# Prolog

CSE 595 – Semantic Web

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http://www3.cs.stonybrook.edu/~pfodor/courses/cse595.html

## Relations/Predicates

- Predicates are building-blocks in predicate calculus: p(a<sub>1</sub>, a<sub>2</sub>, ..., a<sub>k</sub>)
  - parent (X, Y): X is a parent of Y.

```
parent(pam, bob). parent(bob, ann).
parent(tom, bob). parent(bob, pat).
parent(tom, liz). parent(pat, jim).
```

• male (X) : X is a male.

```
male(tom).
male(bob).
```

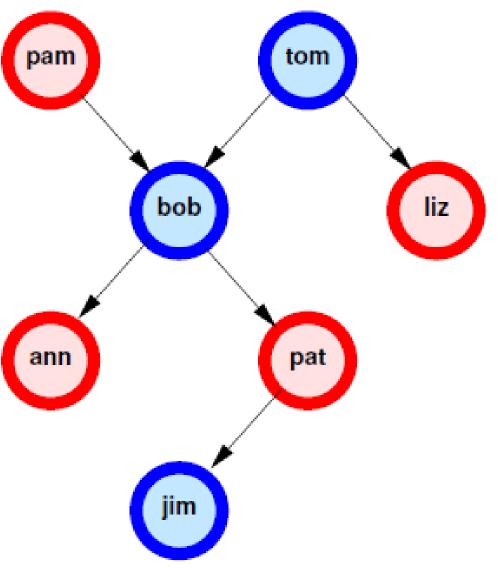
male(jim).

We attach meaning to them, but within the logical system they are simply structural building blocks, with no meaning beyond that provided by explicitly-stated interrelationships

• female (X): X is a female.
 female (pam).
 female (pat).
 female (ann).

female(liz).

```
parent(pam, bob).
parent(tom, bob).
parent(tom, liz).
parent(bob, ann).
parent(bob, pat).
parent(pat, jim).
female (pam).
female (pat).
female (ann).
female(liz).
male(tom).
male (bob).
```



male(jim).

- Rules:
  - mother (X, Y): X is the mother of Y.
    - -In First Order Logic (FOL or predicate calculus):

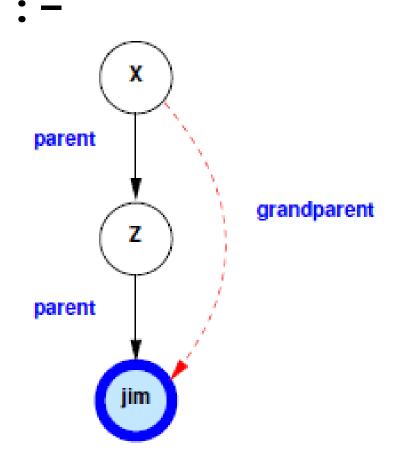
```
\forall X, Y \text{ (parent}(X,Y) \land \text{ female}(X) => \text{mother}(X,Y))
-In Prolog:
```

```
mother(X,Y) :-
   parent(X,Y),
   female(X).
```

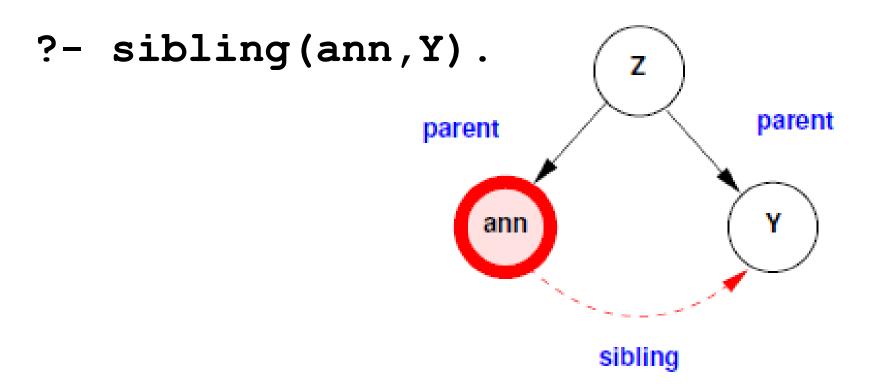
- all variables are universally quantified outside the rule
- "," means and (conjunction), ":-" means if (implication) and ";" means or (disjunction).

• More Relations:

grandparent(X,Y)
 parent(X,Z),
 parent(Z,Y).



```
sibling(X,Y) :- parent(Z,X),
   parent(Z,Y), X \= Y.
```



• More Relations:

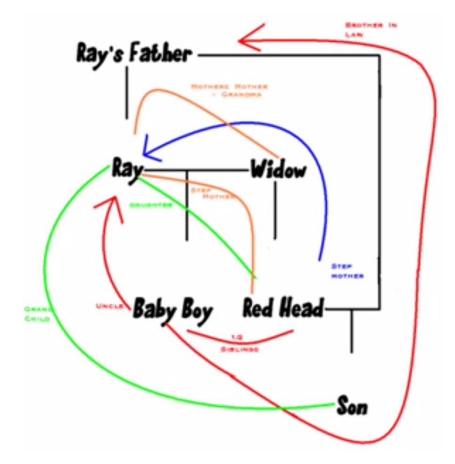
```
cousin(X,Y) :- ...
greatgrandparent(X,Y) :- ...
greatgreatgrandparent(X,Y) :- ...
```

#### Recursion

```
ancestor(X,Y) :-
  parent(X,Y).
ancestor(X,Y) :-
  parent(X,Z),
  ancestor (Z,Y).
?- ancestor(X,jim).
?- ancestor(pam,X).
?- ancestor(X,Y).
```

• How to implement "I'm My Own Grandpa"?

https://www.youtube.com/watch?v=eYlJH81dSiw

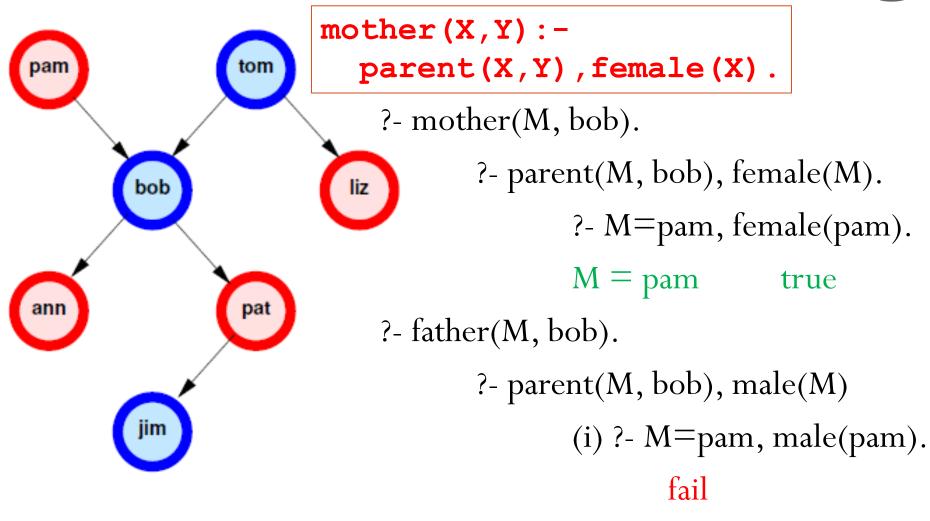


#### Recursion

• What about:

```
ancestor(X,Y) :-
  ancestor(X,Z),
  parent(Z,Y).
ancestor(X,Y) :-
  parent(X,Y).
?- ancestor(X,Y).
  INFINITE LOOP
```

# Computations in Prolog



(ii) ?- M=tom, male(tom).

