Recursion

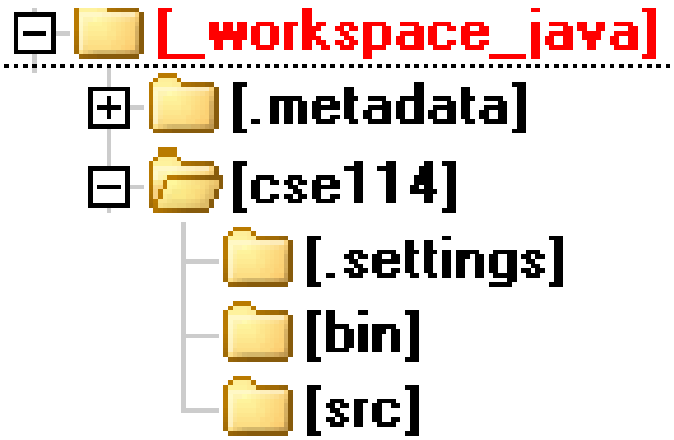
CSE 114, Computer Science 1

Stony Brook University

http://www.cs.stonybrook.edu/~cse114

Motivation

- Suppose you want to find all the files under a directory that contains a particular word.



  - The directory contains subdirectories that also contain subdirectories, and so on.
- The solution is to use recursion by searching the files in the subdirectories recursively.

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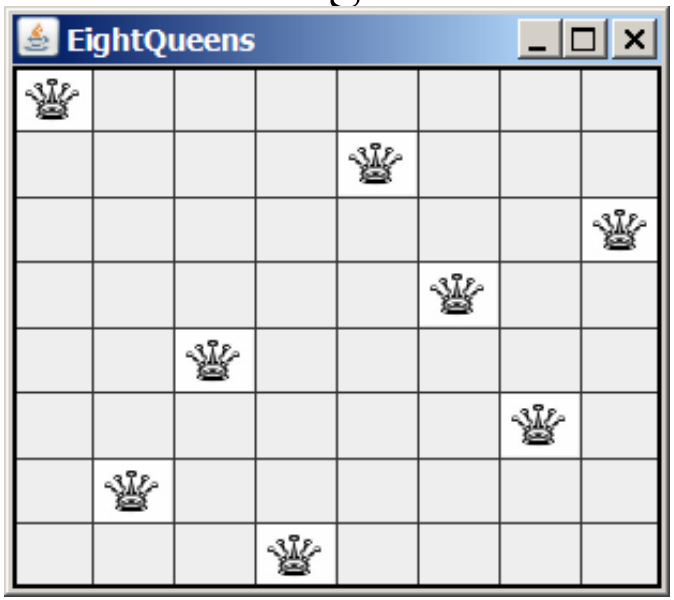
Motivation

- The Eight Queens puzzle is to place eight queens on a chessboard such that no two queens are on the same row, same column, or same diagonal:



We solve this problem using recursion:

- we place the 8th queen after we placed 7 queens on the chessboard.
- we place the 7th queen after we placed 6 queens on the chessboard.

…

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Computing Factorial

$n! = 1 * 2 * 3 * 4 * 5 * \ldots * (n-1) * n$

$(n-1)! = 1 * 2 * 3 * 4 * 5 * \ldots * (n-1)$

So:

$$n! = n * (n-1)!$$

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

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```
import java.util.Scanner;
public class ComputeFactorial {
  public static void main(String[] args) {
    // Create a Scanner
    Scanner input = new Scanner(System.in);
    System.out.print("Enter a non-negative integer: ");
    int n = input.nextInt();
    // Display factorial
    System.out.println("Factorial of "+n+" is "+factorial(n));
  }
  /** Return the factorial for a specified number */
  public static int factorial(int n) {
    if (n == 0) // Base case
      return 1;
    else
      return n * factorial(n - 1); // Recursive call
  }
}
```


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Computing Factorial

factorial(3) =

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

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Computing Factorial

factorial(3) = 3 * factorial(2)

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

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Computing Factorial

factorial(3) = 3 * factorial(2)

= 3 * (2 * factorial(1))

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

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Computing Factorial

factorial(3) = 3 * factorial(2)

= 3 * (2 * factorial(1))

= 3 * (2 * (1 * factorial(0)))

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

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Computing Factorial

factorial(3) = 3 * factorial(2)

= 3 * (2 * factorial(1))

= 3 * (2 * (1 * factorial(0)))

= 3 * (2 * (1 * 1)))

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

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Computing Factorial

factorial(3) = 3 * factorial(2)

= 3 * (2 * factorial(1))

= 3 * (2 * (1 * factorial(0)))

= 3 * (2 * (1 * 1)))

= 3 * (2 * 1)

factorial(0) = 1;

factorial(n) = n*factorial(n-1);

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Computing Factorial

factorial(3) = 3 * factorial(2)
= 3 * (2 * factorial(1))
= 3 * (2 * (1 * factorial(0)))
= 3 * (2 * (1 * 1)))
= 3 * (2 * 1)
= 3 * 2

factorial(0) = 1;
factorial(n) = n*factorial(n-1);

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Computing Factorial

factorial(3) = 3 * factorial(2)
= 3 * (2 * factorial(1))
= 3 * (2 * (1 * factorial(0)))
= 3 * (2 * (1 * 1)))
= 3 * (2 * 1)
= 3 * 2
= 6

factorial(0) = 1;
factorial(n) = n*factorial(n-1);

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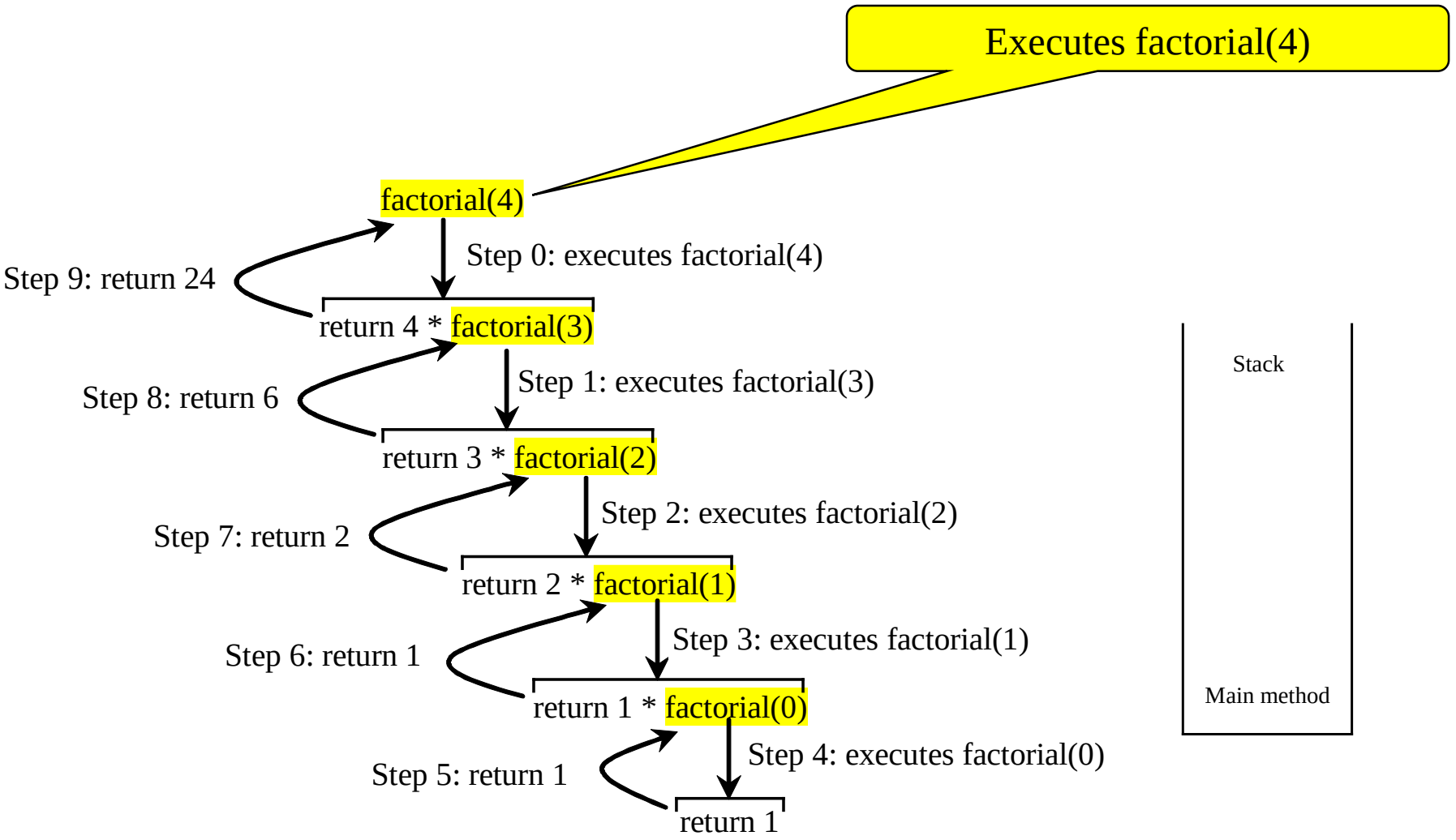
Trace Recursive factorial



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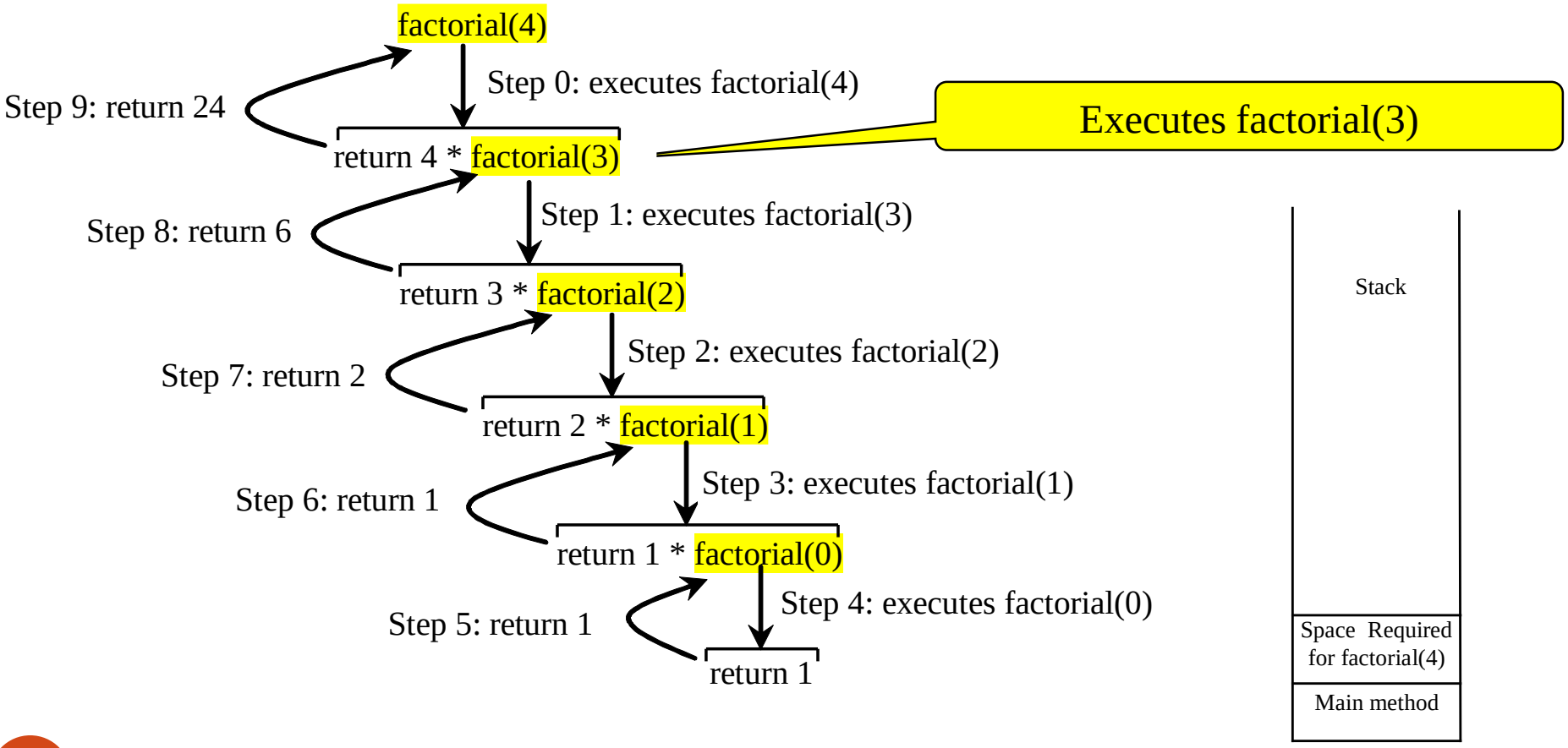
