Logic Programming Negation

CSE 595 – Semantic Web

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Negation in Logic Programs

```
above(X, Y) :- on(X, Y).
above(X, Y) :- on(X, Z), above(Z, Y).
on(c, b).
on(b, a).
```

?- above(c,a).

- Yes, since **above(c,a)** is in the least Herbrand model of the program.

?- above(b,c).

- There are models which contain **above(b, c)**, but it is not in the least Herbrand model of the program.
- Not a logical consequence of the program.

?- ¬ above(b,c).

- Yes, since **above(b,c)** is not a logical consequence of the program.

Closed World Assumption

"… the truth, the whole truth, and nothing but the truth …"

- the truth: anything that is the logical consequence of the program is true.
- "the whole truth, and nothing but the truth": anything that is not a logical consequence of the program is false.
- Closed World Assumption (CWA):

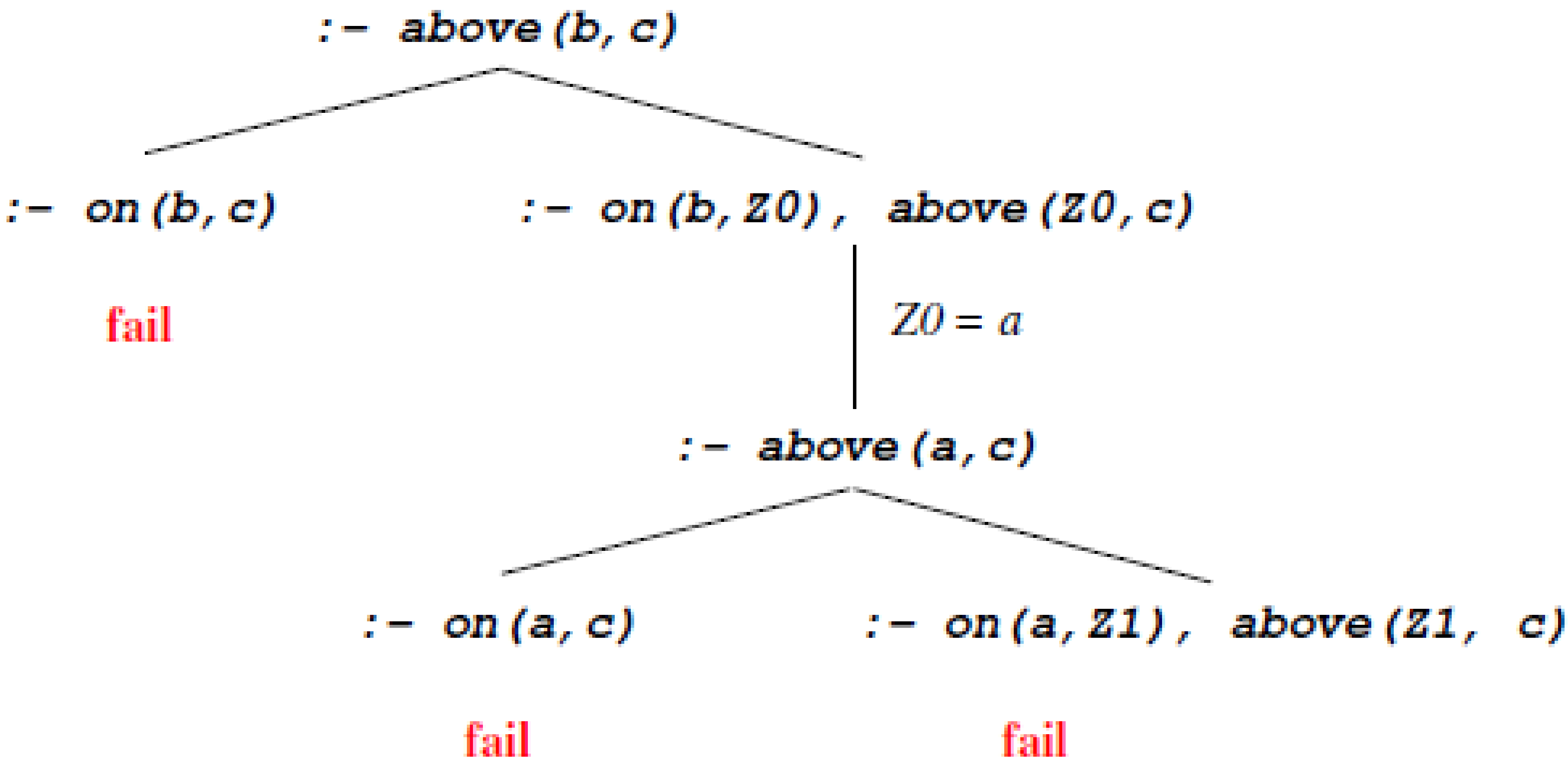
$$\frac{P \not\models A}{\neg A} \qquad \frac{P \nvdash A}{\neg A}$$