# Prolog Execution

- Call: Call a predicate (invocation)
- •Exit: Return an answer to the caller
- Fail: Return to caller with no answer
- Redo: Try next path to find an answer

# The XSB Prolog System

- <a href="http://xsb.sourceforge.net">http://xsb.sourceforge.net</a>
  - Developed at Stony Brook by David Warren and many contributors
- Overview of Installation:
  - Unzip/untar; this will create a subdirectory XSB
  - Windows: you are done
  - Linux:

```
cd XSB/build
```

- ./configure
- ./makexsb

That's it!

• Cygwin under Windows: same as in Linux

#### Use of XSB

• Put your ruleset and data in a file with extension .P (or .pl)
p(X) :- q(X,\_).
q(1,a).
q(2,a).
q(b,c).

- Don't forget: all rules and facts end with a period (.)
- Comments: /\*...\*/ or %.... (% acts like // in Java/C++)
- Type
   .../XSB/bin/xsb (Linux/Cygwin)
   ...\XSB\config\x86-pc-windows\bin\xsb (Windows)
   where ... is the path to the directory where you downloaded XSB
- You will see a prompt
  - **?**-

and are now ready to type queries

#### Use of XSB

• Loading your program, myprog.P or myprog.pl

```
?- [myprog].
```

XSB will compile myprog.P (if necessary) and load it. Now you can type further queries, e.g.

```
P = p(X).
P = p(1).
```

- Some Useful Built-ins:
  - write (X) write whatever X is bound to
  - writeln (X) write then put newline
  - **n1** output newline
  - Equality: =
  - Inequality: \=

```
http://xsb.sourceforge.net/manual1/index.html (Volume 1)
http://xsb.sourceforge.net/manual2/index.html (Volume 2)
```

### Use of XSB

- Some Useful Tricks:
  - XSB returns only the first answer to the query
  - To get the next, type ; <Return>. For instance:
    | ?- q(X).
    X = 2;
    X = 4
    yes
  - Usually, typing the ;'s is tedious. To do this programmatically, use this idiom:

```
|?-(q(_X), write('X='), writeln(_X), fail; true).
```

\_X here tells XSB to not print its own answers, since we are printing them by ourselves. (XSB won't print answers for variables that are prefixed with a \_.)

- In logic, most statements can be written many ways
  - That's great for people but a nuisance for computers.
  - It turns out that if you make certain <u>restrictions</u> on the <u>format of statements</u> you can prove theorems mechanically
    - Most common restriction is to have a single conclusion implied by a conjunction of premises (i.e., *Horn clauses*)
      - Horn clauses are named for the logician Alfred Horn, who first pointed out their significance in 1951
    - That's what logic programming systems do!

# Syntax of Prolog Programs

- A *Prolog* program is a sequence of clauses
- Each *clause* (sometimes called a *rule* or *Horn rule*) is of the form:

```
Head :- Body.
```

- •**Head** is one term
- •Body is a comma-separated list of terms
- A clause with an empty body is called a *fact*

- Operators:
  - conjunction, disjunction, negation, implication
- Universal and existential quantifiers
- Statements
  - sometimes true, sometimes false, sometimes unknown
  - axioms assumed true
  - theorems provably true
  - goals things we'd like to prove true

- A *term* can be a *constant*, *variable*, or *structure* (consisting of a *functor* and a parenthesized list of arguments)
- A *constant* is either an *atom* or a *number* 
  - An *atom* is either what looks like an identifier <u>beginning</u> with a <u>lowercase letter</u>, or a <u>single quoted string</u>
  - A *number* looks like an integer or real from some more ordinary language
- A *variable* looks like an identifier <u>beginning with an upper-case</u> <u>letter</u>
- There are NO declarations (vars, terms, or predicates)
  - All types are discovered implicitly

- The Prolog interpreter has a collection of facts and rules in its DATABASE
  - Facts (i.e., clauses with empty bodies):

```
raining(ny). raining(seattle).
```

- Facts are axioms (things the interpreter assumes to be true)
  - Prolog provides an automatic way to deduce true results from facts and rules
- A rule (i.e., a clause with both sides):

```
wet(X) :- raining(X).
```

The meaning of a rule is that the conjunction of the structures in the body implies the head.

Note: Single-assignment variables: X must have the same value on both sides.

• Query or goal (i.e., a clause with an empty head):

```
?- wet(X).
```

- So, rules are theorems that allow the interpreter to infer things
- To be interesting, <u>rules generally contain variables</u>
   employed(X): employs(Y,X).

can be read as:

"for all X, X is employed if there exists a Y such that Y employs X"

- Note the direction of the implication
- Also, the example does NOT say that X is employed ONLY IF there is a Y that employs X
  - there can be other ways for people to be employed
    - like, we know that someone is employed, but we don't know who is the employer or maybe they are self employed:

employed (bill).

- The scope of a variable is the clause in which it appears:
  - Variables whose first appearance is on the left hand side of the clause (i.e., the <u>head</u>) have implicit <u>universal</u> quantifiers
    - For example, we infer for all possible X that they are employed
       employed(X): employs(Y,X).
  - Variables whose <u>first appearance is in the body</u> of the clause have implicit <u>existential</u> quantifiers <u>in that body</u>
    - For example, there exists some Y that employs X
    - Note that these variables are also universally quantified outside the rule (by logical equivalences)

```
grandmother(A, C) :-
     mother (A, B),
     mother (B, C).
  can be read as:
"for all A, C [A is the grandmother of C if there exists a B
such that A is the mother of B and B is the mother of C]"
• We probably want another rule that says:
grandmother(A, C) :-
     mother (A, B),
```

father (B, C).

#### Recursion

- Transitive closure:
  - Example: a graph declared with facts (true statements)

```
edge (1,2).
```

edge (2,3).

edge (2,4).

1) if there's an **edge** from **X** to **Y**, we can **reach Y** from **X**: **reach (X,Y) :- edge (X,Y)**.