Trace Recursive factorial



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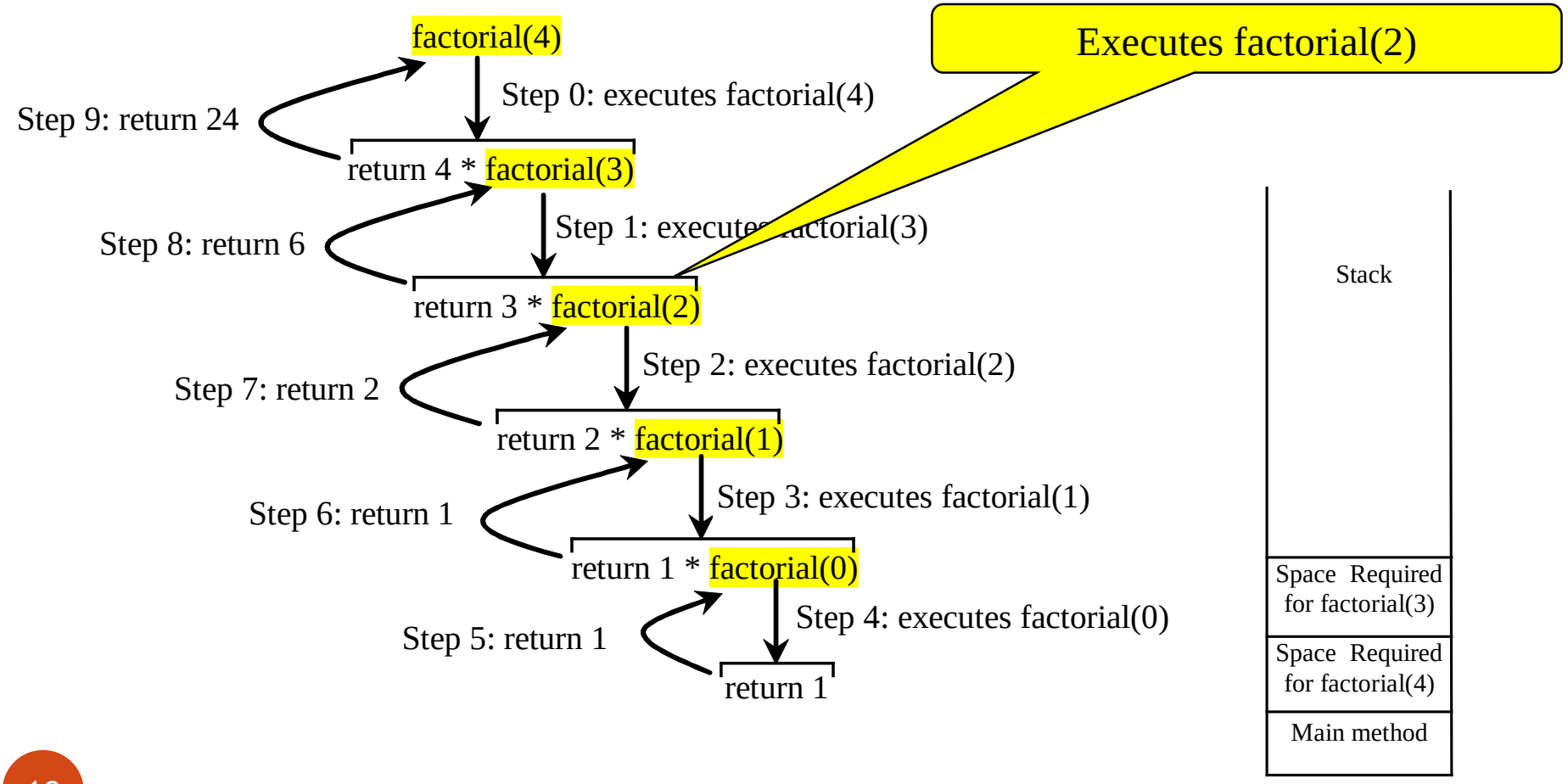
Trace Recursive factorial



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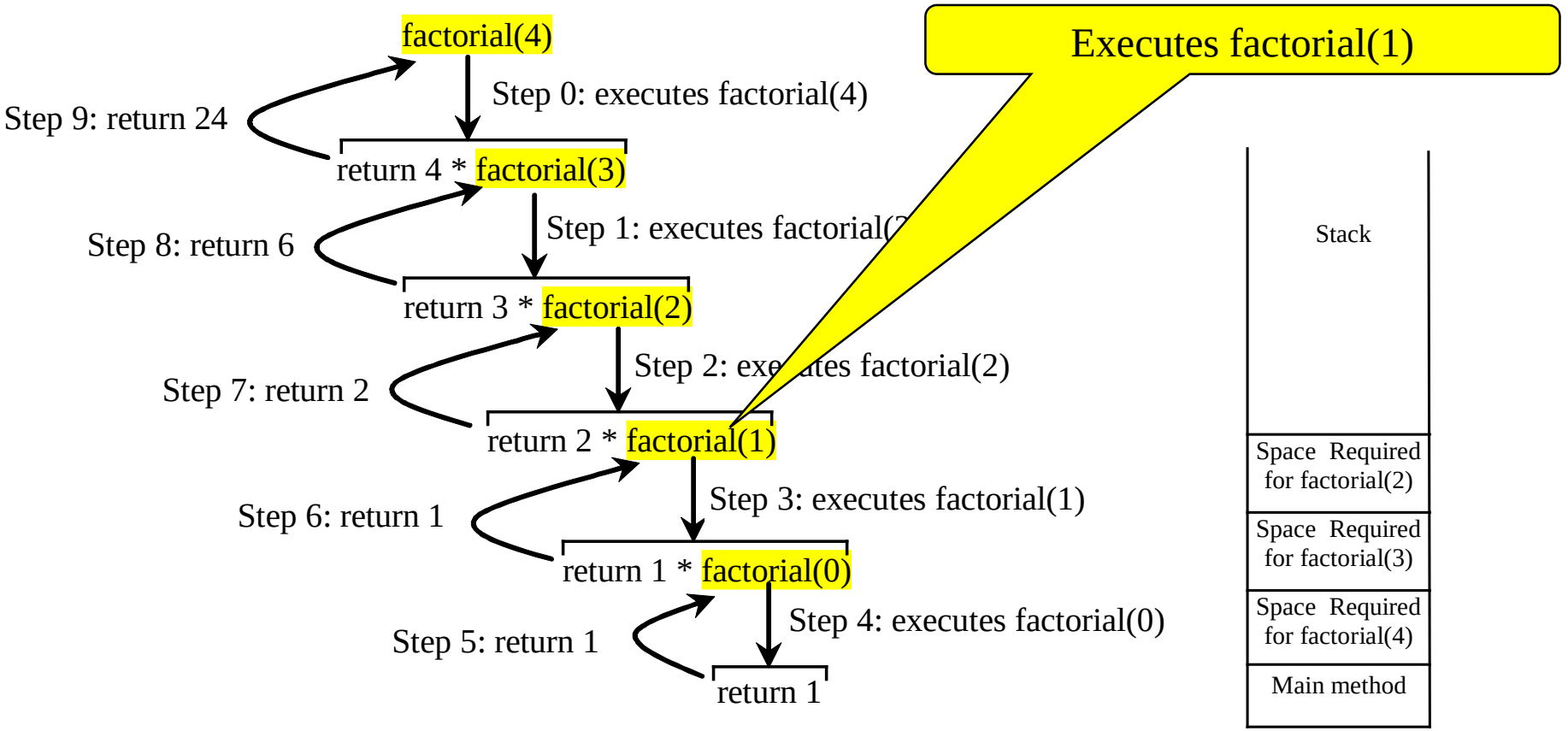
Trace Recursive factorial



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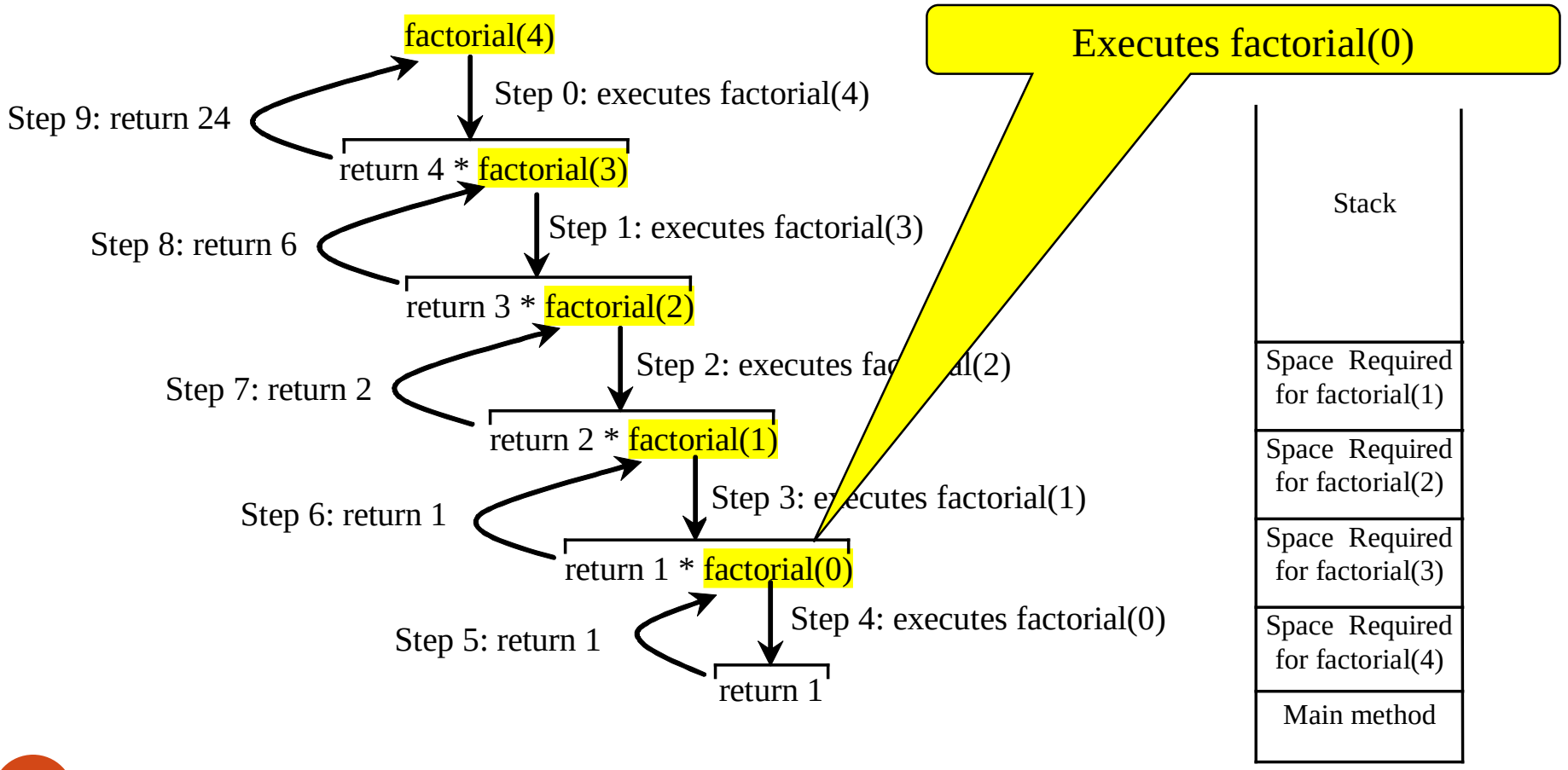
Trace Recursive factorial



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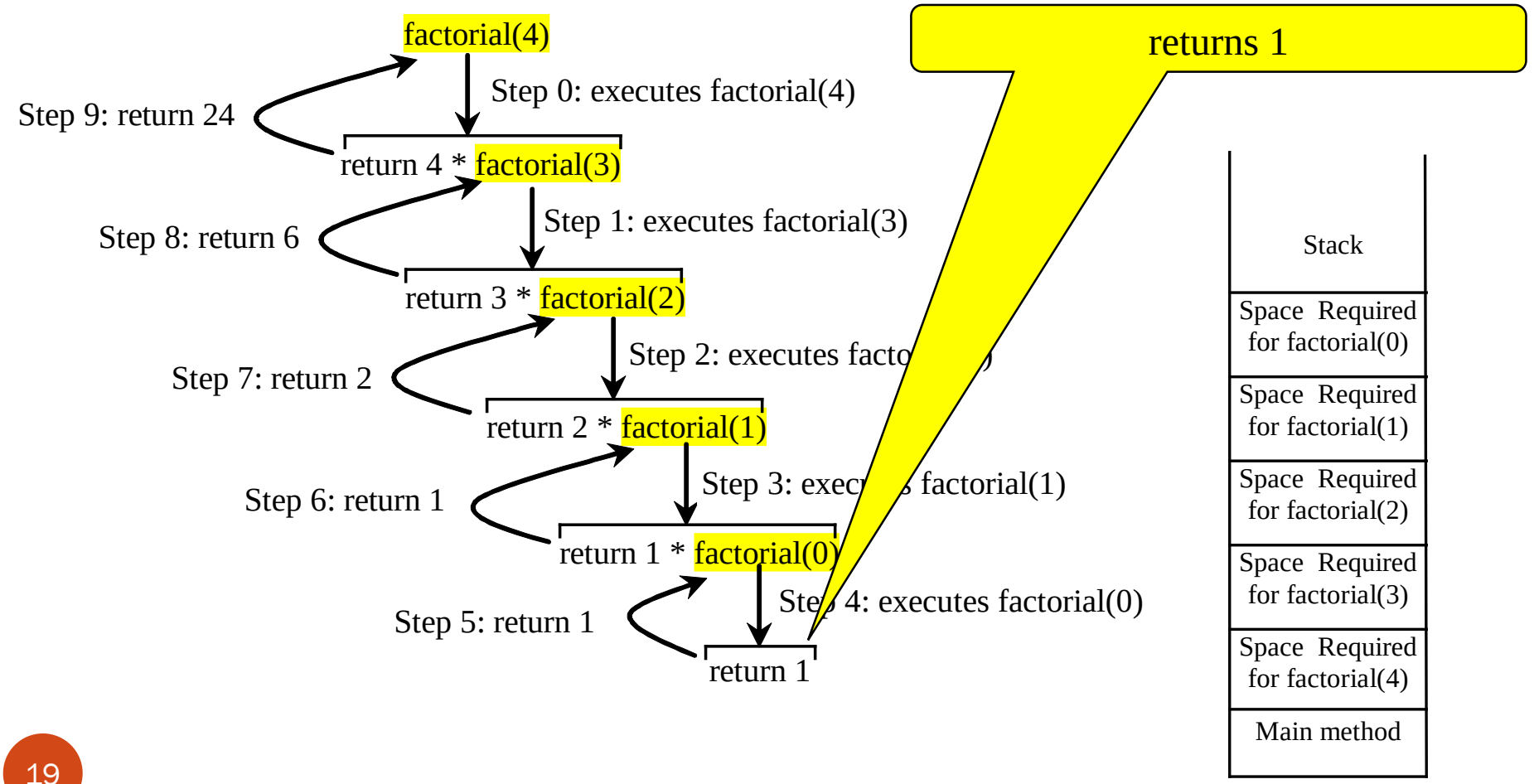
Trace Recursive factorial



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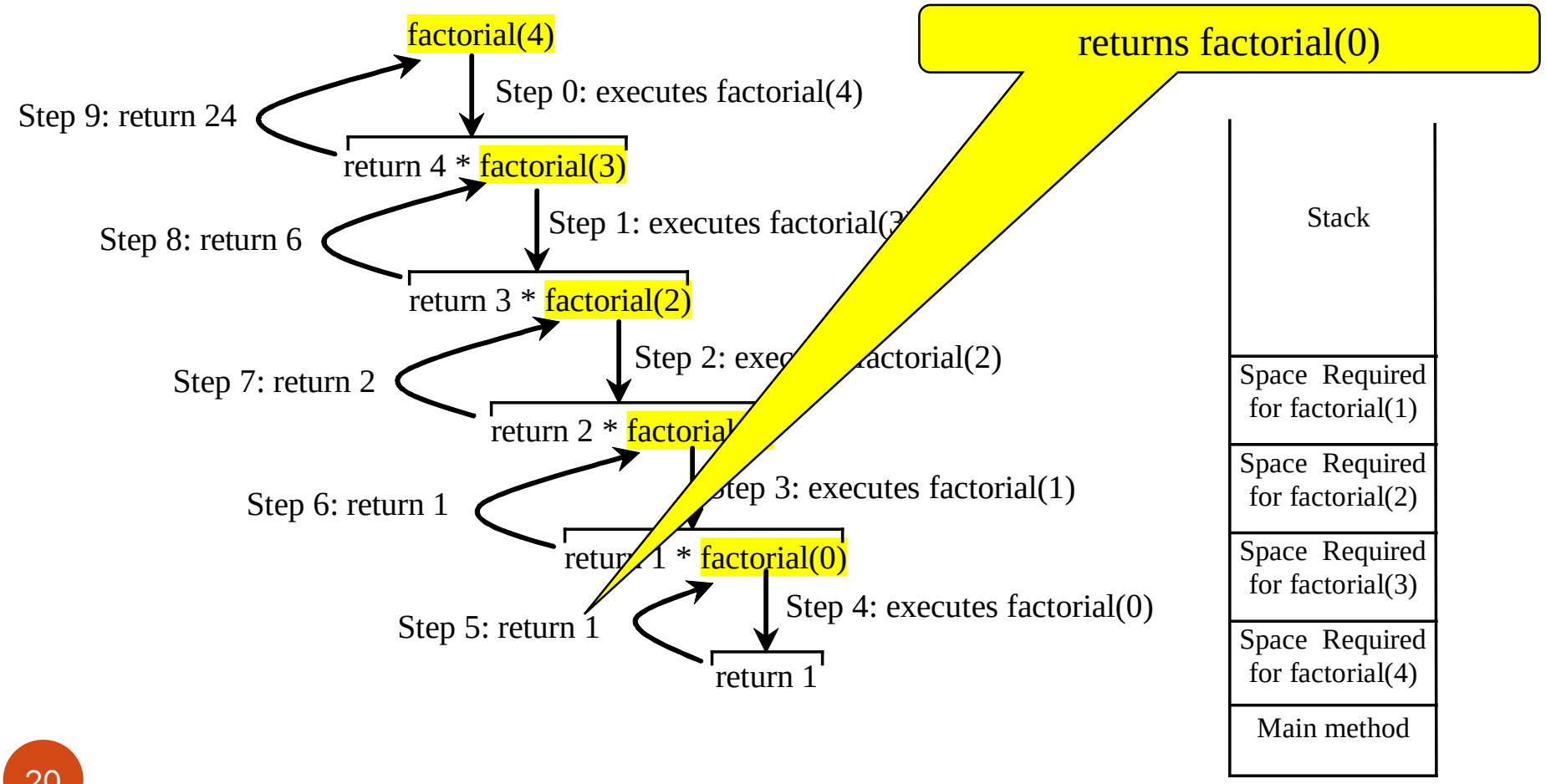
Trace Recursive factorial



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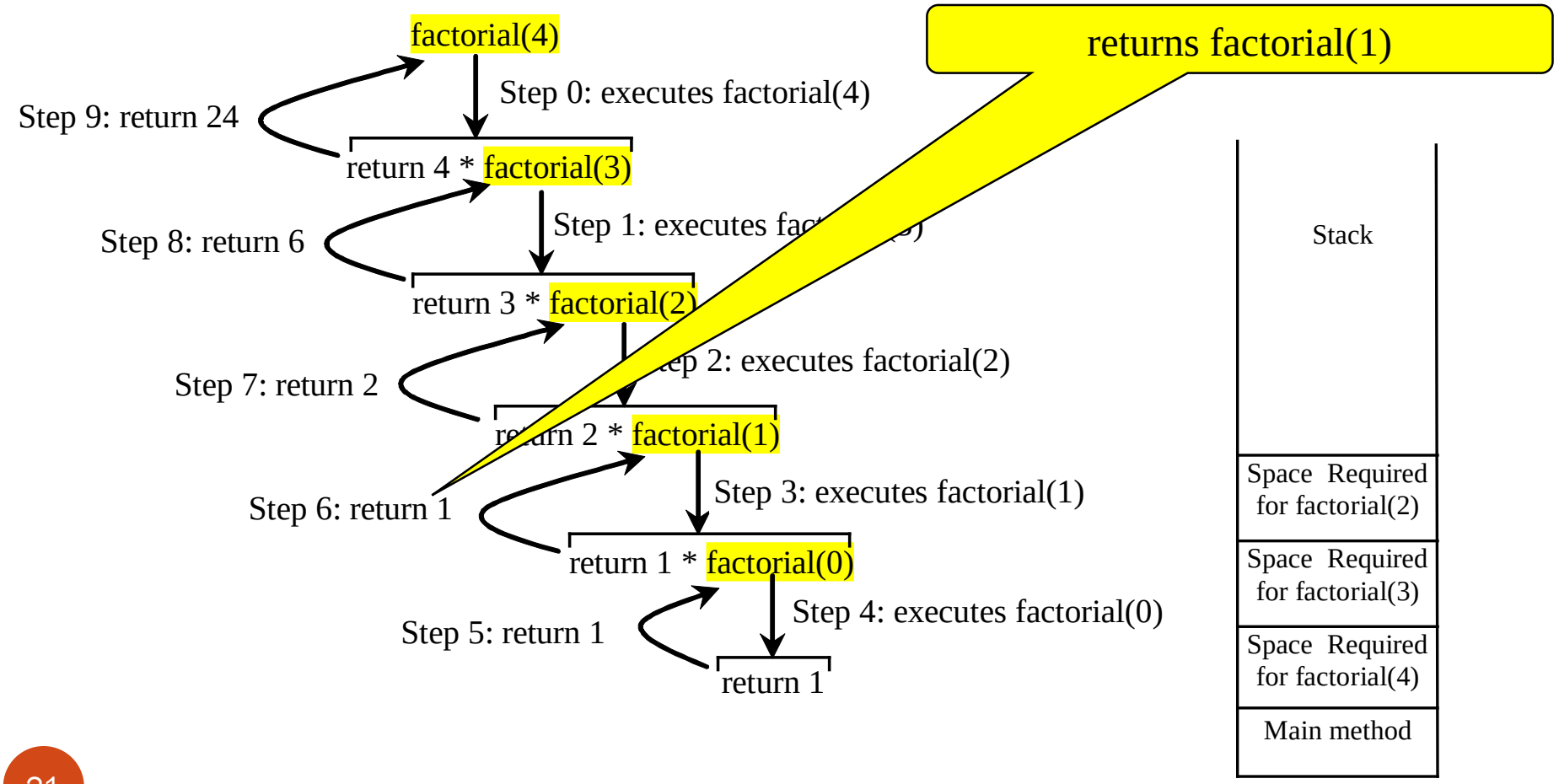
Trace Recursive factorial



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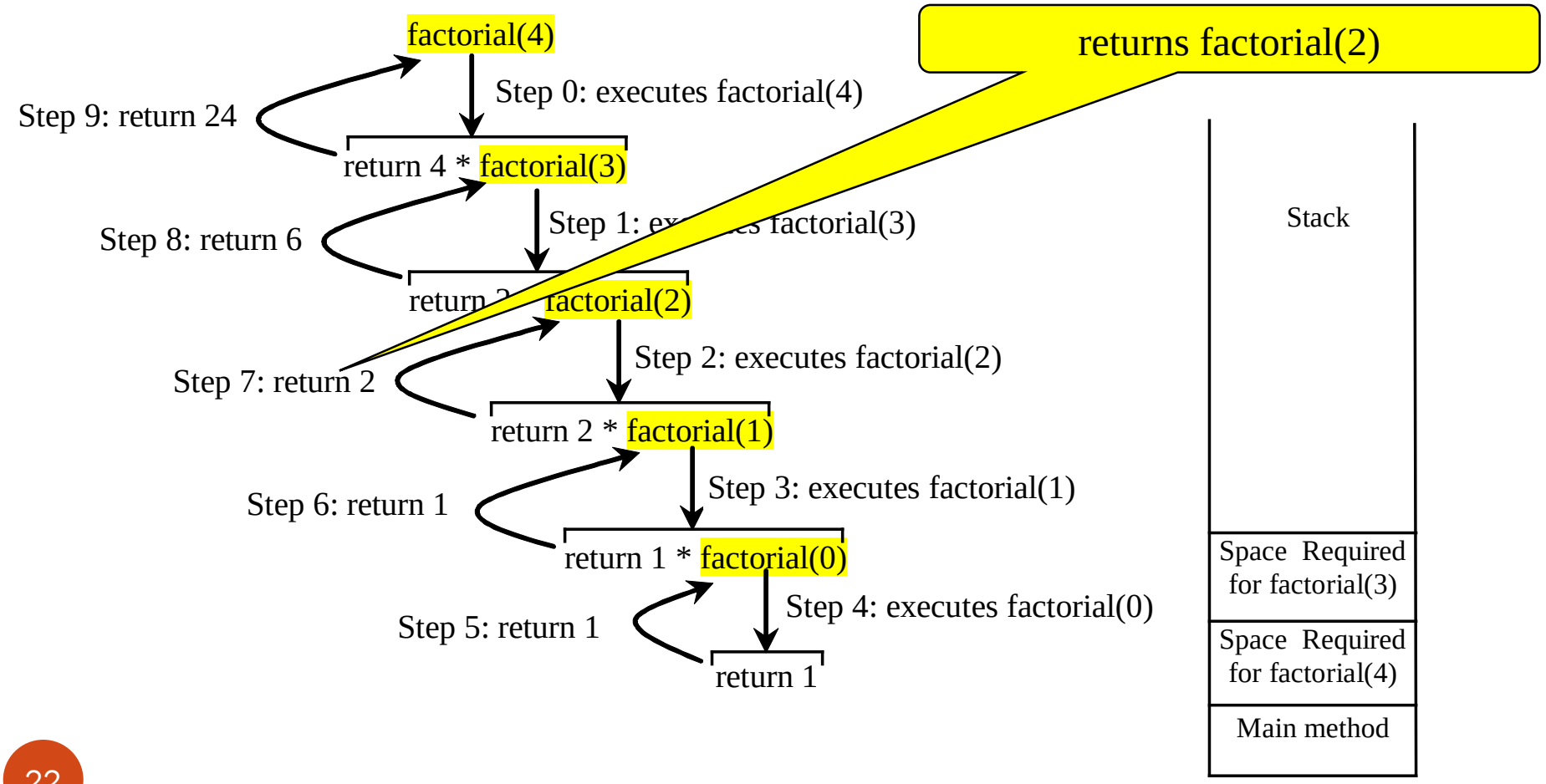
Trace Recursive factorial



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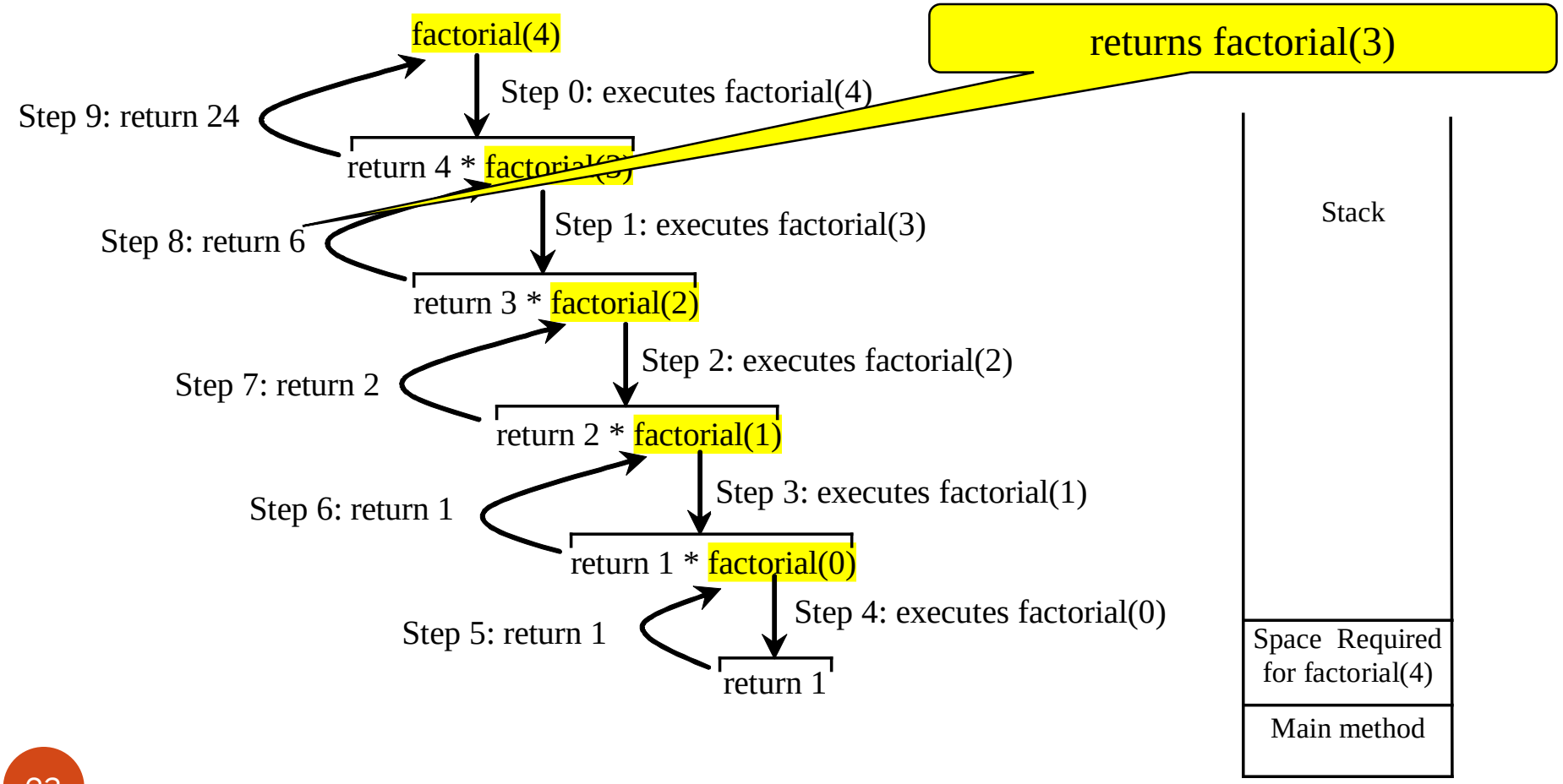
Trace Recursive factorial



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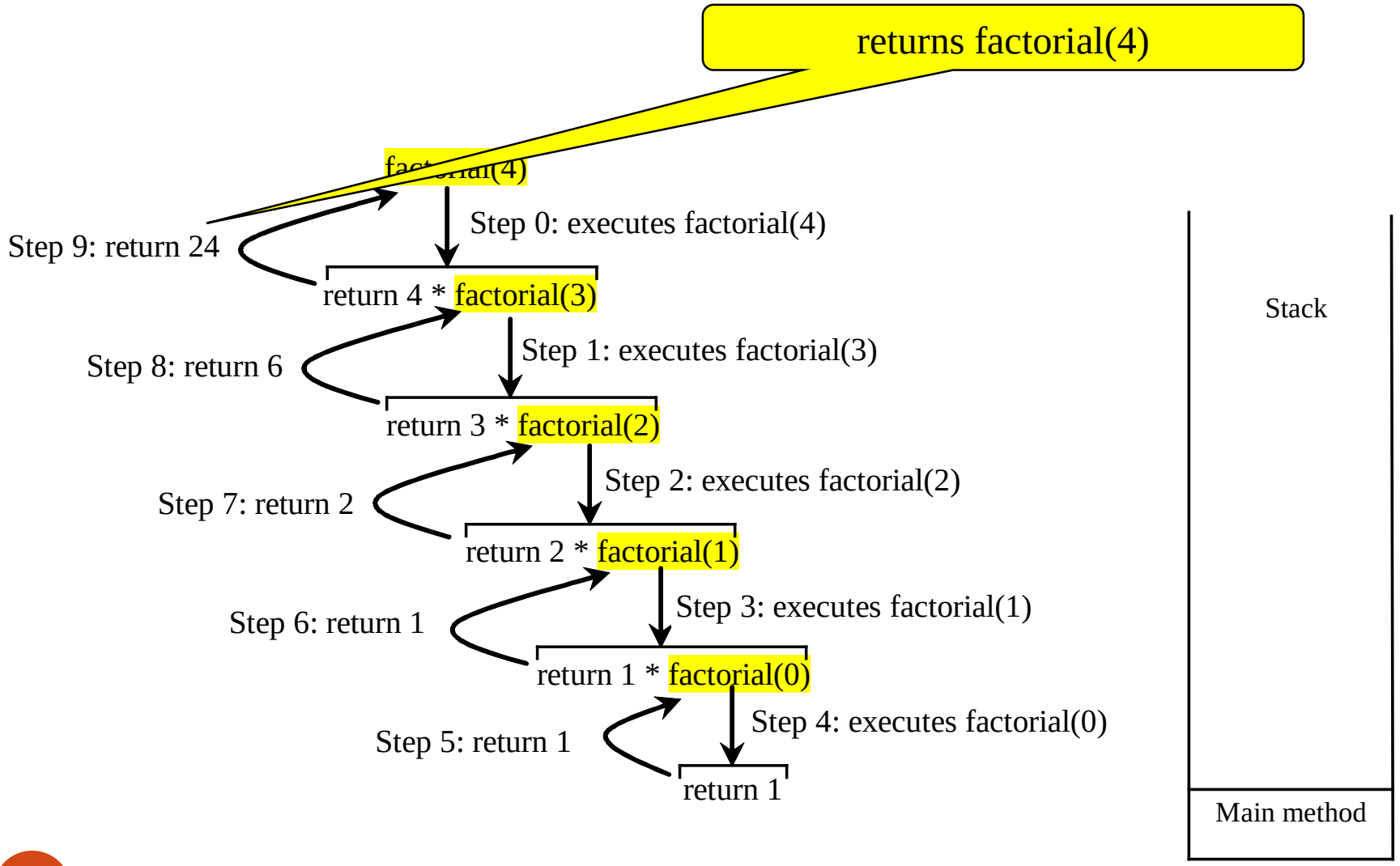
Trace Recursive factorial



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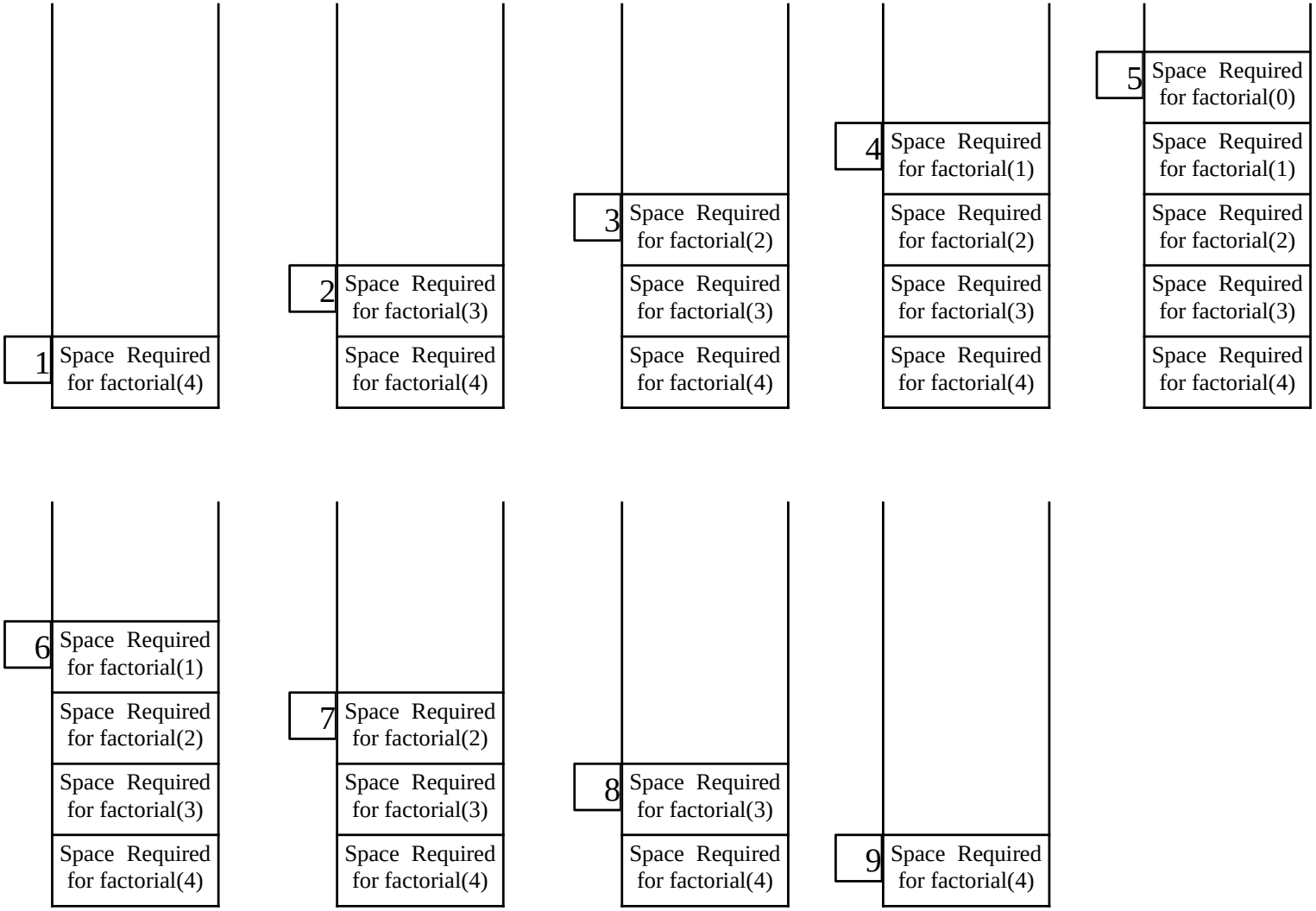
factorial(4) Stack Trace



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Fibonacci Numbers

