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Hybrid modeling of multiphysical processes for particle-based volcano animation

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Abstract

Many complex natural phenomena with dramatic spatial and temporal variation are difficult to animate accurately with anticipated performance in many graphics tasks and applications, because oftentimes in prior art, a single type of physical process could not afford high fidelity and effective scene production. Volcano eruption and its subsequent interaction with earth is one such complicated phenomenon that must depend on multiphysical processes and their tight coupling. This paper documents a novel and effective particle-based solution for volcano animation that embraces multiphysical processes and their tight unification. First, we introduce a governing physical model consisting of multiphysical processes enabling flexible state transition among solid, fluid, and gas. This computational physics model is dictated by temperature and accommodates dynamic viscosity that is changing according to the temperature. Second, we propose an augmented smoothed particle hydrodynamics as the underlying numerical model to simulate the behavior of lava and smoke with several required physical attributes. Third, multiphysical quantities are tightly coupled to support the interaction with surroundings including fluid–solid coupling, ground friction, and lava–smoke coupling. We also develop a temperature-directed rendering technique with nearly no extra computational cost and demonstrate realistic graphics effects of volcano eruption and its interaction with earth with visual appeal.

KEYWORDS
heat transfer, multiphysical interaction and coupling, multiphysical processes, volcano animation

1 | INTRODUCTION AND MOTIVATION

Volcano eruption is one of the most horrific and dramatic natural phenomena on earth, which usually results in terrible disaster and huge economic loss. Its high-fidelity simulation thus has attracted great scientific attention in many relevant fields ranging from geophysics, entertainment industry, to emergency management. Despite earlier progresses, realistic simulation of volcano eruption still remains a great interest in graphics and animation for its indispensability in movie and game production that requires rapid and accurate creation of disaster scenes.

In graphics, there have been lots of natural phenomena that can be well animated with high precision so far, such as water, smoke, fire, debris flow, ice, and sand. However, compared with the aforementioned scenes, volcano eruption and its interaction with surroundings are much more complex than what one single type of physical process could handle. There exists many different types of participating media including lava, mountain, and smoke. So the coupling of different materials and multiphysical processes have to be involved simultaneously. New models with multiphysical processes must be invoked to better handle the scene production; therefore, designing such realistic models, as well as handling their effective integration, is a much more challenging task.

In recent years, prior works tend to focus on lava simulation only, rather than the entire volcano animation involving other media, resulting in the incapability of producing complete eruption scenes. However, we would like to solve this problem, and a multiphysical processes model is proposed to handle complex volcano phenomena such as lava–lava, lava–rigid interaction, state transition, and smoke generation. Our key contributions include

- Multiphysical processes modeling with state transition dictated by temperature;
- An augmented particle-based model involving all the participating media for the animation of complex phenomenon;
- Tight coupling of multiphysical quantities and their interaction with surroundings;
- A novel temperature-based rendering technique integrating concepts of Marching Cubes and SPH with nearly no extra cost.

2 | RELATED WORK

Volcano eruption includes different phenomena, for example, mainly the lava flow and its solidification and melting. Recent progresses have been made in such fields as shown below. We will also talk about smoke animation as we have involved it for the completeness of our model.

Lava animation. Extensive studies on volcano have been conducted in physics, geology, and so forth. Some numerical models can deal with certain kinds of aspects of flow, for example, Keszthelyi[1] demonstrated how the velocity, volumetric flow rate, and length of lava are influenced by relevant rheological parameters, lava cooling, channel dimensions, and crystallization. Other methods made use of temperature, crystallinity, and rheology.[2] Harris et al.[3] demonstrated a self-adaptive, kinematic, and numerical model to describe the downflow thermal and theological evolution of channel-contained lava. However, most of previous methods were based on empirically obtained equations and were difficult to apply for general conditions due to complex processes. Instead of analyzing flow dynamics, others intended to provide an aid for lava path forecast and hazard assessment, for example, SCIARA[4] model, DOWNFLOW[5] model, and model in Negro et al.[6]

