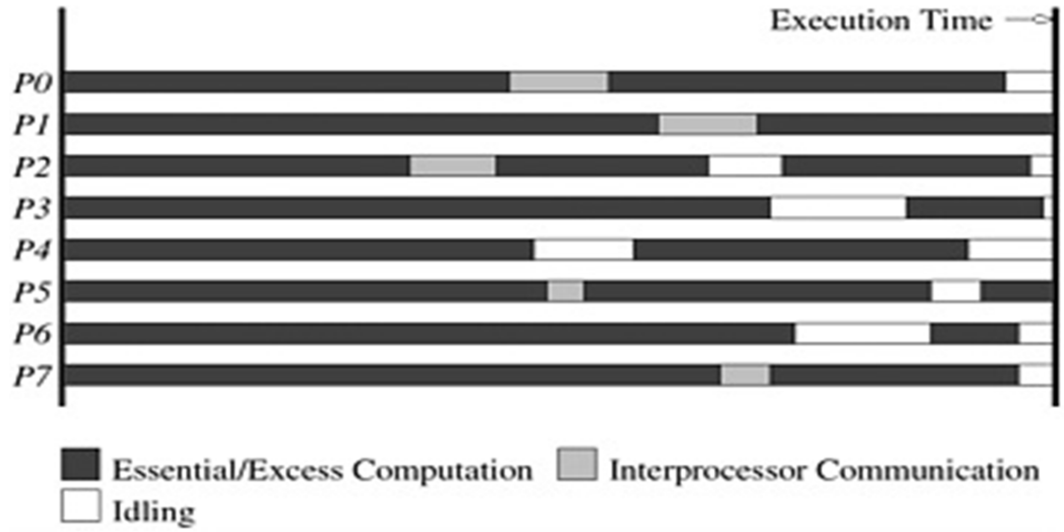
CSE 613: Parallel Programming

Lecture 2
(Analytical Modeling of Parallel Programs)

Rezaul A. Chowdhury

Department of Computer Science
SUNY Stony Brook
Spring 2012

Parallel Execution Time & Overhead



Source: Grama et al., "Introduction to Parallel Computing", 2nd Edition

Serial running time = ***T***

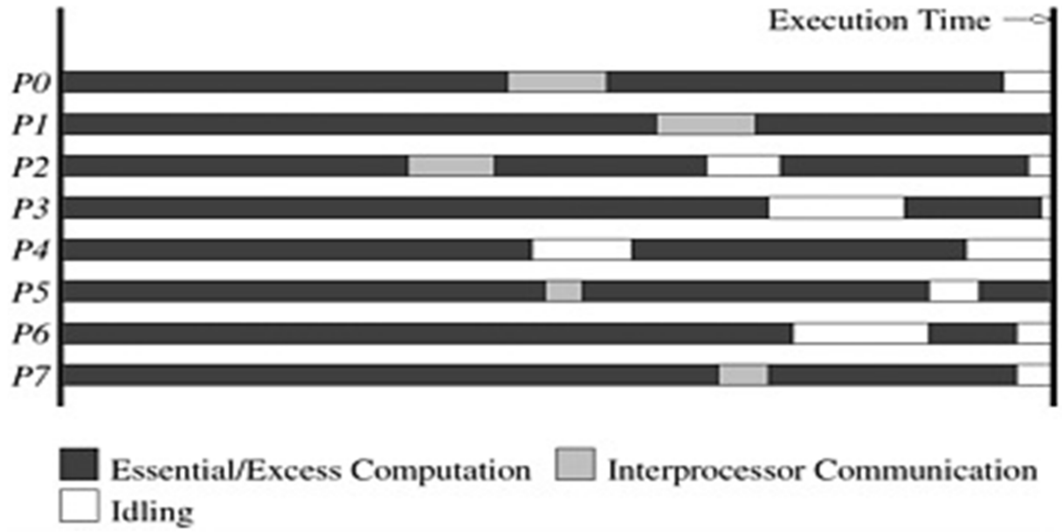
Parallel running time on *p* processing elements,

$$\boldsymbol{T_P = t_{end} - t_{start}},$$

where, $\boldsymbol{t_{start}}$ = starting time of the processing element that starts first

$\boldsymbol{t_{end}}$ = termination time of the processing element that finishes last