```
indices:          0 1 2 3 4 5 6 7  8  9  10 11 …
Fibonacci series: 0 1 1 2 3 5 8 13 21 34 55 89 …
```

fib(0) = 0;

fib(1) = 1;

fib(index) = fib(index -1) + fib(index -2); for integers index >=2

fib(3) = fib(2) + fib(1) = (fib(1) + fib(0)) + fib(1)

= (1 + 0) +fib(1) = 1 + fib(1) = 1 + 1 = 2

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```
import java.util.Scanner;
public class ComputeFibonacci {
public static void main(String args[]) {
    // Create a Scanner
    Scanner input = new Scanner(System.in);
    System.out.print("Enter an index for the Fibonacci number: ");
    int index = input.nextInt();
    // Find and display the Fibonacci number
    System.out.println("Fibonacci(" + index + ") is " + fib(index));
  }
  /** The method for finding the Fibonacci number */
  public static long fib(long index) {
    if (index == 0) // Base case
      return 0;
    else if (index == 1) // Base case
      return 1;
    else  // Reduction and recursive calls
      return fib(index - 1) + fib(index - 2);
  }
}
```


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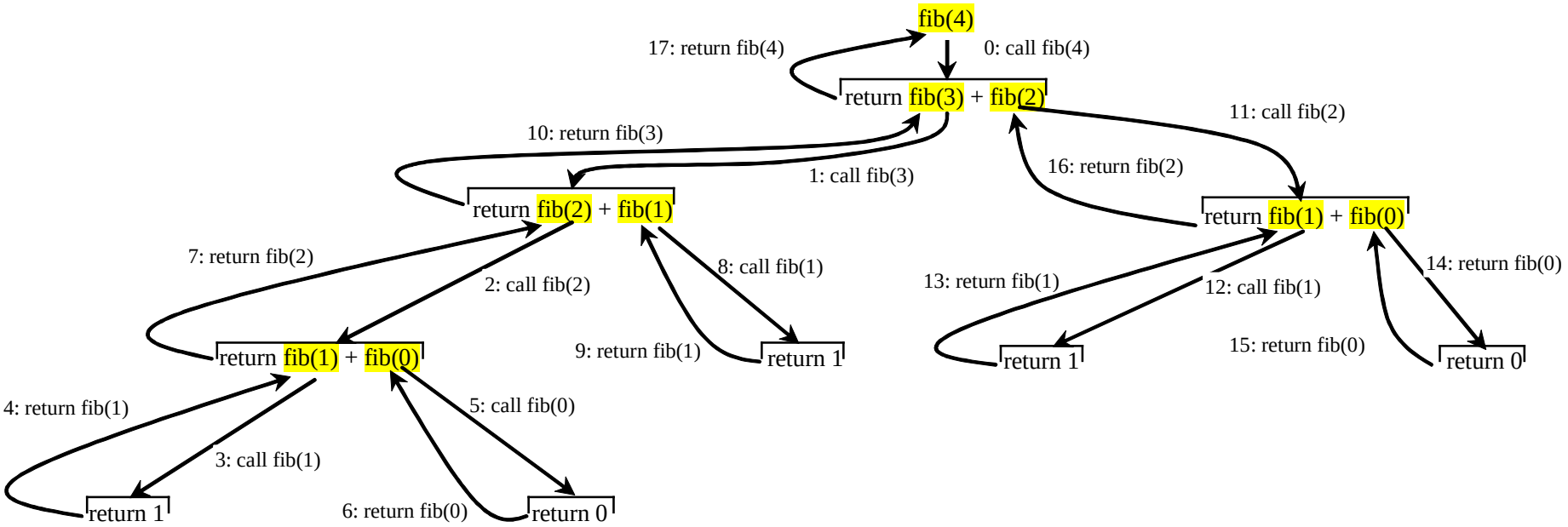
Fibonnaci Numbers



