# CSE 638: Advanced Algorithms 

## Department of Computer Science SUNY Stony Brook Spring 2013

> "For every complex problem, there is a solution that is simple, neat, and wrong."
> - Henry Louis Mencken

## Course Information

- Lecture Time: TuTh 2:30 pm - 3:50 pm
- Location: Melville Library E4540, West Campus
- Instructor: Rezaul A. Chowdhury
- Office Hours: TuTh 12:30 pm - 2:00 pm, 1421 Computer Science
- Email: rezaul@cs.stonybrook.edu
- TA: Vikas Ganjigunte Ashok
- TA Office Hours: Tu 4:00 pm - 5:00 pm, 2110 Computer Science
- TA Email: vganjiguntea@cs.stonybrook.edu
- Class Webpage: http://www.cs.sunysb.edu/~cse638


## Prerequisites

- Required: Background in algorithms analysis (e.g., CSE 548 )
- Required: Background in programming languages ( C / C++ )
- Helpful but Not Required: Background in computer architecture


## Topics to be Covered

Various topics from the following areas will be covered

- Parallel algorithms ( most emphasis )
- Randomized algorithms
- External-memory and cache-efficient algorithms
- Streaming algorithms
- Resilient algorithms


## Grading Policy

- Homeworks ( three: highest 15\%, lowest 5\%, other 10\% ): 30\%
- Exam (one ): 25\%
- Final (in-class ): May 9
- Group project ( one ): 30\%
- Proposal: Feb 28
- Progress report (in-class ): April 2-4
- Final report: May 10
- Scribe note ( one lecture ): 10\%
- Class participation \& attendance: 5\%


## Programming Environment

This course is supported by educational grants from

- Extreme Science and Engineering Discovery Environment ( XSEDE ): https://www.xsede.org

We have access to the following supercomputers

- Lonestar ( Texas Advanced Computing Center ): 1,800+ nodes with 12 cores ( two Intel Westmere processors ) per node
- Trestles ( San Diego Supercomputer Center ): 300+ nodes with 32 cores ( four AMD Magny Cours processors ) per node
- Kraken ( National Institute for Computational Sciences ): 9,000+ nodes with 12 cores ( two AMD Opteron Istanbul processors ) per node
- Keeneland KIDS ( Georgia Tech ): 120 nodes with 16 cores ( two Intel Sandy Bridge processors ) and three NVIDIA Fermi GPU’s per node


## Programming Environment

World's Most Powerful Supercomputers in June, 2008 (www.top500.org)

| Rank | Site | System | Cores | Rmax <br> (TFlop/s) | Reak <br> (TFlop/s) | Power (kW) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (1) | DOE/NNSALLANL United States | Roadrunner - BladeCenter QS22/LS21 Cluster, PowerXCell 8i 3.2 Ghz / Opteron DC 1.8 GHz, Voltaire Infiniband IBM | 122400 | 1026.0 | 1375.8 | 2345 |
| (2) | DOE/NNSA/LLNL United States | BlueGene/L - eServer Blue <br> Gene Solution <br> IBM | 212992 | 478.2 | 596.4 | 2329 |
| (3) | National Institute for Computational Sciences/University of Tennessee United States | $\begin{aligned} & \text { Kraken XT5 - Cray XT5 QC } 2.3 \\ & \text { GHz } \\ & \text { Cray Inc. } \end{aligned}$ | 66000 | 463.3 | 607.2 |  |

## Programming Environment

## World's Most Powerful Supercomputers in November, 2012 (www.top500.org)

| Rank | Site | System | Cores | Rmax <br> (TFlop/s) | Rpeak <br> (TFlop/s) | Power (kW) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (1) | DOE/SC/Oak Ridge <br> National Laboratory United States | Titan - Cray XK7, Opteron 6274 16C 2.200 GHz , Cray Gemini interconnect, NVIDIA K20x Cray Inc. | 560640 | 17590.0 | 27112.5 | 8209 |
| (2) | DOE/NNSA/LLNL <br> United States | Sequoia - BlueGene/Q, Power BQC 16C 1.60 GHz, Custom IBM | 1572864 | 16324.8 | 20132.7 | 7890 |
| (3) | RIKEN Advanced Institute for Computational Science (AICS) Japan | K computer, SPARC64 VIIIfx <br> 2.0 GHz , Tofu interconnect Fujitsu | 705024 | 10510.0 | 11280.4 | 12660 |
| (25) | National Institute for <br> Computational <br> Sciences/University of <br> Tennessee <br> United States | Kraken XT5 - Cray XT5-HE <br> Opteron Six Core 2.6 GHz Cray Inc. | 112800 | 919.1 | 1173.0 | 3090 |
| 96 | Texas Advanced Computing Center/Univ. of Texas United States | Lonestar 4 - Dell PowerEdge <br> M610 Cluster, Xeon 5680 <br> 3.3Ghz, Infiniband QDR <br> Dell | 22656 | 251.8 | 301.8 |  |