From the perspective of physically based simulation, lava can be seen as liquid whose viscosity increases exponentially when the material cools down. Lagrangian approaches such as SPH[7] are more widely used in volcano animation. For example, Stora et al.[8] proposed a method that relies on smoothed particles governed by a state equation for animating the flow, while ignoring smoke simulation. Both Bilotta et al.[9] and Hrault et al.[10] presented a lava simulation model and used SPH method with GPU implementation on CUDA and enjoyed higher efficiency. In addition, Stomakhin et al.[11] documented an augmented material point (MPM) method and achieved better visual results, which could simulate the lava solidification and was then used by Jiang et al.[12] with APIC to provide interesting lava scene. However, they did not handle other phenomena during the volcano eruption. In this paper, the entire volcano animation can be handled including eruption, solidification, melting, and lava–mountain and lava–smoke coupling.

Solidification and melting. Solidification and melting appear in lava animation for the existence of heat transfer. Phase transition with SPH were introduced in Wicke et al.,[13] where particles restored forces in a locally defined lattice. Unified particle model was presented to simulate solidification and melting in Keiser et al.,[14] and they combined the solid mechanics equations with Navier-Stokes equations using a particle-based Lagrangian approach. In Solenthaler et al.,[15] all phases were represented by particles, so it handled state transition only by changing the attribute values of the underlying particles, and our method is similar while we introduce the intermediate state.

Smoke animation. Volcano usually erupts with dark, thick smoke rising. For smoke animation, both grid-based and Lagrangian methods have caught much attention. Fedkiw et al.[16] applied inviscid Euler equations that cut much computational resource, and they also introduced a physical vorticity to model turbulence. Stam et al.[17] used smoke particles and simulated smoke via smoke density, while lacking details. Selle et al.[18] introduced a hybrid technique using vortex particles that achieved better results. Recently, Macklin et al.[19] wrapped the smoke particle with fluid particles and added drag force and vorticity confinement with position-based dynamics method. Because we model our lava system based on particles, we simulate the smoke using SPH as well for convenience and add external forces.

3 | MULTIPHYSICAL PROCESSES MODEL

Volcano eruption and its subsequent interaction with earth is a complex phenomenon that includes different participating media. Here, a multiphysical processes model with particles dictated by temperature is designed to cope with such kind of animation.

3.1 | Particle-based model

Our multiphysical processes model involves different media and processes that enables to handle various phenomena in volcano eruption. Temperature is the key attribute, which dictates the appearances and evolution of different phenomena.

Figure 1 shows the overview of our method that is completely based on particles. The multiphysical processes model can be divided into 3 main parts: lava system, mountain, and smoke, and they couple with each other in different ways. Inside the lava system itself, fluid–solid coupling exists and heat transfer occurs and lava may change its state between fluid and solid accordingly. The mountain then interacts with the lava system and transfer heat. Fluid–rigid coupling and ground friction should be handled at this stage. Smoke is also generated when eruption happens. Table 1 shows the exact attributes used in our particle-based model.

For lava simulation, we describe it as a free surface fluid with complicated boundary conditions and choose SPH as our fluid solver. Basic SPH represents fluid as particles carrying attributes such as mass m_i, density ρ_i, position $\mathbf{x}_i$, pressure p_i, and velocity $\mathbf{v}_i$ and interpolates fluid quantity A_i with a set of known quantities A_j at neighboring particle positions $\mathbf{x}_j$:

$$A_i = \sum_j \frac{m_j}{\rho_j} A_i W_{ij}, \tag{1}$$

where W_{ij} is a smoothing kernel function of the form $W_{ij} = W(\mathbf{x}_{ij}, h)$ with $\mathbf{x}_{ij} = \mathbf{x}_i - \mathbf{x}_j$ and h is the supporting radius. At each simulation step, pressure force, viscosity force, and other external forces $\mathbf{F}_i^{other}$ compose the overall force and govern the acceleration as below:

$$\mathbf{a}_i = -\frac{1}{\rho_i}\nabla p_i + \mu\nabla^2\mathbf{v}_i + \frac{\mathbf{F}_i^{other}}{m_i}. \tag{2}$$

Because lava is incompressible, we choose predictive-corrective incompressible SPH (PCISPH)[20] to enforce the incompressibility.