Parallel Execution Time & Overhead

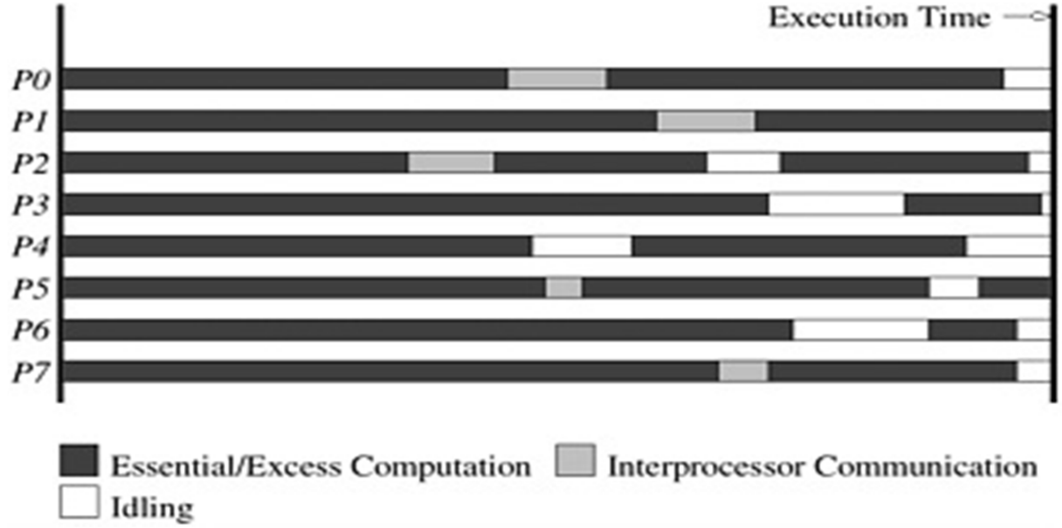


Source: Grama et al., "Introduction to Parallel Computing", 2nd Edition

Sources of overhead (w.r.t. serial execution)

- Interprocess interaction
  - Interact and communicate data (e.g., intermediate results)
- Idling
  - Due to load imbalance, synchronization, presence of serial computation, etc.
- Excess computation
  - Fastest serial algorithm may be difficult/impossible to parallelize
  - Reuse of intermediate results may be difficult (e.g., FFT)

Parallel Execution Time & Overhead



Source: Grama et al., "Introduction to Parallel Computing", 2nd Edition

Overhead function or total parallel overhead,

$$T_O = pT_p - T,$$

where, p = number of processing elements

T = time spent doing useful work

(often execution time of the fastest serial algorithm)

Speedup

Speedup, $\boldsymbol{S_p} = \frac{\boldsymbol{T}}{\boldsymbol{T_p}}$

where, $\boldsymbol{T}$ = time to solve on a single processing element

(often the runtime of the fastest serial algorithm)

$\boldsymbol{T_p}$ = time to solve in parallel on p identical processing

Theoretically, $\boldsymbol{S_p} \leq \boldsymbol{p}$ (why?)

Perfect or linear or ideal speedup if $\boldsymbol{S_p} = \boldsymbol{p}$

Speedup

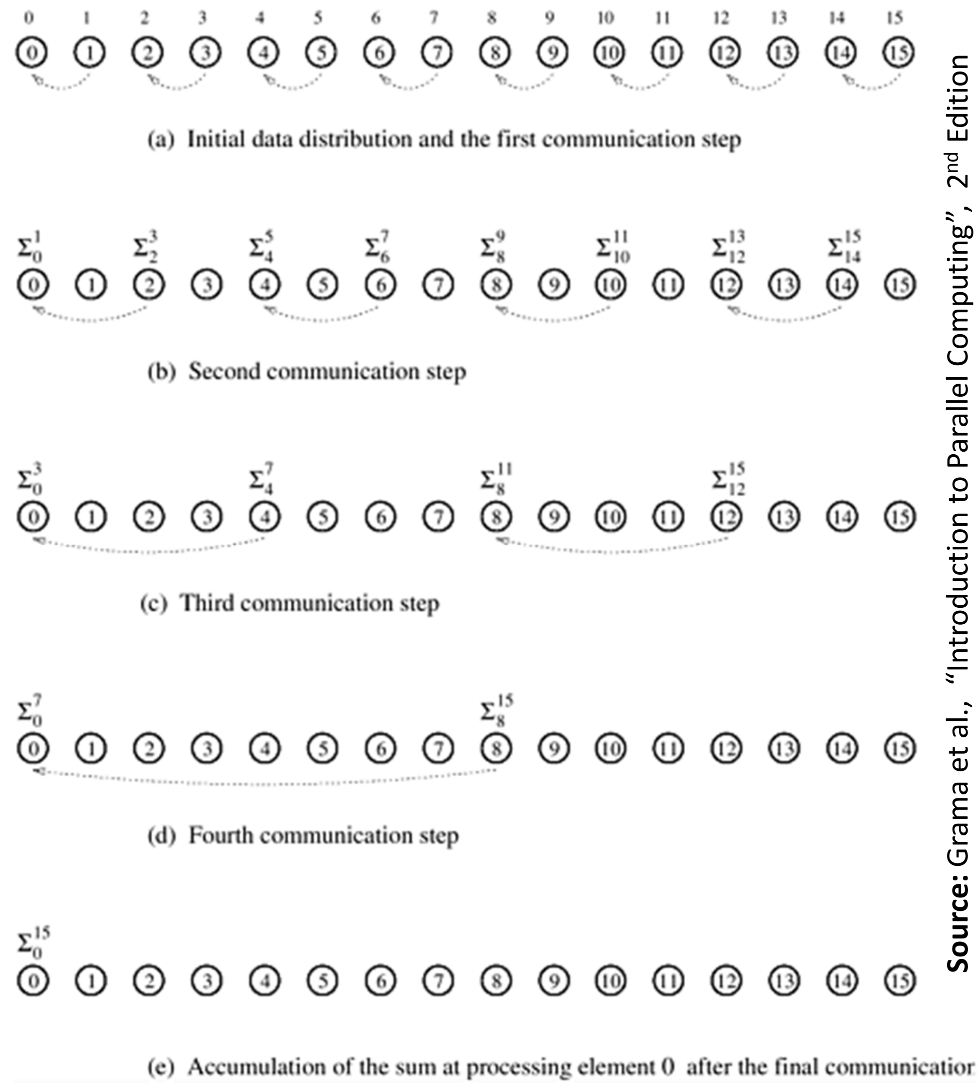
Consider adding *n* numbers using *n* identical processing elements.