## Recommended Textbooks

- A. Grama, G. Karypis, V. Kumar, and A. Gupta. Introduction to Parallel Computing (2nd Edition), Addison Wesley, 2003.
- J. JáJá. An Introduction to Parallel Algorithms (1st Edition), Addison Wesley, 1992.
- T. Cormen, C. Leiserson, R. Rivest, and C. Stein. Introduction to Algorithms (3rd Edition), MIT Press, 2009.
- R. Motwani and P. Raghavan. Randomized Algorithms (1st Edition), Cambridge University Press, 1995.
- J. Vitter. Algorithms and Data Structures for External Memory, Series on Foundations and Trends in Theoretical Computer Science, Now Publishers, Hanover, MA, 2008.
- U. Meyer, P. Sanders, and J. Sibeyn (Editors). Algorithms for Memory Hierarchies: Advanced Lectures (1st Edition), Lecture Notes in Computer Science, Springer, 2003.


## Why Parallelism?

## Moore's Law



## Unicore Performance



Source: Chung-Ta King, Department of Computer Science, National Tsing Hua University

## Unicore Performance Has Hit a Wall!

Some Reasons

- Lack of additional ILP
(Instruction Level Hidden Parallelism)
- High power density
- Manufacturing issues
- Physical limits
- Memory speed


## Unicore Performance: No Additional ILP

> "Everything that can be invented has been invented."
> - Charles $\mathcal{H}$. Duell
> Commissioner, U.S. patent office, 1899

Exhausted all ideas to exploit hidden parallelism?

- Multiple simultaneous instructions
- Dynamic instruction scheduling
- Branch prediction
- Out-of-order instructions
- Speculative execution
- Pipelining
- Non-blocking caches, etc.


## Unicore Performance: High Power Density

- Dynamic power, $P_{d} \propto V^{2} f C$
- $V=$ supply voltage
- $f=$ clock frequency
- C = capacitance
- But $V \propto f$
- Thus $P_{d} \propto f^{3}$



## Unicore Performance: High Power Density

- Changing $f$ by 20\% changes performance by 13\%
- So what happens if we overclock by $20 \%$ ?
- And underclock by 20\%?



## Unicore Performance: High Power Density

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Source: Andrew A. Chien, Vice President of Research, Intel Corporation

## Unicore Performance: High Power Density

- Changing $f$ by $20 \%$ changes performance by $13 \%$
- So what happens if we overclock by $20 \%$ ?
- And underclock by 20\%?


Source: Andrew A. Chien, Vice President of Research, Intel Corporation

## Unicore Performance: Manufacturing Issues

- Frequency, $f \propto 1 / s$
- $s=$ feature size ( transistor dimension )
- Transistors / unit area $\propto 1 / s^{2}$
- Typically, die size $\propto 1 / s$
- So, what happens if feature size goes down by a factor of $x$ ?
- Raw computing power goes up by a factor of $x^{4}$ !
- Typically most programs run faster by a factor of $x^{3}$ without any change!


## Unicore Performance: Manufacturing Issues

- Manufacturing cost goes up as feature size decreases
- Cost of a semiconductor fabrication plant doubles every 4 years (Rock's Law )
- CMOS feature size is limited to 5 nm ( at least 10 atoms )


Source: Kathy Yelick and Jim Demmel, UC Berkeley

## Unicore Performance: Physical Limits

Execute the following loop on a serial machine in 1 second:

$$
\begin{aligned}
& \text { for }\left(i=0 ; i<10^{12} ;++i\right) \\
& \quad z[i]=x[i]+y[i] ;
\end{aligned}
$$

- We will have to access $3 \times 10^{12}$ data items in one second
- Speed of light is, $c \approx 3 \times 10^{8} \mathrm{~m} / \mathrm{s}$
- So each data item must be within $c / 3 \times 10^{12} \approx 0.1 \mathrm{~mm}$ from the CPU on the average
- All data must be put inside a $0.2 \mathrm{~mm} \times 0.2 \mathrm{~mm}$ square
- Each data item ( $\geq 8$ bytes ) can occupy only $1 \AA^{2}$ space! ( size of a small atom! )