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```
import java.util.Scanner;
public class ComputeFibonacciTabling {  // NO REPEATED COMPUTATION
public static void main(String args[]) {
    Scanner input = new Scanner(System.in);
    System.out.print("Enter an index for the Fibonacci number: ");
    int index = input.nextInt();
    f = new long[index+1];
    System.out.println("Fibonacci(" + index + ") is " + fib(index));
  }
  public static long[] f;
  public static long fib(long index) {
    if (index == 0)        return 0;
    if (index == 1) {       f[1]=1;       return 1;   }
    if(f[index]!=0)
      return f[index];
   else  // Reduction and recursive calls
      f[index] = fib(index - 1) + fib(index - 2);
   return f[index];
  }
}
```


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Characteristics of Recursion

All recursive methods have the following characteristics:

- One or more base cases (the simplest case) are used to stop recursion.
- Every recursive call reduces the original problem, bringing it increasingly closer to a base case until it becomes that case.

In general, to solve a problem using recursion, you break it into subproblems.

- If a subproblem resembles the original problem, you can apply the same approach to solve the subproblem recursively.
- This subproblem is almost the same as the original problem in nature with a smaller size.

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Problem Solving Using Recursion

- Print a message for n times
  - break the problem into two subproblems:
    - print the message one time and
    - print the message for n-1 times
      - This new problem is the same as the original problem with a smaller size.
      - The base case for the problem is n==0.