- **<u>Negation as (finite) failure:</u>**

$$\frac{\leftarrow A \text{ has a finitely failed SLD tree}}{\neg A}$$

Finite Failure

- Every SLD derivation that fails in a finite number of resolution steps



A problem with CWA

```
above(X, Y) :- on(X, Y).
above(X, Y) :- on(X, Z), above(Z, Y).
on(c, b).
on(b, a).
```

```
?- ¬above(b,c).
```

above(b,c) is not a logical consequence of the program so **¬above(b,c)** must be true.

- But **¬above(b,c)** is not a logical consequence of the program either!
  - (There are models with **¬above(b,c)**)
- Must strengthen what we mean by a program (NORMAL INTUITION.)

Completion

```
above(X, Y) :- on(X, Y).
above(X, Y) :- on(X, Z), above(Z, Y).
```

- Logical meaning of the program:

```
above(X, Y) ←
    on(X, Y) ∨ ( on(X,Z) ∧ above(Z, Y) )
```

- above(X,Y) cannot be true in any other way (by CWA)!
- Hence the above program is equivalent to:

```
above(X, Y ) ↔
    on(X, Y) ∨ ( on(X,Z) ∧ above(Z, Y) )
```

Called the "*completion*" (also "***Clark's completion***") of the program

How to complete a program

1. Rewrite each rule of the form

$$p(t_1, \ldots, t_m) \leftarrow L_1, \ldots, L_n.$$

to

$$p(X_1, \ldots, X_m) \leftarrow X_1 = t_1, \ldots, X_m = t_m, L_1, \ldots, L_n.$$

2. For each predicate symbol **p** which is defined by rules:

$$p(X_1, \ldots, X_m) \leftarrow B_1.$$

. . .

$$p(X_1, \ldots, X_m) \leftarrow B_n.$$

replace the rules by:

- If $n > 0$:

$$\forall X_1, \ldots, X_m \; p(X_1, \ldots, X_m) \leftrightarrow B_1 \lor B_2 \lor B_3 \lor \ldots \lor B_n.$$

- If $n = 0$:

$$\forall X_1, \ldots, X_m \; \neg p(X_1, \ldots, X_m)$$

Negation in Logic Programs

- The negation-as-failure '**not**' predicate could be defined in Prolog as follows:

```
not(P) :- call(P), !, fail.
not(P).
```

  - Quintus, SWI, and many other prologs use '**\+**' rather than **'not**'.
- Another way one can write the **'not**' definition is using the Prolog *implication* operator '**->**' (if-then-else):

```
not(P) :- (call(P) -> fail ; true)
```

Negation in Logic Programs

```
bachelor(P) :- male(P), not(married(P)).
male(henry).
male(tom).
married(tom).
?- bachelor(henry).
yes
?- bachelor(tom).
no
?- bachelor(Who).
Who= henry ;
no
?- not(married(Who)).
no.
```

not(married(Who)) fails because for the variable binding **Who=tom**, **married(Who)** succeeds, and so the negative goal fails.

This might not be intuitive!

Negation in Logic Programs

```
p(X) :- q(X), not(r(X)).
r(X) :- w(X), not(s(X)).
q(a).
q(b).
q(c).
s(a) :- p(a).
s(c).
w(a).
w(b).
?-p(a).
```

Negation in Logic Programs

```
u(X) :- not(s(X)).
s(X) :- s(f(X)).
?-u(1).
```