To satisfy the needs of our multiphysical processes model, we also augment the particle of traditional SPH with extra attributes for lava as shown in Table 1 including its temperature, current state to handle state transition, thermal conductivity according to its state, and dynamic viscosity coefficient varying with temperature.

Temperature influences the lava mainly on viscosity, and it is common that the viscosity of fluid tends to decrease as its temperature rises and increase as its temperature drops such as melting chocolate and honey and lava flow. Hence, dynamic viscosity is introduced, and details can be found in Section 3.2.

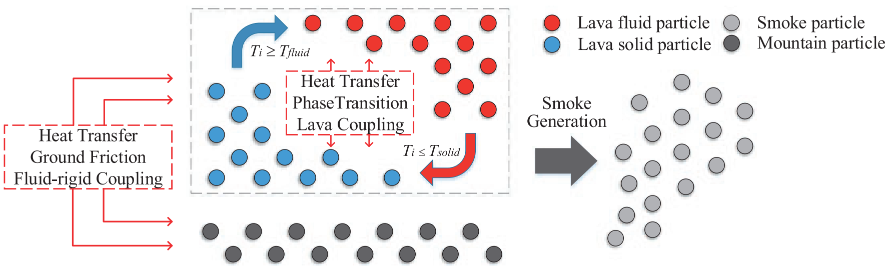

FIGURE 1 Overview of the whole framework

TABLE 1 Required attributes and their notations of different particles

Attributes	Description	Lava	Mountain	Smoke
m	Mass	√	√	√
$\mathbf{x}$	Position	√	√	√
$\mathbf{v}$	Velocity	√		√
$\mathbf{a}$	Acceleration	√		√
state	Particle state	√	√	√
ρ	Density	√	√	√
p	Pressure	√		√
k	Thermal conductivity Coefficient	√	√	
T	Temperature	√	√	
μ	Dynamic viscosity Coefficient	√		
$\mathbf{n}$	Surface normal		√	
lifetime	How long since the particle was generated			√

Besides, temperature also controls the generation of smoke that appears during the eruption due to the high temperature. Hence, introducing smoke is not only a complement but also a must to ensure the realistic animation, whereas previous works rarely focused on this topic. In this paper, we simulate the motion of smoke mainly using SPH with buoyancy while ignoring the viscosity force. To introduce turbulent motion, we add a driving force according to Macklin et al.[19]

Although volcanoes may move along with the Earth's crust, we make an assumption that the mountain keeps still in our animation considering its relatively slow movement compared with others substances. Thus, the mountain is represented as stationary particles in animation. Its temperature can influence the behavior of lava through heat transfer (Section 3.3) and cause state transition (Section 3.4).

3.2 | Dynamic viscosity

The temperature of lava changes in a wide range that reveals an obvious viscosity changing. To simulate such viscous fluid, Takahashi et al.[21] proposed an implicit method. In consideration of such effect, the temperature–viscosity correlation is introduced to capture the characteristic of such material using the equation in Seeton[22]:

$$log(log(\mu_i + \gamma)) = q - y * log(T_i), \tag{3}$$

where μ_i is the viscosity coefficient of particle i, γ denotes an additive constant usually ranging from 0.6 to 0.9, and q, y are specific parameters.