2) if there's an **edge** from **X** to **Z**, and we can **reach Y** from

**Z**, then we can **reach Y** from **X**:

```
reach(X,Y):-
    edge(X,Z),
    reach(Z, Y).
```

## Prolog Programs

- We will now explore Prolog programs in more detail:
  - Syntax of Prolog Programs
    - *Terms* can be:
      - Atomic data
      - Variables
      - Structures

## **Atomic Data**

- Numeric constants: integers, floating point numbers (e.g. 1024, -42, 3.1415, 6.023e23,...)
- Atoms:
  - Identifiers: sequence of letters, digits, underscore, <u>beginning with a lower case letter</u> (e.g. **paul**, **r2d2**, **one\_element**).
  - •Strings of characters enclosed in <u>single quotes</u> (e.g. 'Stony Brook')

- Variables are denoted by identifiers <u>beginning with an</u>

  <u>Uppercase letter or underscore</u> (e.g. X, Index, \_param).
- These are Single-Assignment Logical variables:
  - Variables can be assigned only once
  - Different occurrences of the same variable in a clause denote the same data
- Variables are implicitly declared upon first use
  - Variables are not typed
    - All types are discovered implicitly (no declarations in LP)
  - If the variable does not start with underscore, it is assumed that it appears multiple times in the rule.
    - If is does not appear multiple times, then a warning is produced: "Singleton variable"
    - You can use variables preceded with underscore to eliminate this warning

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- Anonymous variables (also called Don't care variables):
   variables beginning with "\_"
  - Underscore, by itself (i.e., \_\_), represents a variable
    - Each occurrence of \_\_corresponds to a different variable; even within a clause, \_\_does not stand for one and the same object.
  - A variable with a name beginning with "\_", but has more characters. E.g.: \_radius, \_Size
    - we want to give it a descriptive name
    - sometimes it is used to create relationships within a clause (and must therefore be used more than once): a warning is produced: "Singleton-marked variable appears more than once"

- Warnings are used to identify bugs (most because of copy-paste errors)
  - Instead of declarations and type checking
  - Fix all the warnings in a program, so you know that you don't miss any logical error

• Variables can be assigned only once, but that value can be further refined:

• The order also does not matter:

?- 
$$Z=2$$
,  
 $X=f(Y)$ ,  
 $Y=g(Z)$ .  
 $X = f(g(2))$ ,  $Y=g(2)$ ,  $Z=2$ 

• Even infinite structures:

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## Logic Programming Queries

- To run a Prolog program, one asks the interpreter a question
  - This is done by asking a query which the interpreter tries to prove:
    - If it can, it says **yes**
    - If it can't, it says **no**
    - If your query contained variables, the interpreter prints the values it had to give them to make the query true

## Meaning of Logic Programs

- **Declarative Meaning:** What are the *logical* consequences of a program?
- **Procedural Meaning:** For what values of the variables in the query can I *prove* the query?
  - The user gives the system a goal:
    - The system attempts to find axioms + inference rules to **prove** that goal
      - If goal contains variables, then also gives the values for those variables for which the goal is proven

## Declarative Meaning

```
brown(bear). big(bear).
gray(elephant). big(elephant).
black(cat). small(cat).
dark(Z) :- black(Z).
dark(Z) :- brown(Z).
dangerous(X) :- dark(X), big(X).
```

- The *logical consequences* of a program L is the <u>smallest</u> set such that
  - All facts of the program are in L,
  - If  $\mathbf{H} : -\mathbf{B_1}, \mathbf{B_2}, \dots, \mathbf{B_n}$  is an instance of a clause in the program such that  $\mathbf{B_1}, \mathbf{B_2}, \dots, \mathbf{B_n}$  are all in L, then  $\mathbf{H}$  is also in L.
  - For the above program we get dark (cat) and dark (bear) and consequently dangerous (bear) in addition to the original facts.

```
brown(bear). big(bear).
gray(elephant). big(elephant).
black(cat). small(cat).
dark(Z):-black(Z).
dark(Z):-brown(Z).
dangerous(X):-dark(X), big(X).
```

- A query is, in general, a conjunction of goals:  $G_1$ ,  $G_2$ , ...,  $G_n$
- To prove  $G_1$ ,  $G_2$ , ...,  $G_n$ :
  - Find a clause  $\mathbf{H} : \mathbf{B_1}, \mathbf{B_2}, \dots, \mathbf{B_k}$  such that  $\mathbf{G_1}$  and  $\mathbf{H}$  match.
  - Under the <u>substitution for variables</u>, prove  $B_1$ ,  $B_2$ , ...,  $B_k$ ,  $G_2$ , ...,  $G_n$  If nothing is left to prove then the proof succeeds!

    If there are no more clauses to match, the proof fails!

```
brown(bear). big(bear).
gray(elephant). big(elephant).
black(cat). small(cat).
dark(Z):-black(Z).
dark(Z):-brown(Z).
dangerous(X):-dark(X), big(X).
```

- To prove: ?- dangerous (Q).
  - 1. Select dangerous (X): -dark(X), big(X) and prove dark(Q), big(Q).
  - 2. To prove dark (Q) select the first clause of dark, i.e. dark (Z):-black (Z), and prove black (Q), big (Q).
  - 3. Now select the fact black (cat) and prove big (cat). This proof fails!
  - 4. Go back to step 2, and select the second clause of dark, i.e. dark (Z):-brown (Z), and prove brown (Q), big (Q).

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```
brown(bear). big(bear).
gray(elephant). big(elephant).
black(cat). small(cat).
dark(Z):-black(Z).
dark(Z):-brown(Z).
dangerous(X):-dark(X), big(X).
```

- To prove: ?- dangerous (Q).
  - 5. Now select brown (bear) and prove big (bear).
  - 6. Select the fact big (bear).

There is nothing left to prove, so the proof succeeds