```
public static void nPrintln(String message,int times) {
  if (times >= 1) {
    System.out.println(message);
    nPrintln(message, times - 1);
  } // The base case is times == 0
}
```


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Think Recursively

- The palindrome problem (e.g., “eye”, “racecar”):

```
public static boolean isPalindrome(String s) {
  if (s.length() <= 1) // Base case
    return true;
  else if (s.charAt(0) != s.charAt(s.length() - 1))
    // Base case
    return false;
  else
    return isPalindrome(s.substring(1, s.length() - 1));
}
```


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Recursive Helper Methods

- The preceding recursive isPalindrome method is not efficient, because it creates a new string for every recursive call.
- To avoid creating new strings, use a helper method:

```
public static boolean isPalindrome(String s) {
  return isPalindrome(s, 0, s.length() - 1);
}
public static boolean isPalindrome(String s, int low, int high) {
  if (high <= low) // Base case
    return true;
  else if (s.charAt(low) != s.charAt(high))
    // Base case
    return false;
  else
    return isPalindrome(s, low + 1, high - 1);
}
```


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Recursive Selection Sort

1. Find the smallest number in the list and swap it with the first number.
2. Ignore the first number and sort the remaining smaller list recursively.

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```
public class SelectionSort {
  public static void sort(double[] list) {
    int low = 0, high = list.length - 1;
    while (low < high) {
      // Find the smallest number and its index in list(low .. high)
      int indexOfMin = low;
      double min = list[low];
      for (int i = low + 1; i <= high; i++)
        if (list[i] < min) {
          min = list[i];
          indexOfMin = i;
        }
      // Swap the smallest in list(low ... high) with list(low)
      list[indexOfMin] = list[low];
      list[low] = min;
      low = low + 1;
    }
  }
  public static void main(String[] args) {
    double[] list = { 2, 1, 3, 1, 2, 5, 2, -1, 0 };
    sort(list);
    for (int i = 0; i < list.length; i++)
      System.out.print(list[i] + " ");
  }
}
```


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```
public class RecursiveSelectionSort {
  public static void sort(double[] list) {
    sort(list, 0, list.length - 1); // Sort the entire list
  }
  public static void sort(double[] list, int low, int high) {
    if (low < high) {
      // Find the smallest number and its index in list(low .. high)
      int indexOfMin = low;
      double min = list[low];
      for (int i = low + 1; i <= high; i++) {
        if (list[i] < min) {
          min = list[i];
          indexOfMin = i;
        }}
      // Swap the smallest in list(low .. high) with list(low)
      list[indexOfMin] = list[low];
      list[low] = min;
      // Sort the remaining list(low+1 .. high)
      sort(list, low + 1, high);
    }}
  public static void main(String[] args) {
      double[] list = {2, 1, 3, 1, 2, 5, 2, -1, 0};
      sort(list);
      for (int i = 0; i < list.length; i++)
        System.out.print(list[i] + " ");
  }
}
```


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Recursive Binary Search

- Case 1: If the key is less than the middle element, recursively search the key in the first half of the array.
- Case 2: If the key is equal to the middle element, the search ends with a match (Base case).
- Case 3: If the key is greater than the middle element, recursively search the key in the second half of the array.

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```
public class BinarySearch {
  public static int binarySearch(int[] list, int key) {
    int low = 0;
    int high = list.length - 1;
    while(low <= high){
      int mid = (low + high) / 2;
      if (key < list[mid])
        high = mid - 1;
      else if (key == list[mid])
        return mid;
      else
        low = mid + 1;
    }
    // The list has been exhausted without a match
    return -low - 1;
  }
  public static void main(String[] args) {
    int[] list = { 1,2,3,4,5,6,10 };
    System.out.print(binarySearch(list,6));
  }
}
```


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```
public class RecursiveBinarySearch {
  public static int recursiveBinarySearch(int[] list, int key) {
    int low = 0;
    int high = list.length - 1;
    return recursiveBinarySearch(list, key, low, high);
  }
  public static int recursiveBinarySearch(int[] list, int key,
      int low, int high) {
    if (low > high)  // The list has been exhausted without a match
      return -low - 1;
    int mid = (low + high) / 2;
    if (key < list[mid])
      return recursiveBinarySearch(list, key, low, mid - 1);
    else if (key == list[mid])
      return mid;
    else
      return recursiveBinarySearch(list, key, mid + 1, high);
  }
}
```


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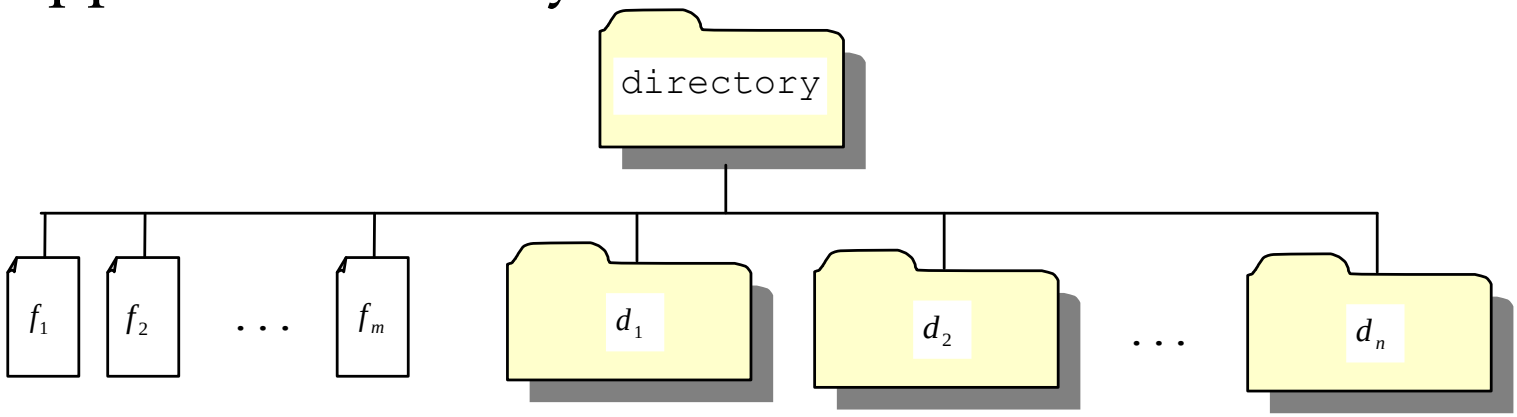
Directory Size

- Some problems are impossible to solve without recursion.
- Example: find the size of a directory.
  - The size of a directory is the sum of the sizes of all files in the directory.
  - A directory may contain subdirectories.
  - Suppose a directory contains files and subdirectories



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```
import java.io.File;
import java.util.Scanner;
public class DirectorySize {
  public static void main(String[] args) {
    System.out.print("Enter a directory or a file: ");
    Scanner input = new Scanner(System.in);
    String directory = input.nextLine();
    System.out.println(getSize(new File(directory)) + " bytes");
  }
  public static long getSize(File file) {
    long size = 0; // Store the total size of all files
    if (file.isDirectory()) {
      File[] files = file.listFiles(); // All files and subdirectories
      for (int i = 0; i < files.length; i++) {
        size += getSize(files[i]); // Recursive call
      }
    }else { // Base case
      size += file.length();
    }
    return size;
  }}
```


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Towers of Hanoi

- There are *n* disks labeled 1, 2, 3, . . ., *n*, and three towers labeled A, B, and C.
- No disk can be on top of a smaller disk at any time.
- All the disks are initially placed on tower A.
- Only one disk can be moved at a time, and it must be the top disk on the tower.