## Unicore Performance: Memory Wall



Source: Rick Hetherington, Chief Technology Officer, Microelectronics, Sun Microsystems

## Moore's Law Reinterpreted



Source: Report of the 2011 Workshop on Exascale Programming Challenges

## Cores / Processor (General Purpose)



## No Free Lunch for Traditional Software



Source: Simon Floyd, Workstation Performance: Tomorrow's Possibilities (Viewpoint Column)

## Top 500 Supercomputing Sites

## Cores per Socket - Systems Share



## Insatiable Demand for Performance



Weather Prediction


Genomics Research


Oil Exploration


Financial Analysis


Design Simulation


Medical Imaging

## Numerical Weather Prediction

Problem: ( temperature, pressure, ..., humidity, wind velocity )
$\leftarrow f($ longitude, latitude, height, time )

## Approach ( very coarse resolution ):

- Consider only modeling fluid flow in the atmosphere
- Divide the entire global atmosphere into cubic cells of size 1 mile $\times 1$ mile $\times 1$ mile each to a height of 10 miles $\approx 2 \times 10^{9}$ cells
- Simulate 7 days in 1 minute intervals
$\approx 10^{4}$ time-steps to simulate
- 200 floating point operations ( flop ) / cell / time-step
$\approx 4 \times 10^{15}$ floating point operations in total
- To predict in 1 hour $\approx 1$ Tflop/s ( Tera flop / sec )


## Some Useful Classifications of Parallel Computers

## Parallel Computer Memory Architecture (Shared Memory)

- All processors access all memory as global address space
- Changes in memory by one processor are visible to all others
- Tow types:

- Uniform Memory Access ( UMA )
- Non-Uniform Memory Access ( NUMA)


NUMA

## Parallel Computer Memory Architecture (Shared Memory)

Advantages

- User-friendly programming perspective to memory
- Fast data sharing

Disadvantages

- Difficult and expensive to scale
- Correct data access is user responsibility


UMA


NUMA

## Parallel Computer Memory Architecture (Distributed Memory)

- Each processor has its own local memory - no global address space
- Changes in local memory by one processor have no effect


Source: Blaise Barney, LLNL

- Communication network to connect inter-processor memory


## Parallel Computer Memory Architecture (Distributed Memory)

## Advantages

- Easily scalable
- No cache-coherency needed among processors

- Cost-effective

> Source: Blaise Barney, LLNL

Disadvantages

- Communication is user responsibility
- Non-uniform memory access
- May be difficult to map shared-memory data structures to this type of memory organization


## Parallel Computer Memory Architecture (Hybrid Distributed-Shared Memory)

- The share-memory component can be a cache-coherent SMP or a Graphics Processing Unit (GPU)
- The distributed-memory
 component is the networking of multiple SMP/GPU machines
- Most common architecture for the largest and fastest computers in the world today



## Flynn's Taxonomy of Parallel Computers

Flynn's classical taxonomy ( 1966 ):
Classification of multi-processor computer architectures along two independent dimensions of instruction and data.

|  | Single Data <br> (SD ) | Multiple Data <br> (MD ) |
| :---: | :---: | :---: |
| Single Instruction <br> ( SI ) | SISD | SIMD |
| Multiple Instruction <br> $($ MI $)$ | MISD | MIMD |

## Flynn's Taxonomy of Parallel Computers

## SISD

- A serial ( non-parallel ) computer
- The oldest and the most common type of computers
- Example: Uniprocessor unicore machines


Source: Blaise Barney, LLNL

## Flynn's Taxonomy of Parallel Computers

| prev instruct |
| :---: |
| load $A(1)$ |
| load $B(1)$ |
| $C(1)=A(1) * B(1)$ |
| store $C(1)$ |
| next instruct |
| $P 1$ |


| prev instruct |
| :---: |
| load $A(2)$ |
| load $B(2)$ |
| $C(2)=A(2)^{*} B(2)$ |
| store $C(2)$ |
| next instruct |
| $\mathbf{P} 2$ |


| prev instruct |
| ---: |
| $\operatorname{load} A(n)$ |
| $\operatorname{load} B(n)$ |
| $C(n)=A(n)^{*} B(n)$ |
| store $C(n)$ |
| next instruct |
| Pn |