Serial runtime, $T = \Theta(n)$

Parallel runtime, $T_n = \Theta(\log n)$

Speedup, $S_n = \frac{T}{T_n} = \Theta\left(\frac{n}{\log n}\right)$

Speedup not ideal. (why?)



Source: Grama et al., "Introduction to Parallel Computing", 2nd Edition

Superlinear Speedup

Theoretically, $\boldsymbol{S_p} \leq \boldsymbol{p}$

But in practice *superlinear speedup* is sometimes observed, that is, $\boldsymbol{S_p} > \boldsymbol{p}$ (why?)

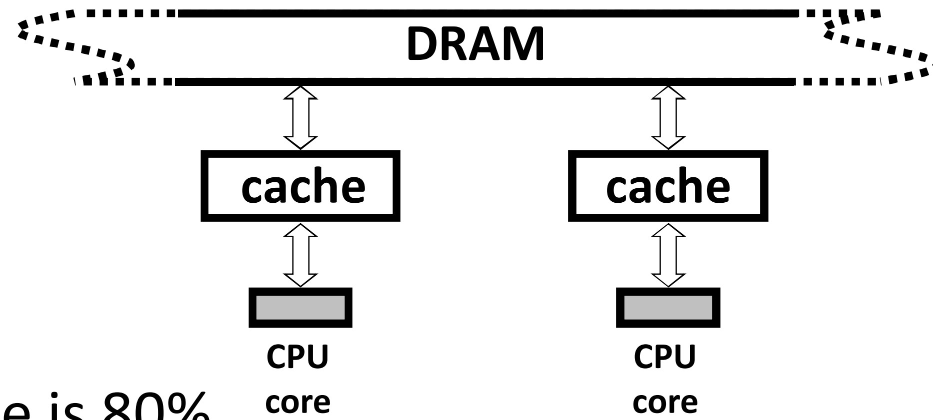
Reasons for superlinear speedup

- Cache effects
- Exploratory decomposition

Superlinear Speedup
(Cache Effects)

Let cache access latency = 2 ns

DRAM access latency = 100 ns



Suppose we want solve a problem instance that executes *k* FLOPs.

With 1 Core: Suppose cache hit rate is 80%.

If the computation performs 1 FLOP/memory access, then each FLOP will take 2 × 0.8 + 100 × 0.2 = 21.6 ns to execute.

With 2 Cores: Cache hit rate will improve. (why?)

Suppose cache hit rate is now 90%.

Then each FLOP will take 2 × 0.9 + 100 × 0.1 = 11.8 ns to execute.

Since now each core will execute only *k* / 2 FLOPs,

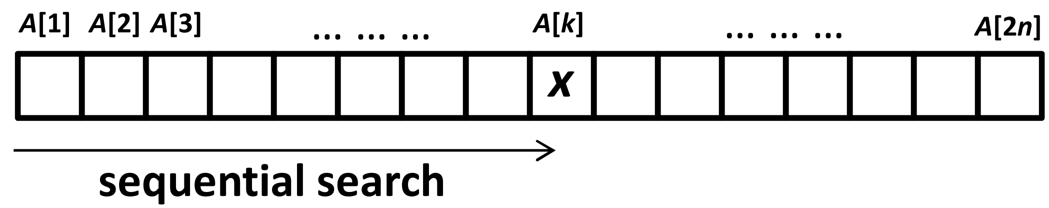
Speedup, $S_n = \frac{k \times 21.6}{(k/2) \times 11.8} \approx 3.66 > 2!$

Superlinear Speedup
(Due to Exploratory Decomposition)

Consider searching an array of $2n$ unordered elements for a specific element x.

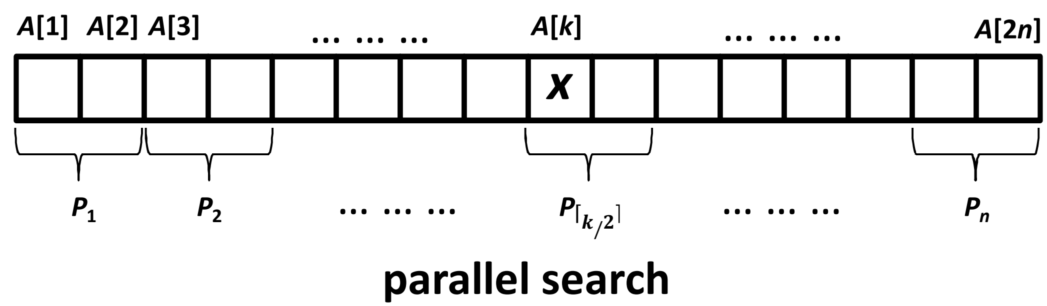
Suppose x is located at array location $k > n$ and k is odd.