3.3 | Heat transfer

To simulate the temperature change, we model the heat transfer that occurs when lava interacts with each other or the external world analogously to the general heat equation in Stora et al.[8]:

$$\frac{dT}{dt} = k\nabla^2 T. \tag{4}$$

As different thermal conductivity coefficient k_i is applied for particles of different states (see Section 3.4), the right term is adapted to the SPH formalism as below:

$$\begin{aligned}\nabla^2(k_iT_i) = 2\sum_j \frac{m_j}{\rho_j}(k_jT_j - k_iT_i)\frac{\mathbf{x}_{ij}\cdot\nabla W_{ij}}{\mathbf{x}_{ij}\cdot\mathbf{x}_{ij} + 0.01h^2} \\ + 2\sum_b \frac{m_b}{\rho_b}(k_bT_b - k_iT_i)\frac{\mathbf{x}_{ib}\cdot\nabla W_{ib}}{\mathbf{x}_{ib}\cdot\mathbf{x}_{ib} + 0.01h^2},\end{aligned} \tag{5}$$

where j and b represent the fluid and boundary particle neighboring, and $\mathbf{x}_{ib} = \mathbf{x}_i - \mathbf{x}_b$. With temperature updated, we recompute the dynamic viscosity coefficient according to Equation 3 to reflect the influence of temperature on viscosity.

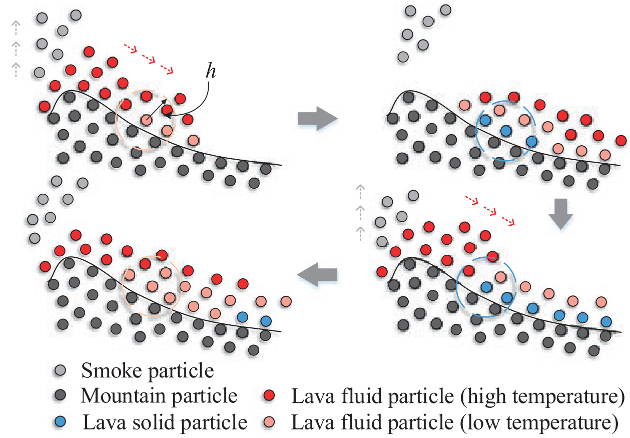

FIGURE 2 State transition during the animation. First, hot lava generates smoke and turns into solid after interacting with the Earth. Then during another eruption, the solid lava melts again as new hot lava comes

3.4 | State transition

In reality, lava cools down as flowing downhill, and it shares two different states, namely, fluid and solid lava. Our method to handle solidification and melting is similar to Solenthaler et al.[15]: each sort of particle stores a melting temperature T_{melt} and solidification temperature $T_{solidify}$.

While they chose $T_{melt} = T_{solidify}$ so that particle changes its state directly between liquid and solid, we allow $T_{melt} > T_{solidify}$ and introduce intermediate state considering that state transition cannot occur at exactly the same temperature everytime. Particles at the intermediate state keep the previous state with the thermal conductivity coefficient k_i interpolated linearly between liquid and solid conductivity. As lava turns solid, the interaction between lava fluid and lava solid is handled as fluid–rigid coupling in Section 4.1.

Considering that high-temperature lava usually creates plenty of smoke when bursting out, we set a threshold T_{smoke} as well, and lava particles can generate smoke particles once their temperature reaches T_{smoke}. Details of smoke generation are illustrated in Section 4.3.

Figure 2 shows how state transition happens, and we execute it as Algorithm 1. In this step, we embrace all the states together and integrate the whole system under control of temperature.

4 | COUPLING IN VOLCANO ANIMATION

The complexity of volcano animation is mainly due to its interaction with the environment. Because we have involved multiphysical processes, we also focus on the synchronization of the coupling between different participating media.