Source: Blaise Barney, LLNL

SIMD

- A type of parallel computer
- All PU's run the same instruction at any given clock cycle
- Each PU can act on a different data item
- Synchronous (lockstep) execution
- Two types: processor arrays and vector pipelines
- Example: GPUs (Graphics Processing Units )


## Flynn's Taxonomy of Parallel Computers

## MISD

- A type of parallel computer
- Very few ever existed

| prev instruct |
| :---: |
| $\operatorname{load} \mathrm{A}(1)$ |
| $\mathrm{C}(1)=\mathrm{A}(1)^{* 1}$ |
| store $\mathrm{C}(1)$ |
| next instruct |
| P1 |


| prev instruct |
| :---: |
| load $A(1)$ |
| $\mathbf{C}(2)=A(1)^{*} 2$ |
| store $\mathbf{C}(2)$ |
| next instruct |
| $\mathbf{P 2}$ |


| prev instruct |
| :---: |
| load $A(1)$ |
| $\mathrm{C}(\mathrm{n})=\mathrm{A}(1)^{*} \mathrm{n}$ |
| store $\mathrm{C}(\mathrm{n})$ |
| next instruct |
| Pn |

## MIMD

- A type of parallel computer
- Synchronous /asynchronous execution
- Examples: most modern supercomputers, parallel computing clusters, multicore PCs

| prev instruct |
| :---: |
| load $\mathrm{A}(1)$ |
| load $\mathrm{B}(1)$ |
| $\mathrm{C}(1)=\mathrm{A}(1)^{*} \mathrm{~B}(1)$ |
| store $\mathrm{C}(1)$ |
| next instruct |
| $\mathbf{P 1}$ |


| prev instruct |
| :---: |
| call funcD |
| $x=y^{* z}$ |
| sum=x*2 |
| call sub1 $(\mathbf{i}, \mathrm{j})$ |
| next instruct |
| $\mathbf{P 2}$ |

## Parallel Algorithms Warm-up

"The way the processor industry is going, is to add more and more cores, but nobody knows how to program those things. I mean, two, yeah; four, not really; eight, forget it."
—Steve Jobs, $\mathcal{N} \Upsilon$ Times interview, June 102008

## Parallel Algorithms Warm-up (1)

Consider the following loop:

$$
\begin{aligned}
& \text { for } i=1 \text { to } n \text { do } \\
& \qquad C[i] \leftarrow A[i] \times B[i]
\end{aligned}
$$

- Suppose you have an infinite number of processors/cores
- Ignore all overheads due to scheduling, memory accesses, communication, etc.
- Suppose each operation takes a constant amount of time
- How long will this loop take to complete execution?


## Parallel Algorithms Warm-up (1)

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- Suppose each operation takes a constant amount of time
- How long will this loop take to complete execution?
- O( 1 ) time


## Parallel Algorithms Warm-up (2)

Now consider the following loop:

$$
\begin{aligned}
& c \leftarrow 0 \\
& \text { for } i=1 \text { to } n \text { do } \\
& \quad c \leftarrow c+A[i] \times B[i]
\end{aligned}
$$

- How long will this loop take to complete execution?


## Parallel Algorithms Warm-up (2)

Now consider the following loop:

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\begin{aligned}
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& \quad c \leftarrow c+A[i] \times B[i]
\end{aligned}
$$

- How long will this loop take to complete execution?

$$
-O(\log n) \text { time }
$$

## Parallel Algorithms Warm-up (3)

Now consider quicksort:

$$
\begin{aligned}
& \operatorname{QSort}(A) \\
& \text { if }|A| \leq 1 \text { return } A \\
& \text { else } p \leftarrow A[\operatorname{rand}(|A|)] \\
& \quad \operatorname{return} \operatorname{QSort}(\{x \in A: x<p\}) \\
& \quad \#\{p\} \# \\
& \quad \operatorname{QSort}(\{x \in A: x>p\})
\end{aligned}
$$

- Assuming that $A$ is split in the middle everytime, and the two recursive calls can be made in parallel, how long will this algorithm take?


## Parallel Algorithms Warm-up (3)

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\end{aligned}
$$

- Assuming that $A$ is split in the middle everytime, and the two recursive calls can be made in parallel, how long will this algorithm take?
$-O\left(\log ^{2} n\right)$ (if partitioning takes logarithmic time )
$-O(\log n) \quad($ but can be partitioned in constant time )