Serial runtime, $T = k$



Parallel running time with n processing elements, $T_n = 1$

Speedup, $S_n = \frac{T}{T_n} = k > n$



Speedup is superlinear!

Efficiency

Efficiency, $\boldsymbol{E_p} = \dfrac{\boldsymbol{S_p}}{\boldsymbol{p}}$

Efficiency is a measure of the fraction of time for which a processing element is usefully employed.

In an ideal parallel system, $S_p = p$ and $E_p = 1$.

Consider again the example of adding *n* numbers using *n* identical processing elements.

Speedup, $S_n = \dfrac{T}{T_n} = \Theta\left(\dfrac{n}{\log n}\right)$

Efficiency, $E_n = \dfrac{S_n}{n} = \Theta\left(\dfrac{1}{\log n}\right)$

Cost or Work

The cost of solving (or work performed for solving) a problem:

On a Serial Computer: is the execution time T of the fastest known sequential algorithm for solving the problem.

On a Parallel Computer: is given by pT_p.

A parallel algorithm is *cost-optimal* or *work-optimal* provided

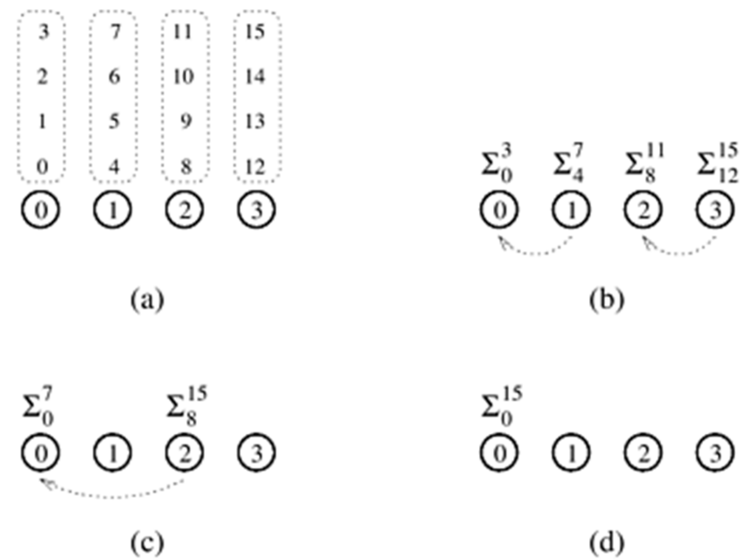
$$pT_p = \Theta(T)$$

For a work-optimal parallel algorithm: $E_p = \frac{S_p}{p} = \frac{T}{pT_p} = \Theta(1)$

Our algorithm for adding *n* numbers using *n* identical processing elements is clearly not cost optimal.

<u>Adding *n* Numbers Work-Optimality</u>

We reduce the number of processing elements which in turn increases the granularity of the subproblem assigned to each processing element.



Source: Grama et al., "Introduction to Parallel Computing", 2nd Edition

Suppose we use *p* processing elements.

First each processing element locally adds its $\frac{n}{p}$ numbers in time $\Theta\left(\frac{n}{p}\right)$.