Algorithm 1 State Transition

if $state_i$ ==FLUID **then**
  if $T_i \le T_{solidify}$ **then**
    $state_i$ =SOLID
  if $T_i \ge T_{smoke}$ **then**
    Generate smoke particles
if $state_i$ ==SOLID & $T_i \ge T_{melt}$ **then**
  $state_i$ =FLUID
Interpolate k_i linearly

4.1 | Fluid–rigid coupling

When lava moves downhill, it interacts with the mountain and solid lava that can be regarded as fluid–rigid coupling. Some methods have been proposed on boundary handling[23,24] and so forth. The fluid particle at $\mathbf{x}_i$ is considered to penetrate the boundary at position $\mathbf{x}_b$ if $\|\mathbf{x}_i - \mathbf{x}_b\| < r_0$, where r_0 is the spacing of the boundary particles. As proposed in Ihmsen et al,[23] we compute the average normal $\mathbf{n}_i^{ave}$ of all boundary particles penetrated by particle i and correct the position of particle i towards $\mathbf{n}_i^{ave}$, whereas the new velocity is computed as

$$\mathbf{v}_i(t+\Delta t) = \varepsilon[\mathbf{v}_i^*(t+\Delta t)]_t, \tag{6}$$

where $[\mathbf{v}_i^*(t+\Delta t)]_t$ is the tangential velocity of the predicted velocity according to $\mathbf{n}_i^{ave}$ and ε is a parameter controlling the elasticity of collision. The densities of lava and mountain particles are then updated in each prediction step as

$$\rho_i^*(t+\Delta t) = \sum_j m_j W_{ij}^* + \sum_b m_b W_{ib}^*, \tag{7}$$

where $W_{ij}^* = W(\mathbf{x}_i^* - \mathbf{x}_j^*, h)$, $W_{ib}^* = W(\mathbf{x}_i^* - \mathbf{x}_b, h)$ with $\mathbf{x}_i^*$, $\mathbf{x}_j^*$ being the predicted position.

4.2 | Ground friction

Because lava moves slowly along the mountain, ground fiction should also be invoked. Unlike Hérault et al,[25] which used tangent velocity and contact surface to compute friction force, we apply the momentum law to first obtain the collision force $\mathbf{F}^{col}$ that mountain exerting on the fluid lava. As the conservation of momentum, $\mathbf{F}^{col}$ is related with the collision time, mass, and velocity variation. Here, we take the animation time step Δt as collision time, $\mathbf{v}(t)$ as the before-collision velocity, and $\mathbf{v}(t+\Delta t)$ computed in Equation 6 as the after-collision velocity. Hence, the collision force $\mathbf{F}^{col}$ on a lava fluid particle i is

$$\mathbf{F}^{col} = \frac{m_i(\mathbf{v}_i(t+\Delta t) - \mathbf{v}_i(t))}{\Delta t}. \tag{8}$$

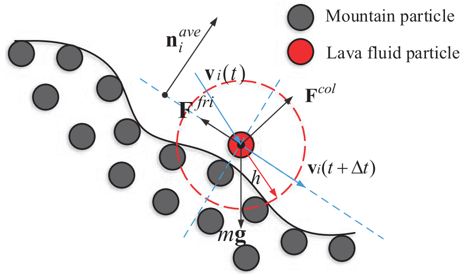

FIGURE 3 A fluid particle contacting the Earth

Together with the gravity force, particle i receives a ground friction:

$$\mathbf{F}^{fri} = -\lambda((m_i g - \mathbf{F}^{col}) \cdot \mathbf{n}_i^{ave}) \frac{\mathbf{v}_i(t+\Delta t)}{\|\mathbf{v}_i(t+\Delta t)\|}, \tag{9}$$

where $\mathbf{g}$ is the gravity and λ is the friction factor that artists can control the speed of lava flow by tuning it. Figure 3 shows how a lava fluid particle contacts the Earth.

4.3 | Lava–smoke coupling

Concerning the characteristic of smoke, we add *lifetime* to smoke particle as additional attribute to indicate how long the smoke particle has been generated, and we delete the smoke particle once it lives beyond the life cycle.

