

```
inc1(X,Y) :-
    Y is X+1.
```

Control / Meta-predicates

```
map(_BinaryCall,[],[]).
map(BinaryCall,[X|T],[Y|T2]) :-
     copy_term(BinaryCall, BinaryCall2),
     BinaryCall2 =.. [_F,X,Y],
     call(BinaryCall2),
     map(BinaryCall, T, T2).
incl(X,Y) :-
     Y is X+1.
?- map(incl(X,Y), [5,6], R).
R = [6,7]
```

Control / Meta-predicates

```
square(X,Y) :-
    Y is X*X.
?- map(square(E, E2), [2,3,1], R).
R = [4,9,1];
no
```

Control / Meta-predicates

- Use the relation **`map`** to implement a relation **`pairAll(E,L,L2)`** which pairs the element **`E`** with each element of the list **`L`** to obtain **`L2`**. Examples:

`?- pairAll(1,[2,3,1], L2).`

returns **`L2=[[1,2],[1,3],[1,1]]`**

`?- pairAll(1,[], L2).`

returns **`L2=[]`**.

Control / Meta-predicates

```
pair(E2, (_E1,E2)).

pairAll(E,L,L2):-
    map(pair(E2, (E,E2)), L, L2).

?- pairAll(1, [2,3,1], R).
R = [(1,2),(1,3),(1,1)]
```

Assert and retract

- **`asserta(C)`**

  Assert clause **C** into database above other clauses with the same predicate.

- **`assertz(C), assert(C)`**

  Assert clause **C** into database below other clauses with the same predicate.

- **`retract(C)`**

  Retract **C** from the database. **C** must be sufficiently instantiated to determine the predicate.

Prolog terms

- **`functor(E,F,N)`**
  `E` must be bound to a functor expression of the form '**`f(...)`**'. **`F`** will be bound to '**`f`**', and **`N`** will be bound to the number of arguments that **`f`** has.
- **`arg(N,E,A)`**
  `E` must be bound to a functor expression, **`N`** is a whole number, and **`A`** will be bound to the **`N`**th argument of **`E`**

Prolog terms and clauses

- `=..`

  converts between term and list. For example,

```
?- parent(a,X) =.. L.
L = [parent, a, _X001]

?- [1] =.. X.
X = [.,1,[]]
```