```
brown (bear).
                        big(bear).
                       big(elephant).
  gray(elephant).
  black(cat).
                       small(cat).
  dark(Z) :- black(Z).
  dark(Z) := brown(Z).
  dangerous(X) := dark(X), big(X).
                      dangerous (Q)
dangerous(Q) :-
     dark(Q), big(Q).
                    dark(0), big(0)
dark(Q) := black(Q).
                                   dark(0) := brown(0).
           black(Q), big(Q)
                                brown(Q), big(Q)
    black(cat).
                                      brown (bear).
               big(cat)
                               big(bear)
                  fail
                                 succeed
```

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- The Prolog interpreter works by what is called **BACK WARD CHAINING** (also called **top-down**, **goal directed**)
  - It begins with the thing it is trying to prove and works backwards looking for things that would imply it, until it gets to facts.
- It is also possible to work forward from the facts trying to see if any of the things you can prove from them are what you were looking for
  - This methodology is called *bottom-up resolution*
  - It can be very time-consuming
  - Example: Answer set programming, DLV, Potassco (the Potsdam Answer Set Solving Collection), OntoBroker
- Fancier logic languages use both kinds of chaining, with special smarts or hints from the user to bound the searches

- When it attempts resolution, the Prolog interpreter pushes the current goal onto a stack, makes the first term in the body the current goal, and goes back to the beginning of the database and starts looking again.
- If it gets through the first goal of a body successfully, the interpreter continues with the next one.
- If it gets all the way through the body, the goal is satisfied and it backs up a level and proceeds.

- The Prolog interpreter starts at the <u>beginning of your database</u> (**this ordering is part of Prolog**, NOT of logic programming in general) and looks for something with which to unify the current goal
  - If it finds a fact, great; it succeeds,
  - If it finds a rule, it attempts to satisfy the terms in the body of the rule depth first.
  - This process is motivated by the *RESOLUTION PRINCIPLE*, due to Robinson, 1965:



It says that if C1 and C2 are Horn clauses, where C2 represents a true statement and the head of C2 unifies with one of the terms in the body of C1, then we can replace the term in C1 with the body of C2 to obtain another statement that is true if and only if C1 is true

- If it fails to satisfy the terms in the body of a rule, the interpreter **undoes** the unification of the left hand side (BACKTRACKING) (this includes un-instantiating any variables that were given values as a result of the unification) and keeps looking through the database for something else with which to unify
- If the interpreter gets to the end of database without succeeding, it **backs** out a level (that's how it might **fail** to satisfy something in a body) and continues from there.

- PROLOG IS NOT PURELY DECLARATIVE:
  - The <u>ordering of the database</u> and <u>the left-to-right pursuit of sub-goals</u> gives a deterministic imperative semantics to searching and backtracking
  - Changing the order of statements in the database can give you different results:
    - It can lead to infinite loops
    - It can result in inefficiency

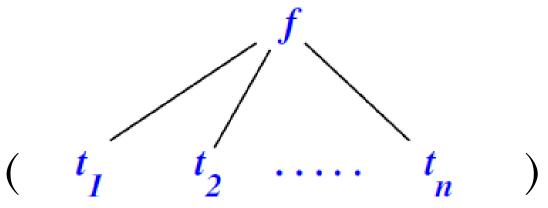
• Transitive closure with <u>left recursion</u> in Prolog will run into an infinite loop:

```
reach(X,Y) :-
    reach(X,Z),
    edge(Z, Y).
reach(X,Y) :-
    edge(X,Y).
```

```
?- reach(A,B).
Infinite loop
```

## Structures

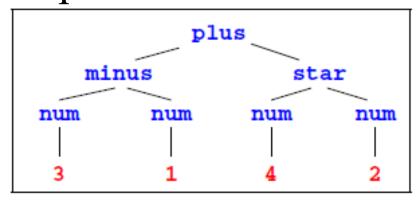
• If  $\mathbf{f}$  is an identifier and  $\mathbf{t_1}$ ,  $\mathbf{t_2}$ , ...,  $\mathbf{t_n}$  are terms, then  $\mathbf{f}(\mathbf{t_1}, \mathbf{t_2}, \ldots, \mathbf{t_n})$  is a term



- In the above, f is called a functor and t<sub>i</sub>s are called arguments
- Structures are used to group related data items together (in some ways similar to struct in C and objects in Java)
  - Structures are used to construct trees (and, as a special case of trees, **lists**)

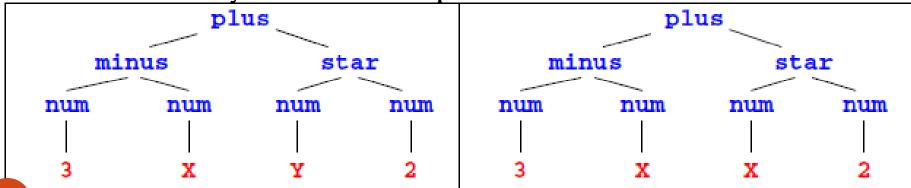
### Trees

• Example: expression trees:

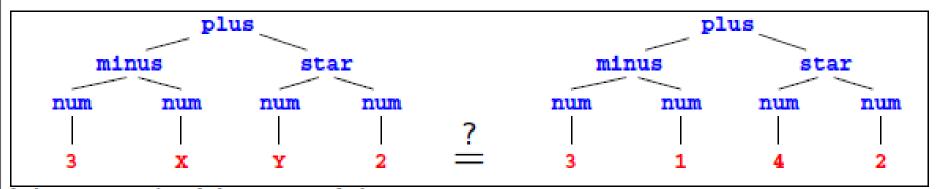


plus(minus(num(3), num(1)), star(num(4), num(2)))

 Data structures may have variables AND the same variable may occur multiple times in a data structure



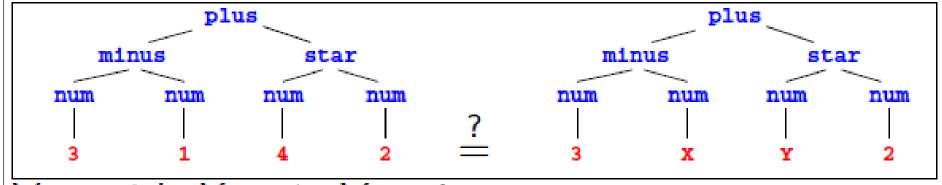
- t1 = t2: finds substitutions for variables in t1 and t2 that make the two terms identical
  - (We'll later introduce *unification*, a related operation that has logical semantics)



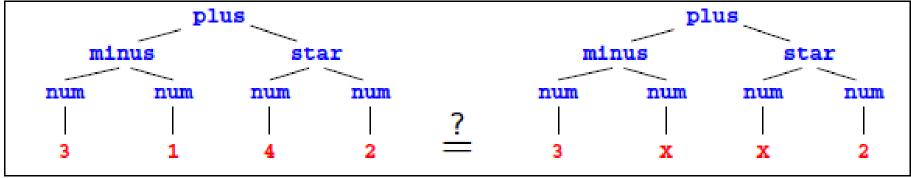
Yes, with  $X=1,\ Y=4$ .

- Matching: given two terms, we can ask if they "match" each other
  - A constant matches with itself: **42** unifies with **42**
  - A variable matches with anything:
    - if it matches with something other than a variable, then it instantiates,
    - if it matches with a variable, then the two variables become associated.
      - A=35,  $A=B \rightarrow B$  becomes 35
      - A=B, A=35 → B becomes 35
  - Two structures match if they:
    - Have the same functor,
    - Have the same arity, and
    - Match recursively
      - foo (g (42), 37) matches with foo (A, 37),
         foo (g (A), B), etc.

- The general Rules to decide whether two terms **S** and **T** *match* are as follows:
  - •If **S** and **T** are constants, **S=T** if both are same object
  - •If **S** is a variable and **T** is anything, **T=S**
  - •If **T** is variable and **S** is anything, **S=T**
  - •If **S** and **T** are structures, **S=T** if
    - S and T have same functor, same arity, and
    - All their corresponding arguments components have to match (c) Paul Fodor (CS Stony Brook) and Elsevier



Yes, with X = 1, Y = 4.



No! X cannot be 1 and 4 at the same time.

- Which of these match?
  - A
  - 100
  - func(B)
  - func (100)
  - func(C, D)
  - func(+(99, 1))

- Which of these match?
  - A
  - 100
  - func (B)
  - func (100)
  - func(C, D)
  - func(+(99, 1))
- A matches with 100, func (B), func (100), func (C,D), func (+(99, 1)).
- 100 matches only with **A**.
- func (B) matches with A, func (100), func (+ (99, 1))
- func (C, D) matches with A.
- func (+ (99, 1)) matches with A and func (B).

### Accessing arguments of a structure

- Matching is the predominant means for accessing structures arguments
  - Let date ('Sep', 1, 2020) be a structure used to represent dates, with the month, day and year as the three arguments (in that order!)