When dealing with the smoke generation, we are inspired by the work of Ihmsen et al.[26] Their generation of diffuse particles to enhance details is quite similar to the idea of creating smoke particles. In our model, only lava fluid particles with temperature higher than a certain threshold can generate smoke particles. As Figure 4 shows, according to the position $\mathbf{x}_i$ and velocity $\mathbf{v}_i$ of fluid particle i, we obtain a reference plane spanned by e_1 and e_2 and form a cylindrical volume. Three random variables X_r, X_θ, X_h ranging from 0 to 1 uniformly are designed to add randomness into the smoke system. The smoke particle is then initialized at

$$\mathbf{x}_s = \mathbf{x}_i + r\cos\theta e_1 + r\sin\theta e_2 + d\hat{\mathbf{v}}_i, \tag{10}$$

where $\theta = 2\pi X_\theta$, $d = \mathbf{v}_i \Delta t\, X_d$, and $r = R\sqrt{X_r}$ and R is the volume radius, $\hat{\mathbf{v}}_i$ is the normalized velocity. Its initial velocity is

$$\mathbf{v}_s = \alpha(\mathbf{v}_i + r\cos\theta e_1 + r\sin\theta) + \frac{\mathbf{F}^{buo}}{m_s}\Delta t, \tag{11}$$

with $\mathbf{F}^{buo}$ being the buoyancy and α being a positive parameter.

5 | IMPLEMENTATION AND RESULTS

5.1 | Rendering and simulation details

Because we modeled lava carefully considering the impact of temperature, a novel efficient method to render the lava

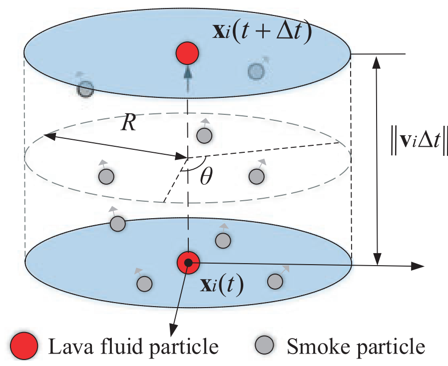

FIGURE 4 Smoke generation step. Lava fluid particle *i* generates smoke particles uniformly in a cylinder according it current position and velocity

FIGURE 5 The temperature color map

according to its local temperature is also proposed. To obtain the temperature of each mesh, we combine the idea of both Marching Cubes (MC)[27] and SPH. While computing the scalar values for a MC grid vertex, we also regard the weighted sum of temperature T_j of contributing particles j around as the temperature of the MC grid vertex T_v^{grid}, which depends on the interpolation idea of SPH:

Algorithm 2 Pseudo code of volcano animation

```
while animating do
    Search neighborhood
    for lava and mountain particles do
        Transfer heat and update temperature
    for lava particle i do
        Handle state transition
        Obtain viscosity coefficient μ_i
    for smoke particle s do
        Update position and velocity
    for lava fluid particle i do
        Apply PCISPH
        if i contacts the ground then
            Handle lava-rigid collisions
            Compute ground friction
            Correct its position and velocity
    Surface extraction and rendering
```

$$T_v^{grid} = \sum_j T_j W_{ij} / \sum_j W_{ij}. \tag{12}$$

The temperature of each surface vertex is then interpolated linearly at the triangulation stage, and we take the average temperature of the three triangle mesh vertices as the mesh temperature and render lava as Figure 5 shows. Because the temperature computation step can be executed in the same loop of the scalar field computation of MC, its computational cost is negligible.

Algorithm 2 details our algorithmic pipeline. We have documented a complete simulation step and involved all the processes above. In addition to the previous parallel SPH implementation, for example, Ihmsen et al.[28] and Harada et al.,[29] the heat transfer part is parallelized as well with CUDA to speed up.