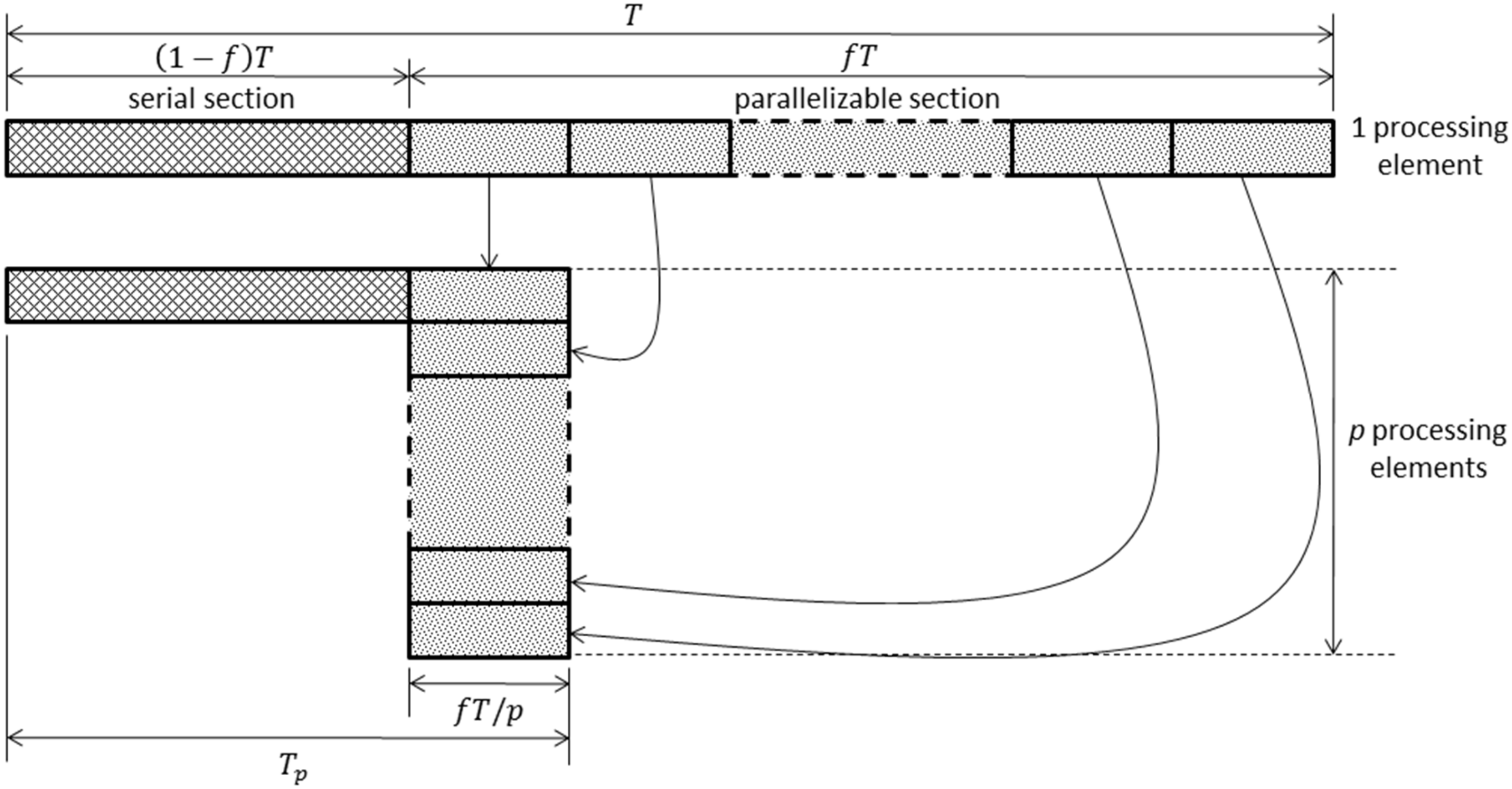
Then *p* processing elements adds these *p* partial sums in time $\Theta(\log p)$.

Thus $T_p = \Theta\left(\frac{n}{p} + \log p\right)$, and $E_p = \frac{S_p}{p} = \frac{T}{pT_p} = \frac{\Theta(n)}{\Theta(n+p\ \log p)}$.

So the algorithm is work-optimal, i.e., $E_p = \Theta(1)$, provided

$$n = \Omega(p \log p)$$

Scaling of Parallel Algorithms (Amdahl's Law)



Suppose only a fraction f of a computation can be parallelized.

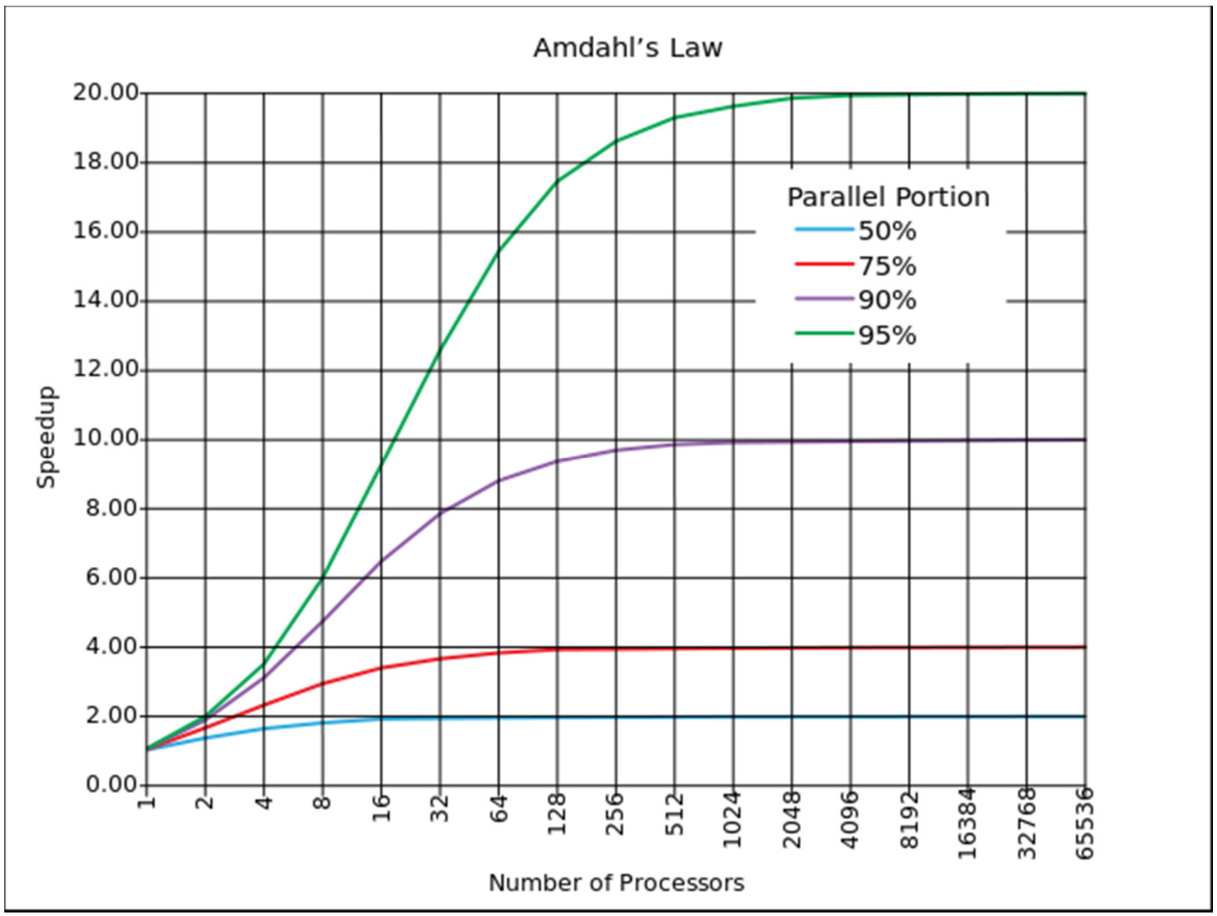
Then parallel running time, $T_p \geq (1-f)T + f\frac{T}{p}$