```
then date(M,D,Y) = date('Sep',1,2020).

makes
```

```
M = 'Sep', D = 1, Y = 2020
```

• If we want to get only the day, we can write

```
date(, D, ) = date('Sep', 1, 2020).
```

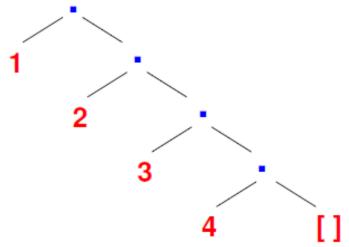
Then we only get: D = 1

### Lists

- Prolog uses a special syntax to represent and manipulate lists
  - [1,2,3,4]: represents a list with 1, 2, 3 and 4, respectively.
  - This can also be written as [1|[2,3,4]]: a list with 1 as the head (first element) and [2,3,4] as its tail (the list of remaining elements).
    - If X = 1 and Y = [2,3,4] then [X|Y] is same as [1,2,3,4].
  - The empty list is represented by [] or nil
  - The symbol " | " (pipe) and is used to separate the beginning elements of a list from its tail
    - For example: [1,2,3,4] = [1|[2,3,4]] = [1|[2|[3,4]]] = [1,2|[3,4]] = [1,2,3|[4]] = [1|[2|[3|[4|[]]]]

### Lists

- Lists are special cases of trees (syntactic sugar, i.e., internally, they use structures)
  - For instance, the list [1,2,3,4] is represented by the following structure:



• where the function symbol ./2 is the list constructor:

[1,2,3,4] is same as .(1,.(2,.(3,.(4,[]))))

### Lists

- Strings: are sequences of characters surrounded by double quotes "abc", "John Smith", "to be, or not to be".
  - A string is <u>equivalent to a list of the (numeric)</u> <u>character codes</u>:

```
?- X="abc".
```

$$X = [97, 98, 99]$$

- member/2 finds if a given element occurs in a list:
  - The program:

```
member(X, [X|_]).
member(X, [_|Ys]) :-
    member(X,Ys).
```

• Example queries:

```
?- member(2,[1,2,3]).
?- member(X,[1,i,s,t]).
```

?- member(f(X),[f(1),g(2),f(3),h(4)]).

?- member (1, L).

- append/3 concatenate two lists to form the third list:
  - The program:
    - Empty list append L is L:append([], L, L).
    - Otherwise, break the first list up into the head **X**, and the tail **L**: if **L** append **M** is **N**, then **[X|L]** append **M** is **[X|N]**:

• Example queries:

```
?- append([1,2],[3,4],X).
```

- ?-append(X, Y, [1,2,3,4]).
- ?- append(X, [3,4], [1,2,3,4]).
- ?- append([1,2], Y, [1,2,3,4]).

- Is the predicate a function?
  - No. We are not applying arguments to get a result. Instead, we are proving that a theorem holds. Therefore, we can leave any variables unbound.

```
?- append(L, [2, 3], [1, 2, 3]).
   L = [1]
?- append([ 1 ], L, [1, 2, 3]).
   L = [2, 3]
?- append(L1, L2, [1, 2, 3]).
 L1 = [],
                  L2 = [1, 2, 3];
                L2 = [2, 3];
 L1 = [1],
 L1 = [1, 2], L2 = [3];
 L1 = [1, 2, 3], L2 = [];
 no
```

```
append([],L,L).
append([X|L], M, [X|N]) :- append(L,M,N).
```

append([1,2],[3,4],X)?

```
append([],L,L).
append([X|L],M,[X|N]) :- append(L,M,N).
  append([1,2],[3,4],A)?
                           X=1, L=[2], M=[3,4], A=[X|N]
```

```
append([],L,L).
append([X|L],M,[X|N]) :- append(L,M,N).
```

```
append([2],[3,4],N)?

append([1,2],[3,4],A)? X=1,L=[2],M=[3,4],A=[X|N]
```

```
append([],L,L).
append([X|L],M,[X|N']) :- append(L,M,N').
   append([2],[3,4],N)?
                          |X=2,L=[],M=[3,4],N=[2|N']
  append([1,2],[3,4],A)?
                          |X=1,L=[2],M=[3,4],A=[1|N]
```

```
append([],L,L).
append([X|L],M,[X|N']) :- append(L,M,N').
```

```
append([],[3,4],N')?

append([2],[3,4],N)? X=2,L=[],M=[3,4],N=[2|N']

append([1,2],[3,4],A)? X=1,L=[2],M=[3,4],A=[1|N]
```

```
append([],L,L).
append([X|I], M, [X|N']) :- append(L, M, N').
   append([],[3,4],N')?
                               L = [3,4], N' = L
   append([2],[3,4],N)?
                           X=2, L=[], M=[3,4], N=[2|N']
                           X=1, L=[2], M=[3,4], A=[1|N]
  append([1,2],[3,4],A)?
```

```
append([],L,L).
append([X|L],M,[X|N']) :- append(L,M,N').

A = [1|N]
N = [2|N']
N' = L
L = [3,4]
Answer: A = [1,2,3,4]
```

```
append([],[3,4],N')? L = [3,4], N' = L
append([2],[3,4],N)? X=2,L=[],M=[3,4],N=[2|N']
append([1,2],[3,4],A)? X=1,L=[2],M=[3,4],A=[1|N]
```

- **len**/2 to find the length of a list (the first argument):
  - The program:

```
len([], 0).
len([_|Xs], N+1) :-
    len(Xs, N).
```

• Example queries:

```
?- len([], X).
    X = 0
?- len([l,i,s,t], 4).
    false
?- len([l,i,s,t], X).
    X = 0+1+1+1
```

## Arithmetic

?- 1+2 = 3.

false

- In Predicate logic, the basis for Prolog, the only symbols that have a meaning are the predicates themselves
  - •In particular, function symbols are uninterpreted: have no special meaning and can only be used to construct data structures

## Arithmetic

• Meaning for arithmetic expressions is given by the built-in predicate "is":

```
?- X is 1 + 2.
    succeeds, binding X = 3.
?- 3 is 1 + 2.
    succeeds.
```

- General form: **R** is **E** where **E** is an expression to be evaluated and **R** is matched with the expression's value
- Y is X + 1, where X is a free variable, will give an error because X does not (yet) have a value, so, X + 1 cannot be evaluated

#### The list length example revisited

- **length**/2 **finds** the length of a list (first argument):
  - The program:

• Example queries:

#### Conditional Evaluation

- Conditional operator: the if-then-else construct in Prolog:
  - if A then B else C is written as ( A -> B; C)
    - To Prolog this means: try **A**. If you can prove it, go on to prove **B** and ignore **C**. If **A** fails, however, go on to prove **C** ignoring **B**.