5.2 | Results and discussion

According to the models and principles above, we set up several scenes to test our method. All experiments ran on a

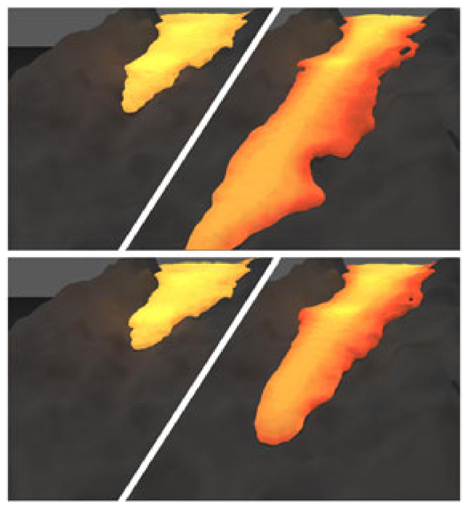

FIGURE 6 **Slope:** Comparison of lava flowing along the slope using constant viscosity (upper) and our dynamic viscosity method (lower), respectively

FIGURE 7 **"CASA"**: Bird's eye view of lava rushing down the mountain with basic fluid–rigid coupling (left) and our ground friction (right)

desktop with 3.3 GHz Intel i5-4590 CPU, 8.0 GB RAM, and an NVIDIA GTX 960 graphic card.

To illustrate our viscosity model, Figure 6 compares our dynamic viscosity method with constant viscosity method. Lava flowed along the slope in this scene. The constant viscosity was set relatively low according to the original temperature, and the lava almost flowed at a steady fast speed. Our method enables the lava to appear viscous as temperature drops and move at a lower speed that distinguishes the behavior of lava of different temperature.

Figure 7 shows the bird's eye view of lava rushing down the mountain that compares our ground friction method with basic fluid–rigid coupling. As we can see in many volcano videos, though the lava may be at a high speed when it breaks out, it quickly slows down once contacts the ground and becomes viscous. Just as shown in Figure 7 and in the supplemental video, our ground friction has slowed down the lava immediately that avoids the lava to look like flood instead as left column shows.

Figure 8 animates the "Eruption" scene with up to 1.3 M lava particles, 0.6 M smoke particles, and 3 M mountain particles. It integrates all the processes aforementioned. Statistics of different frames are in Table 2. Eruption happened intermittently, and the number of lava and smoke particles increased, causing the rise of simulation and surface extraction time. Meanwhile, more fluid lava began to solidify at lower altitudes and caused the increase of lava solid particles. Finally, smoke particles vanished as they reached lifetime. With close-ups and temperature fields, we provide details of how our rendering method works and heat transfer happens.