Speedup, $S_p = \frac{T}{T_p} \leq \frac{p}{f+(1-f)p} \leq \frac{1}{1-f}$

Scaling of Parallel Algorithms (Amdahl's Law)

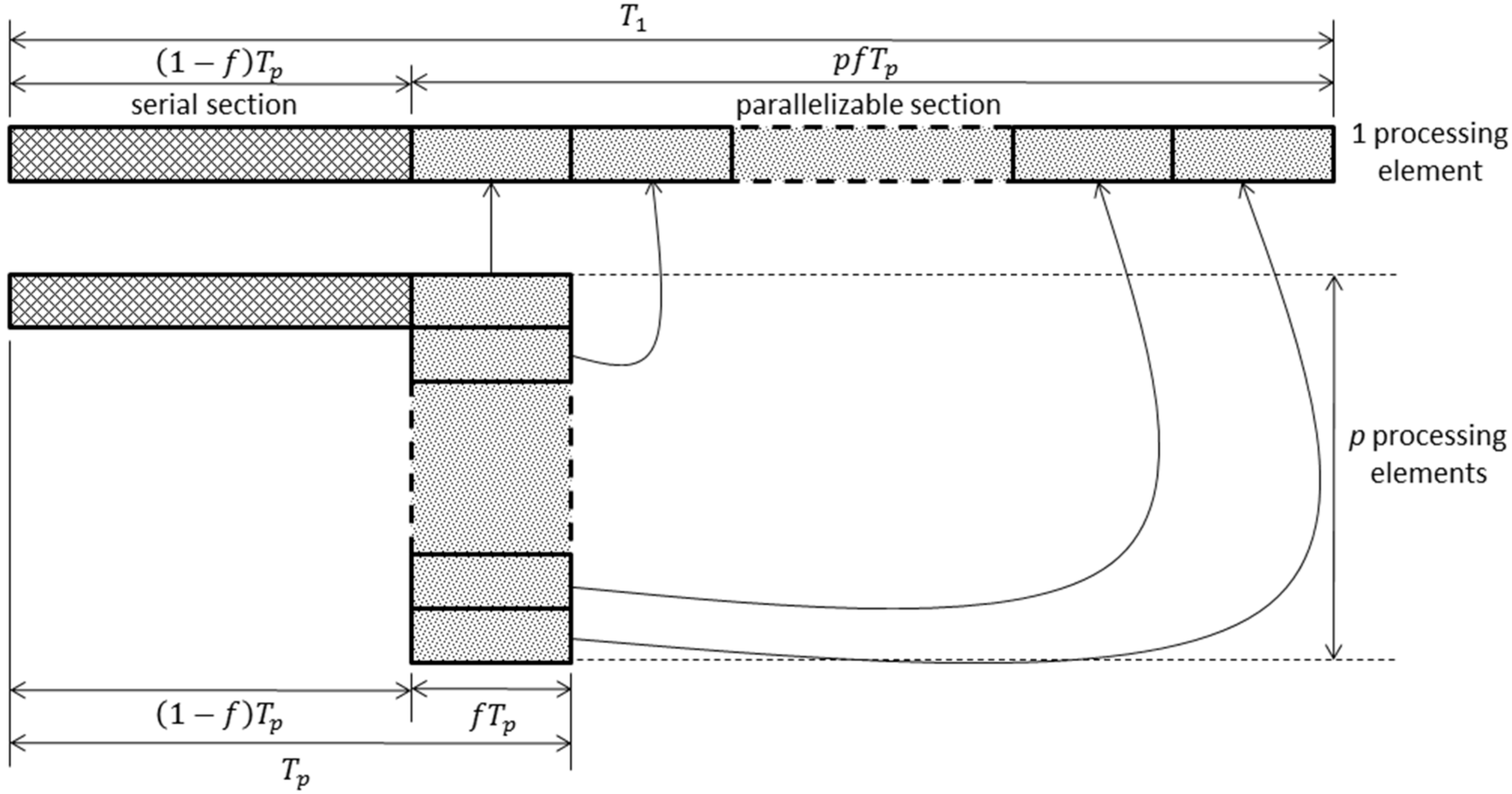
Suppose only a fraction f of a computation can be parallelized.