```
max(X,Y,Z):-
( X =< Y
-> Z = Y
; Z = X
).
```

```
?- \max(1,2,X).
X = 2.
```

#### Conditional Evaluation

Consider the computation of n! (i.e. the factorial of n)
 % factorial (+N, -F)

```
factorial(N, F) :- ...
```

- **N** is the input parameter and **F** is the output parameter!
- The body of the rule species how the output is related to the input
  - For factorial, there are two cases:  $N \le 0$  and N > 0
    - if  $N \le 0$ , then F = 1
    - if N > 0, then F = N \* factorial (N 1)

factorial (N1, F1)

```
factorial(N, F) :-
(N > 0
-> N1 is N-1,
```

F is N\*F1

F = 1

?- factorial(12,X).

X = 479001600

(c) Paul Fodor (CS Stony Brook) and Elsevier

## Imperative features

• Other imperative features: we can think of prolog rules as imperative programs w/ backtracking

```
program :-
    member(X, [1, 2, 3, 4]),
    write(X),
    nl,
    fail;
    true.
```

- ?- program. % prints all solutions
- fail: always fails, causes backtracking
- ! is the cut operator: prevents other rules from matching (we will see it later)

## Arithmetic Operators

- Integer/Floating Point operators: +, -, \*, /
  - Automatic detection of Integer/Floating Point
- Integer operators: mod, // (integer division)
- Comparison operators: <, >, =<, >=,
- Expr1 = := Expr2 (succeeds if expression
  - Expr1 evaluates to a number equal to Expr2),
- $Expr1 = \exists Expr2$  (succeeds if expression
  - Expr1 evaluates to a number non-equal to Expr2)

• Quicksort:

```
quicksort([], []).
quicksort([X0|Xs], Ys) :-
  partition (X0, Xs, Ls, Gs),
  quicksort(Ls, Ys1),
  quicksort(Gs, Ys2),
  append(Ys1, [X0|Ys2], Ys).
partition(Pivot,[],[],[]).
partition(Pivot,[X|Xs],[X|Ys],Zs) :-
   X = < Pivot,
   partition (Pivot, Xs, Ys, Zs).
partition(Pivot, [X|Xs], Ys, [X|Zs]) :-
   X > Pivot
   partition (Pivot, Xs, Ys, Zs).
```

• Quicksort:

```
quicksort([], []).
quicksort([X0|Xs], Ys) :-
  partition (X0, Xs, Ls, Gs),
  quicksort(Ls, Ys1),
  quicksort(Gs, Ys2),
  append(Ys1, [X0|Ys2], Ys).
partition(Pivot,[],[],[]).
partition(Pivot,[X|Xs],[X|Ys],Zs) :-
   X = < Pivot,
   !, % cut
   partition (Pivot, Xs, Ys, Zs).
partition(Pivot,[X|Xs],Ys,[X|Zs]) :-
   partition (Pivot, Xs, Ys, Zs).
```

- We want to define delete/3, to remove a given element from a list (called select/3 in XSB's basics library):
  - delete(2, [1,2,3], X) should succeed with X=[1,3]
  - delete(X, [1,2,3], [1,3]) should succeed with X=2
  - delete(2, X, [1,3]) should succeed with

```
X=[2,1,3]; X = [1,2,3]; X=[1,3,2]; fail
```

• delete(2, [2,1,2], x) should succeed with

```
X=[1,2]; X = [2,1]; fail
```

- Algorithm:
  - •When **X** is selected from **[X|Ys]**, **Ys** results as the rest of the list
  - •When **X** is selected from the tail of [H|Ys], [H|Zs] results, where **Zs** is the result of taking **X** out of **Ys**

• The program: delete(X,[], ) :- fail.% not needed delete(X, [X|Ys], Ys). delete(X, [Y|Ys], [Y|Zs]) :delete(X, Ys, Zs). • Example queries: ?- delete(s, [l,i,s,t], Z). X = [1, i, t]?- delete(X, [l,i,s,t], Z).

?- delete(X, Y, [l,i,s,t]).

?- delete(s, Y, [l,i,t]).

#### Permutations

- Define **permute**/2, to find a permutation of a given list.
  - E.g. **permute** ([1,2,3], X) should return X=[1,2,3] and upon backtracking, X=[1,3,2], X=[2,1,3], X=[2,3,1], X=[3,1,2], and X=[3,2,1].
  - Hint: What is the relationship between the permutations of [1,2,3] and the permutations of [2,3]?

<u> </u>	
permute([2,3],Y)	permute([1,2,3],Y)
[2,3]	[1,2,3]
	[2, <b>1</b> ,3]
	[2,3, <b>1</b> ]
[3,2]	[1,3,2]
	[3, <mark>1</mark> ,2]
	[3,2, <mark>1</mark> ]

• The program:

```
permute([], []).
permute([X|Xs], Ys) :-
   permute(Xs, Zs),
   delete(X, Ys, Zs).
```

• Example query:

```
?- permute([1,2,3], X).
```

## The Issue of Efficiency

- Define a predicate, **rev**/2 that finds the reverse of a given list
  - E.g. rev([1,2,3],X) should succeed with X=[3,2,1]
  - Hint: what is the relationship between the reverse of [1,2,3] and the reverse of [2,3]? Answer: append([3,2],[1],[3,2,1])

```
rev([], []).
```

```
rev([X|Xs], Ys) :- rev(Xs, Zs),
    append(Zs, [X], Ys).
```

- How long does it take to evaluate **rev([1,2,...,n],X)**?
  - T(n) = T(n 1) + time to add 1 element to the end of an n 1 element list = T(n 1) + n 1 =

$$T(n-2) + n-2 + n-1 = ...$$

$$\rightarrow$$
 T(n) = O(n<sup>2</sup>) (quadratic)

## Making rev/2 faster

- Keep an accumulator: stack all elements seen so far
  - i.e. a list, with elements seen so far in reverse order
- The program: rev(L1, L2) :- rev h(L1, [], L2). rev h([X|Xs], AccBefore, Out):rev h(Xs, [X|AccBefore], Out). rev h([], Acc, Acc). % Base case • Example query: ?- rev([1,2,3], X).will call rev h([1,2,3], [], X) which calls rev h([2,3], [1], X)which calls rev h([3], [2,1], X)

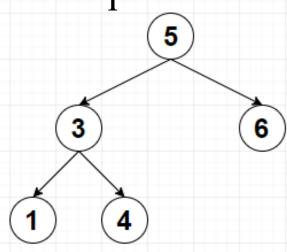
which returns X = [3,2,1]

which calls rev h([], [3,2,1], X)

### Tree Traversal

- Assume you have a binary tree, represented by
  - node/3 facts for internal nodes: node (a,b,c) means that a has b and c as children
  - leaf/1 facts: for leaves: leaf(a) means that a is a leaf

•Example:



```
node(5, 3, 6).
node(3, 1, 4).
leaf(1).
leaf(4).
leaf(6).
```

#### Tree Traversal

• Write a predicate **preorder**/2 that traverses the tree (starting from a given node) and returns the list of nodes in pre-order preorder(Root, [Root]) :leaf (Root) . preorder(Root, [Root|L]) :node (Root, Child1, Child2), preorder(Child1, L1), preorder(Child2, L2), append(L1, L2, L). ?- preorder(5, L). L = [5, 3, 1, 4, 6]• The program takes  $O(n^2)$  time to traverse a tree with nnodes

- The lists in Prolog are singly-linked; hence we can access the first element in constant time, **but** need to scan the entire list to get the **last** element
- However, unlike functional languages like Lisp or SML, we can use **variables** in data structures:
  - We can exploit this to make lists "open tailed" (also called difference lists in Prolog): end the list with a variable tail and pass that variable, so we can add elements at the end of the list

- When **X**=[1,2,3|**Y**], **X** is a list with **1**, **2**, **3** as its first three elements, followed by **Y** 
  - Now if Y = [4 | Z] then X = [1, 2, 3, 4 | Z]
    - We can think of Z as "pointing to" the end of X
  - We can now add an element to the end of X in constant time!!
    - and continue adding more elements, e.g. **Z=[5|W]**

- A *difference list* is represented by two variables: one referring to the entire list, and another to its (uninstantiated) tail
  - $\bullet$  e.g. X = [1,2,3|Z], Z
- Most Prolog programmers use the notation List-Tail to denote a list List with tail Tail.
  - e.g. **X-Z**
  - Note that "-" is used as a data structure symbol (not used for arithmetic here)

• Append 2 open ended lists: dappend(X,T, Y,T2, L,T3) :-T = YT2 = T3L = X. ?- dappend([1,2,3|T],T, [4,5,6|T2],T2, L,T3). L = [1,2,3,4,5,6]T3] • Simplified version: dappend(X,T, T,T2, X,T2). • More simplified **notation** (with "-"): dappend(X-T, T-T2, X-T2). ?- dappend([1,2,3|T]-T, [4,5,6|T2]-T2, L-T3). L = [1,2,3,4,5,6]T2

• Add an element at the end of a list:

```
add(L-T, X, L2-T2) :-
     T = [X|T2],
     L = L2.
?- add([1,2,3|T]-T, 4, L-T2).
L = [1,2,3,4|T2]
• We can simplify it as:
add(L-T, X, L-T2) :-
     T = [X|T2].
• This can be simplified more like:
add(L-[X|T2], X, L-T2).
• Alternative using dappend:
add(L-T, X, L-T2) :-
     dappend (L-T, [X|T2]-T2, L-T2).
```

• Check if a list is a palindrome: palindrome(X) :palindromeHelp(X-[]). palindromeHelp(A-A). % an empty list palindromeHelp([ |A]-A). % 1-element list palindromeHelp([C|A]-D) :palindromeHelp(A-B), B=[C|D].?- palindrome([1,2,2,1]). yes ?- palindrome([1,2,3,2,1]). yes ?- palindrome([1,2,3,4,5]). no

## Tree Traversal, Revisited

```
preorder1(Node, List, Tail) :-
  node (Node, Child1, Child2),
  List = [Node|List1],
  preorder1(Child1, List1, Tail1),
  preorder1 (Child2, Tail1, Tail).
preorder1(Node, [Node|Tail], Tail) :-
  leaf (Node) .
preorder(Node, List) :-
   preorder1(Node, List, []).
```

• The program takes **O(n)** time to traverse a tree with **n** nodes

• The preorder traversal program may be rewritten as:

```
preorder1(Node, [Node|L]-T) :-
  node(Node, Child1, Child2),
  preorder1(Child1, L-T1),
  preorder1(Child2, T1-T).
preorder1(Node, [Node|T]-T).
```

• The inorder traversal program:

```
inorder1(Node, L-T) :-
  node (Node, Child1, Child2),
  inorder1(Child1, L-T1),
  T1 = [Node|T2],
  inorder1(Child2, T2-T).
inorder1(Node, [Node|T]-T).
inorder(Node,L):-
  inorder1(Node, L-[]).
```

• The postorder traversal program:

```
postorder1(Node, L-T) :-
  node (Node, Child1, Child2),
  postorder1(Child1, L-T1),
  postorder1(Child2, T1-T2),
  T2 = [Node|T].
postorder1(Node, [Node|T]-T).
postorder(Node,L):-
  postorder1(Node, L-[]).
```

- There are several ways to represent graphs in Prolog:
  - represent each edge separately as one clause (fact):edge(a,b).edge(b,c).
    - isolated nodes cannot be represented, unless we have also node/1 facts
  - the whole graph as one data object: as a pair of two sets (nodes and edges): graph([a,b,c,d,f,g], [e(a,b), e(b,c),e(b,f)])
    list of arcs: [a-b, b-c, b-f]
  - •adjacency-list: [n(a,[b]), n(b,[c,f]), n(d,[])]

- Path from one node to another one:
  - a predicate **path** (**G**, **A**, **B**, **P**) to find an acyclic path **P** from node **A** to node **B** in the graph **G**.
  - The predicate should return all paths via backtracking.
  - We will solve it using the graph as a data object, like in graph ([a,b,c,d,f,g],[e(a,b),e(b,c),e(b,f)]

• adjacent for directed edges:

```
adjacent(X,Y,graph(_,Es)) :-
member(e(X,Y),Es).
```

• adjacent for undirected edges (no distinction between the two vertices associated with each edge):

```
adjacent(X,Y,graph(_,Es)) :-
    member(e(X,Y),Es).
adjacent(X,Y,graph(_,Es)) :-
    member(e(Y,X),Es).
```

• Path from one node to another one:

```
path(G,A,B,P) :-
   pathHelper(G,A,[B],P).
% Base case
pathHelper( ,A,[A|P1],[A|P1]).
pathHelper(G,A,[Y|P1],P) :-
   adjacent(X,Y,G),
   \+ member(X,[Y|P1]),
   pathHelper(G,A,[X,Y|P1],P).
```

- Cycle from a given node in a directed graph:
  - a predicate **cycle** (**G**, **A**, **P**) to find a closed path (cycle) **P** starting at a given node **A** in the graph **G**.
  - The predicate should return all cycles via backtracking