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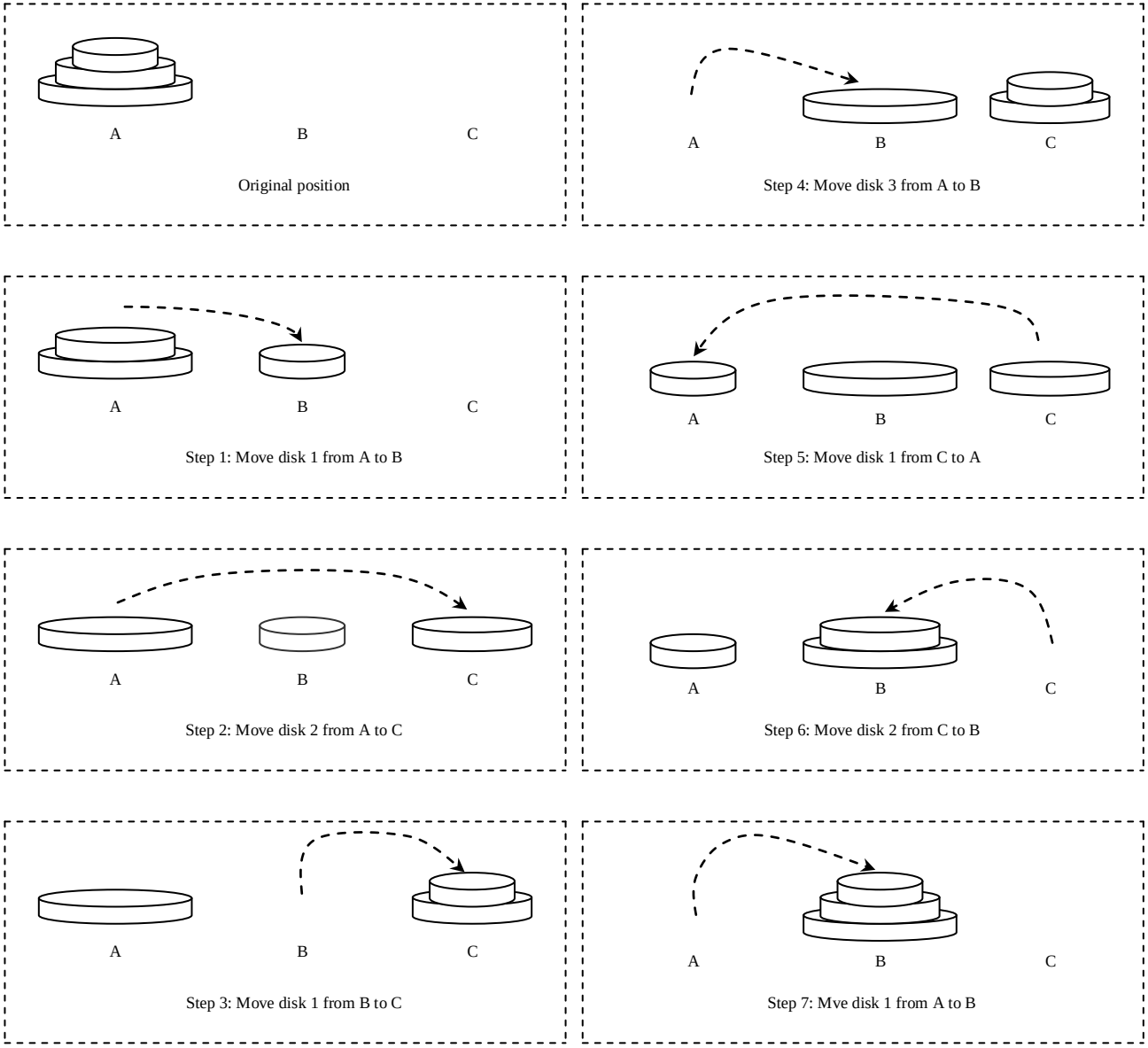

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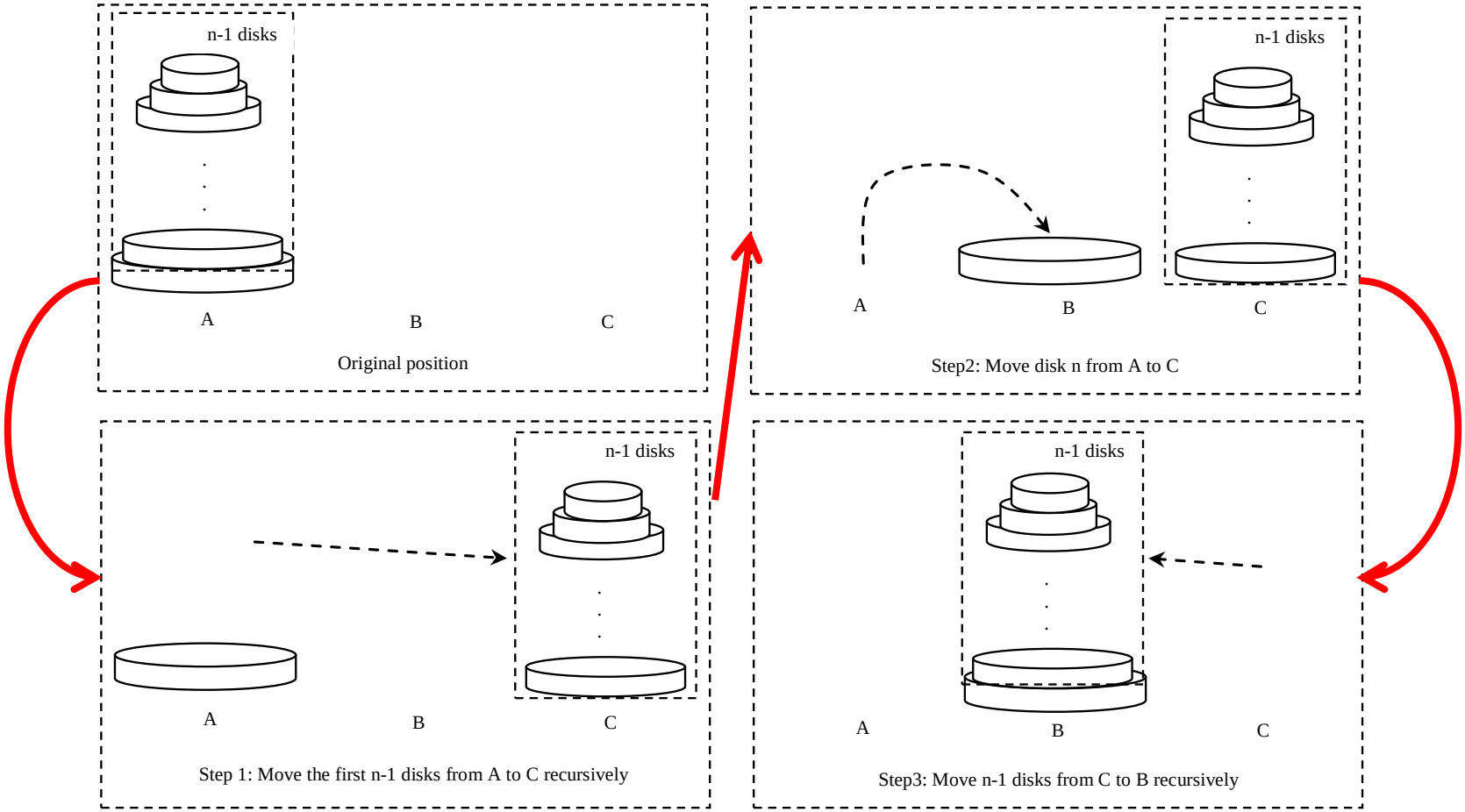
The Towers of Hanoi problem can be decomposed into three subproblems:



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Solution to Towers of Hanoi

- Move the first n - 1 disks from A to C with the assistance of tower B.
- Move disk n from A to B.
- Move n - 1 disks from C to B with the assistance of tower A.

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```
import java.util.Scanner;
public class TowersOfHanoi {
  public static void main(String[] args) {
    Scanner input = new Scanner(System.in);
    System.out.print("Enter number of disks: ");
    int n = input.nextInt(); System.out.println("The moves are:");
    moveDisks(n, 'A', 'B', 'C');
  }
  public static void moveDisks(int n, char fromTower, char toTower,
      char auxTower) {
    if (n == 1) // Stopping condition
      System.out.println("Move disk " + n + " from " +
        fromTower + " to " + toTower);
    else {
      moveDisks(n - 1, fromTower, auxTower, toTower);
      System.out.println("Move disk " + n + " from " +
        fromTower + " to " + toTower);
      moveDisks(n - 1, auxTower, toTower, fromTower);
    }
  }
}
```


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Greatest Common Divisor (GCD)

gcd(2, 3) = 1

gcd(2, 10) = 2

gcd(25, 35) = 5

gcd(205, 5) = 5

gcd(m, n):

- Approach 1: Brute-force, start from min(n, m) down to 1, to check if a number is common divisor for both m and n, if so, it is the greatest common divisor.
- Approach 2: `Euclid's` algorithm
- Approach 3: Recursive method

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Approach 1: GCD

```
public static int gcd(int m,int n){
   int min = n;
  if(m < n) min = m;
  for(int i=min; i>1; i--)
     if(m%i==0 && n%i==0)
          return i;
  return 1;
}
```


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Approach 2: Euclid's algorithm

```
// Get absolute value of m and n;
t1 = Math.abs(m); t2 = Math.abs(n);
// r is the remainder of t1 divided by t2
r = t1 % t2;
while (r != 0) {
  t1 = t2;
  t2 = r;
  r = t1 % t2;
}
// When r is 0, t2 is the greatest
// common divisor between t1 and t2
return t2;
```


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Approach 3: Recursive Method

```
gcd(m, n) = n                    if m % n = 0
gcd(m, n) = gcd(n, m % n);  otherwise
```

```
public static int gcd(int m,int n){
    if (m % n == 0) return n;
    else return gcd(n, m % n);
}
```


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From Iteration to Recursion

```
public static void m(int n){
     for(int i=1; i<=n; i++){
          for(int j=1; j<=n; j++){
               System.out.print(i+j);
          }
          System.out.println(i);
     }
}
public static void main(String[] args) {
     m(10);
}
```


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```
public static void mr(int n){
      mr(1,n);
}
public static void mr(int i, int n){
      if(i<=n){
            mr(1, i, n);
            System.out.println(i);
            mr(i+1,n);
      }
}
public static void mr(int j, int i, int n){
      if(j<=n){
            System.out.print(i+j);
            mr(j+1,i,n);
      }
}
```


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Conclusion

Conclusion for this course

- That is all!
  - I hope that this course has sparked a lot of ideas and encourages you to exercise programming
- Thank you!

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