Speedup, $S_p = \frac{T}{T_p} \leq \frac{p}{f + (1-f)p} \leq \frac{1}{1-f}$



Source: Wikipedia

Scaling of Parallel Algorithms (Gustafson-Barsis' Law)



Suppose only a fraction f of a computation was parallelized.

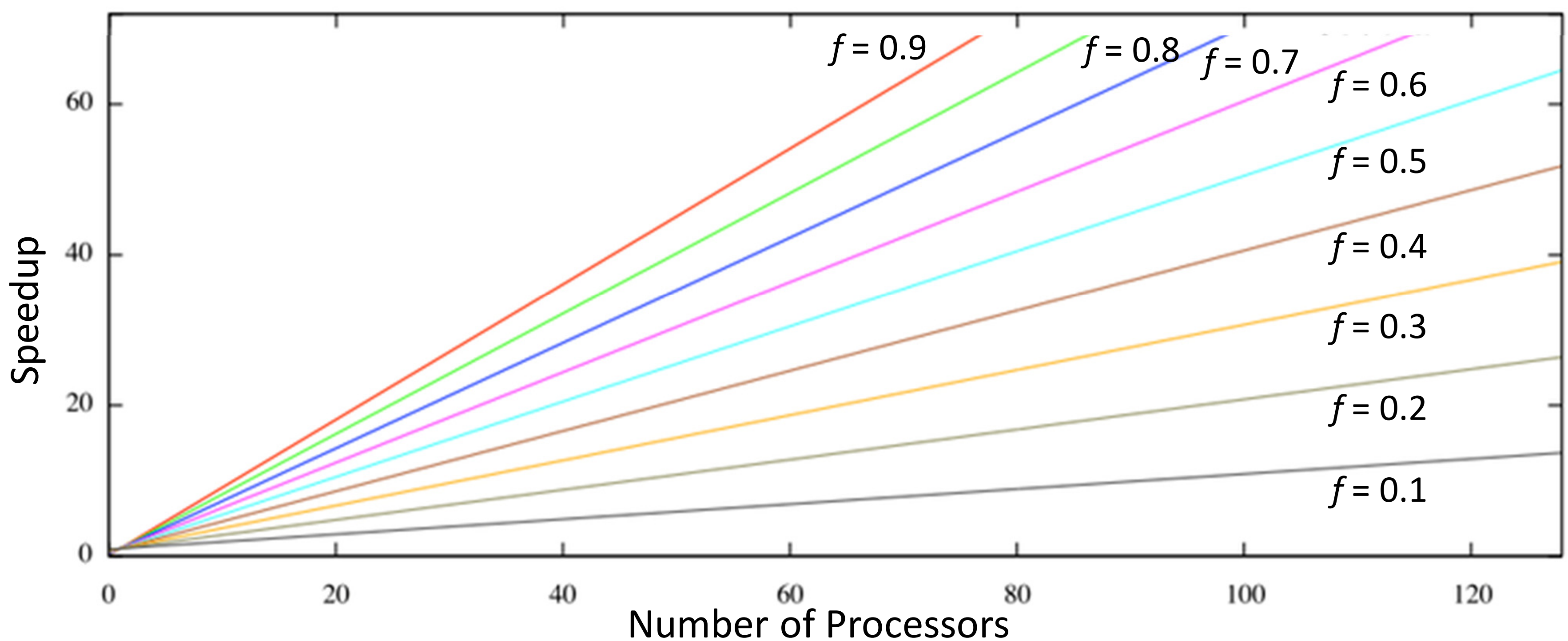
Then serial running time, $T \leq T_1 = (1-f)T_p + pfT_p$

Speedup, $S_p = \frac{T}{T_p} \leq \frac{T_1}{T_p} = \frac{(1-f)T_p + pfT_p}{T_p} = 1 + (p-1)f$

Scaling of Parallel Algorithms (Gustafson-Barsis' Law)

Suppose only a fraction f of a computation was parallelized.

Speedup, $S_p = \frac{T}{T_p} \leq \frac{T_1}{T_p} = \frac{(1-f)T_p + pfT_p}{T_p} = 1 + (p-1)f$



Source: Wikipedia

Scalable Parallel Algorithms

A parallel algorithm is called *scalable* if its efficiency can be maintained at a fixed value by simultaneously increasing the number of processing elements and the problem size.

Scalability reflects a parallel algorithm's ability to utilize increasing processing elements effectively.