```
cycle(G,A,P) :-
   adjacent(A,B,G),
   path(G,B,A,P1),
   P = [A|P1].
```

### Complete program in XSB

```
:- import member/2 from basics.
adjacent(X,Y,graph( ,Es)) :-
        member(e(X,Y),Es).
path(G,A,B,P) :-
        pathHelper(G,A,[B],P).
pathHelper( ,A,[A|P1],[A|P1]).
pathHelper(G,A,[Y|P1],P) :-
        adjacent(X,Y,G),
        \+ member(X,[Y|P1]),
        pathHelper(G,A,[X,Y|P1],P).
cycle(G,A,P) :-
        adjacent(A,B,G),
        path(G,B,A,P1),
        P = [A|P1].
?- Graph = graph([a,b,c,d,f,g],[e(a,b), e(b,c),e(c,a),e(a,e),e(e,a)]),
        cycle (Graph, a, P),
        writeln(P),
        fail;
                        (c) Paul Fodor (CS Stony Brook) and Elsevier
        true.
```

## Aggregates in XSB

- setof (Template, Goal, Set): Set is the set of all instances of Template such that Goal is provable.
- findall (Template, Goal, List) is similar to predicate bagof/3, except that variables in Goal that do not occur in Template are treated as existential, and alternative lists are not returned for different bindings of such variables.
- bagof (Template, Goal, Bag) has the same semantics as setof/3 except that the third argument returns an unsorted list that may contain duplicates. X^Goal will not bind X
- tfindall (Template, Goal, List) is similar to predicate findall/3, but the Goal must be a call to a single tabled predicate.

## Aggregates in XSB

```
p(1,1).
p(1,2).
p(2,1).
?- setof(Y,p(X,Y),L).
L=[1,2]
?- findall(Y,p(X,Y),L).
L=[1,2,1]
?- bagof(Y,p(X,Y),L).
X=1, L=[1,2];
X=2, L=[1];
fail
```

# XSB Prolog

- Negation: \+ is negation-as-failure
- Another negation called tnot (TABLING = memoization)
  - Use: ... :- ..., tnot(foobar(X)).
  - All variables under the scope of **tnot** must also occur to the left of that scope in the body of the rule in other <u>positive</u> relations:
  - Ok: ...: -p(X,Y), tnot(foobar(X,Y)),...
  - Not ok: ...: -p(X, Z), tnot(foobar(X, Y)), ...
- XSB also supports Datalog:
  - :- auto table.

at the top of the program file

# XSB Prolog

- •Read/write from and to files:
  - Edinburgh style:
  - ?- tell('a.txt'),
    write('Hello, World!'), told.
  - ?- see('a.txt'), read(X), seen.

# XSB Prolog

- •Read/write from and to files:
  - •ISO style:

```
?- open('a.txt', write, X),
write(X,'Hello, World!'),
close(X).
```

## Cut (logic programming)

• Cut (! in Prolog) is a goal which always succeeds, but cannot be backtracked past.

```
\max(X,Y,Y) :- X =< Y, !.
\max(X, X).
```

- cut says "stop looking for alternatives"
- no check is needed in the second rule anymore because if we got there, then X =< Y must have failed, so X > Y must be true.
- red cut: if someone deletes!, then the rule is incorrect
- Green cut

```
\max (X,Y,Y) :- X =< Y, !.

\max (X,Y,X) :- X > Y.
```

• by explicitly writing X > Y, it guarantees that the second rule will always work even if the first one is removed by accident or changed (cut is deleted)

## Cut (logic programming)

• No backtracking pass the guard, but ok after:

```
p(a). p(b).
q(a) . q(b) . q(c) .
?-p(X),!
X=a;
no
?-p(X),!,q(Y).
X=a, Y=a;
X=a, Y=b;
X=a, Y=c;
no
```

### Testing types

atom(X)

Tests whether **X** is bound to a symbolic atom.

```
?- atom(a).
yes
?- atom(3).
no
```

integer(X)

Tests whether **X** is bound to an integer.

real(X)

Tests whether **X** is bound to a real number.

### Testing for variables

• is\_list(L)
Tests whether L is bound to a list.

• ground (G)
Tests whether G has unbound logical variables.

• **var** (**X**)
Tests whether **X** is bound to a Prolog variable.

• call (P)
Force P to be a goal; succeed if P does, else fail.

#### clause (H,B)

Retrieves clauses from memory whose head matches **H** and body matches **B**. **H** must be sufficiently instantiated to determine the main predicate of the head.

#### • copy term(P,NewP)

Creates a new copy of the first parameter (with new variables).

• It is used in <u>iteration</u> through non-ground clauses, so that the original calls are not bound to values

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Write a Prolog relation

```
map (BinaryRelation,InputList, OutputList)
which applies a binary relation on each of the elements of the list
InputList as the first argument and collects the second
argument in the result list.
```

Example:

```
?- map(inc1(X,Y),[5,6],R). returns R=[6,7]
where inc1(X,Y) was defined as:
    inc1(X,Y) :-
    Y is X+1.
```

```
map( BinaryCall,[],[]).
map(BinaryCall,[X|T],[Y|T2]) :-
     copy term(BinaryCall, BinaryCall2),
     BinaryCall2 = .. [F,X,Y],
     call (BinaryCall2),
     map(BinaryCall, T, T2).
incl(X,Y) :-
     Y is X+1.
?- map(inc1(X,Y), [5,6], R).
R = [6,7]
```

```
square(X,Y) :-
    Y is X*X.
?- map(square(E, E2), [2,3,1], R).
R = [4,9,1];
no
```

Use the relation map to implement a relation
pairAll (E, L, L2) which pairs the element E
with each element of the list L to obtain L2.
Examples:

```
?- pairAll(1,[2,3,1], L2).
returns L2=[[1,2],[1,3],[1,1]]
?- pairAll(1,[], L2).
returns L2=[].
```

```
pair(E2, (_E1,E2)).

pairAll(E,L,L2):-
    map(pair(E2, (E,E2)), L, L2).

?- pairAll(1, [2,3,1], R).
R = [(1,2),(1,3),(1,1)]
```

#### Assert and retract

#### asserta(C)

Assert clause **C** into database above other clauses with the same predicate.

#### assertz(C), assert(C)

Assert clause **C** into database below other clauses with the same predicate.

#### retract(C)

Retract **C** from the database. **C** must be sufficiently instantiated to determine the predicate.

### Prolog terms

- functor (E,F,N)
  - **E** must be bound to a functor expression of the form '**f(...)**'. **F** will be bound to '**f**', and **N** will be bound to the number of arguments that **f** has.
- arg(N,E,A)
  - **E** must be bound to a functor expression, **N** is a whole number, and **A** will be bound to the **N**th argument of **E**

### Prolog terms and clauses

• = . . converts between term and list. For example,

```
?- parent(a,X) = .. L.
L = [parent, a, _X001]
```

```
?-[1] = ... X.
X = [.,1,[]]
```