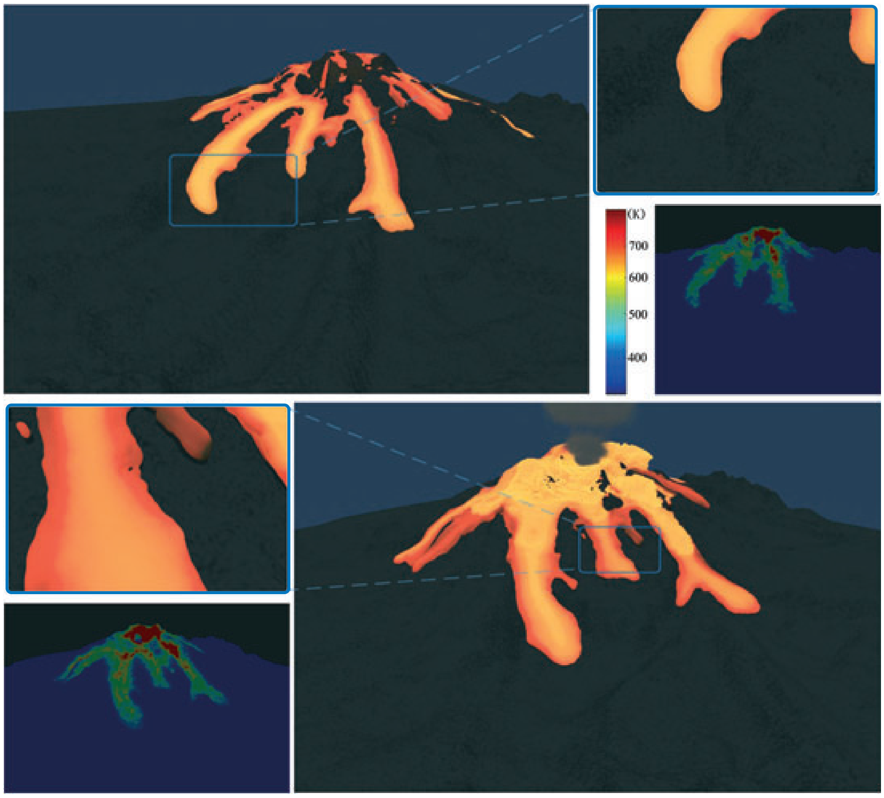

FIGURE 8 **Eruption**: Volcano eruption animation at two different frames with close-ups and temperature fields using our multiphysical processes model. We have involved all the processes mentioned in this paper and maintained tight coupling of the entire system

TABLE 2 Statistics of different frames of "Eruption" using our method

Frame No.	#Lava fluid particles	#Lava solid particles	#Lava particles	#Mountain particles	#Smoke particles	Heat transfer and state transition time (s)	Total simulation time (s)	Surface extraction time (s)
100	378,376	0	378,376	3,001,740	209,281	0.382	3.006	28.531
400	583,155	6,821	589,976	3,001,740	411,050	0.401	3.320	30.582
700	1,265,749	37,227	1,302,976	3,001,740	622,481	0.733	4.108	62.582
1000	1,152,231	150,745	1,302,976	3,001,740	113,255	0.715	4.010	65.190

TABLE 3 Comparison of recent works and our proposed method

Recent works	Model	State transition	Smoke	External interaction	Rendering results
Stora et al.[8]	SPH	No	No	Single	Good
Negro et al.[6]	CNNs	No	No	Single	Only numerical results
Stomakhin et al.[11] and Jiang et al.[12]	MPM	Yes	No	Single	Better
Our method	SPH	Yes	Yes	Multiphysics	Better

Note. CNN = cellular nonlinear network; MPM = material point method; SPH = smoothed particle hydrodynamics.

Table 3 documents the comparison between our method and other methods for volcano or lava animation. We have covered more aspects like state transition and smoke. The multiphysical processes model also considers more external interaction and achieves better performance in results thanks to our rendering method.

6 | CONCLUSION AND FUTURE WORK

This paper has proposed an effective multi-physical processes model for particle-based volcano animation. Necessary physical quantities have been introduced into our model to simulate various accompanying phenomena and its subsequent interaction with the environment. Towards the goal of realistic animation, multiphysical quantities are tightly integrated, and an efficient rendering technique has been designed. In addition, the introduction of dynamic viscosity and intermediate state also helps maintain the details and obey the laws of physics.

Meanwhile, our work still has some limitations. Efficiency may be a great concern for animation of large-scale scenes, and we would like to continue to reduce the computational cost in the near future. The proposed model has simplified the lava as pure fluid; however, in reality, it is a mixture consisting of liquid, ash, stones, and so forth. We could take moving obstacles and crustal movement into consideration and handle two-way coupling in the future. Lava–smoke coupling should also be better handled. More accurate multiphase and multiphysical models shall be explored for precise production of complex scenes like the eruption process, where high-speed lava is bursting out into the air.

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SUPPORTING INFORMATION

Additional Supporting Information may be found online in the supporting information tab for this article.

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