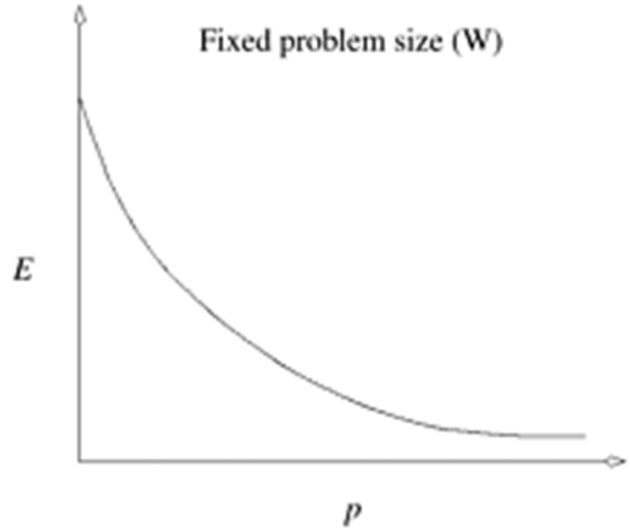
Efficiency, $E_p = \frac{S_p}{p} = \frac{T}{pT_p} = \frac{T}{T+T_O} = \frac{1}{1+\frac{T_O}{T}}$

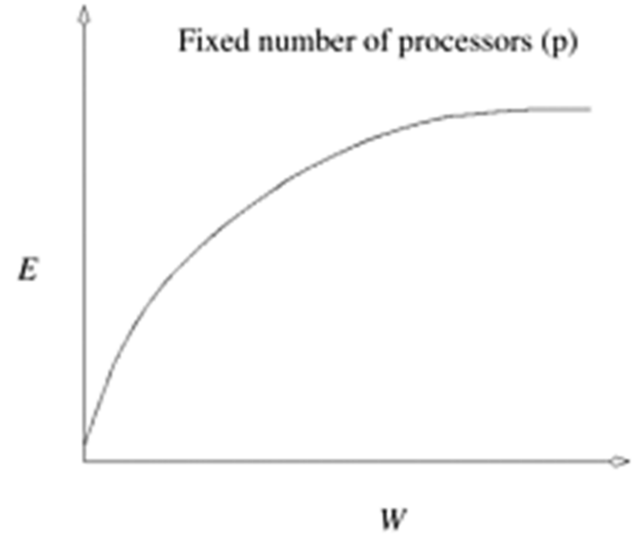
Observe that if the problem size is fixed, T_O increases with *p*. (why?)
So E_p drops as *p* increases.

On the other hand, for many algorithms T_O grows sublinearly w.r.t. *T*.

For such algorithms E_p can be kept fixed by increasing the problem size and *p* simultaneously.

Scalable Parallel Algorithms





Source: Grama et al., "Introduction to Parallel Computing", 2nd Edition

Efficiency, $E_p = \frac{S_p}{p} = \frac{T}{pT_p} = \frac{T}{T+T_O} = \frac{1}{1+\frac{T_O}{T}}$

Observe that if the problem size is fixed, T_O increases with *p*. (why?)
So E_p drops as *p* increases.

On the other hand, for many algorithms T_O grows sublinearly w.r.t. *T*.

For such algorithms E_p can be kept fixed by increasing the problem size and *p* simultaneously.

Scalable Parallel Algorithms

In order to keep E_p fixed at a constant *k*, we need

$$E_p = k \Rightarrow \frac{T}{pT_p} = k \Rightarrow T = kpT_p$$

For the algorithm that adds *n* numbers using *p* processing elements:

$$T = n \text{ and } T_p = \frac{n}{p} + 2\log p$$

So in order to keep E_p fixed at *k*, we must have:

$$n = kp\left(\frac{n}{p} + 2\log p\right) \Rightarrow n = \frac{2k}{1-k} p \log p$$

n	$p = 1$	$p = 4$	$p = 8$	$p = 16$	$p = 32$
64	1.0	***0.80***	0.57	0.33	0.17
192	1.0	0.92	***0.80***	0.60	0.38
320	1.0	0.95	0.87	0.71	0.50
512	1.0	0.97	0.91	***0.80***	0.62

Fig: Efficiency for adding *n* numbers using *p* processing elements

Source: Grama et al., "Introduction to Parallel Computing", 2nd Edition

The Isoefficiency Function

For a given problem, we define *problem size W* as the number of basic computation steps in the fastest sequential algorithm that solves the problem on a serial machine.

Thus $W = T$.

We have already seen, $E_p = \frac{1}{1+\frac{T_O}{T}} = \frac{1}{1+\frac{T_O(W,p)}{W}}$

Rearranging, $W = \frac{E_p}{1-E_p} T_O(W,p) = KT_O(W,p)$, where $K = \frac{E_p}{1-E_p}$

We have already seen how to obtain the isoefficiency function for adding *n* numbers using *p* processing elements.

Isoefficiency for Complex Overhead Functions

Suppose, $T_O = p^{3/2} + p^{3/4}W^{3/4}$.

We balance W against each term of T_O, and the component of T_O that requires W to grow at the highest rate w.r.t. p gives the overall asymptotic isoefficiency function for the algorithm.

Using only the 1st term, $W = Kp^{3/2}$

Using only the 2nd term, $W = K^4p^3$

Hence, the overall isoefficiency function is $\Theta(p^3)$.