# CSE 613: Parallel Programming 

## Department of Computer Science SUNY Stony Brook Spring 2019

"We used to joke that<br>"parallel computing is the future, and always will be," 6ut the pessimists have been proven wrong."<br>- Tony Hey

## Course Information

- Lecture Time: MF 1:00 pm-2:20 pm
- Location: Room 2120, Old CS Building, West Campus
- Instructor: Rezaul A. Chowdhury
- Office Hours: MF 4:00 pm - 5:30 pm, 239 New CS Building
- Email: rezaul@cs.stonybrook.edu
- TA: Unlikely
- Class Webpage:
http://www3.cs.stonybrook.edu/~rezaul/CSE613-S19.html


## Prerequisites

- Required: Background in algorithms analysis

$$
\text { ( e.g., CSE } 373 \text { or CSE } 548 \text { ) }
$$

- Required: Background in programming languages ( C / C++ )
- Helpful but Not Required: Background in computer architecture
- Please Note: This is not a course on
- Programming languages
- Computer architecture
- Main Emphasis: Parallel algorithms


## Topics to be Covered

The following topics will be covered

- Analytical modeling of parallel programs
- Scheduling
- Programming using the message-passing paradigm and for shared address-space platforms
- Parallel algorithms for dense matrix operations, sorting, searching, graphs, computational geometry, and dynamic programming
- Concurrent data structures
- Transactional memory, etc.


## Grading Policy

- Homeworks ( three: lowest score 8\%, highest score 20\%, and the remaining one 12\% ): 40\%
- Group project ( one ): 45\%
- Proposal: Feb 22
- Progress report: Apr 1
- Final demo / report: May 6 - May 10
- Scribe note ( one lecture ): 10\%
- Class participation \& attendance: 5\%


## Programming Environment

This course is supported by an educational grant from

- Extreme Science and Engineering Discovery Environment ( XSEDE ): https://www.xsede.org

We have access to the following supercomputing resources

- Stampede 2 ( Texas Advanced Computing Center ):

4,200 KNL nodes with 68 cores (Intel Xeon Phi 7250 / "Knights Landing") each; 1,736 SKX nodes each with 48 cores (Intel Xeon Platinum / "Skylake") on two sockets.

- Comet (San Diego Supercomputer Center ):

1,984 nodes with 24 cores ( 2 Intel Haswell ) per node. The Comet GPU resource features 36 K80 GPU nodes (with 2 Intel Haswell processors each), and 36 P100 nodes (with 2 Intel Broadwell processors each).

## Programming Environment

## World's Most Powerful Supercomputers in November, 2018 www.top500.org).

| Rank | System | Cores | Rmax <br> (TFlop/s) | Rpeak <br> (TFlop/s) | Power (kW) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Summit - IBM Power System AC922, IBM POWER9 22C 3.07GHz, NVIDIA Volta GV100, Dual-rail Mellanox EDR Infiniband , IBM DOE/SC/Oak Ridge National Laboratory United States | 2,397,824 | 143,500.0 | 200,794.9 | 9,783 |
| 2 | Sierra - IBM Power System S922LC, IBM POWER9 22C 3.1GHz, NVIDIA Volta GV100, Dual-rail Mellanox EDR Infiniband , IBM / NVIDIA / Mellanox DOE/NNSA/LLNL <br> United States | 1,572,480 | 94,640.0 | 125,712.0 | 7,438.3 |
| 3 | Sunway TaihuLight - Sunway MPP, Sunway SW26010 260C 1.45 GHz , <br> Sunway, NRCPC <br> National Supercomputing Center in Wuxi <br> China | 10,649,600 | 93,014.6 | 125,435.9 | 15,371 |
| 4 | Tianhe-2A - TH-IVB-FEP Cluster, Intel Xeon E5-2692v2 12C 2.2GHz, TH Express-2, Matrix-2000 , NUDT National Super Computer Center in Guangzhou China | 4,981,760 | 61,444.5 | 100,678.7 | 18,482 |
| 5 | Piz Daint - Cray XC50, Xeon E5-2690v3 12C 2.6GHz, Aries interconnect , NVIDIA Tesla P100, Cray Inc. <br> Swiss National Supercomputing Centre [CSCS] <br> Switzerland | 387,872 | 21,230.0 | 27,154.3 | 2,384.2 |
| 6 | Trinity - Cray XC40, Xeon E5-2698v3 16C 2.3GHz, Intel Xeon Phi 7250 68C 1.4 GHz , Aries interconnect, Cray Inc. <br> DOE/NNSA/LANL/SNL <br> United States | 979,072 | 20,158.7 | 41,461.2 | 7,578.1 |
| 7 | AI Bridging Cloud Infrastructure (ABCI) - PRIMERGY CX2570 M4, Xeon Gold 6148 20C 2.4 GHz , NVIDIA Tesla V100 SXM2, Infiniband EDR, Fujitsu National Institute of Advanced Industrial Science and Technology (AIST) Japan | 391,680 | 19,880.0 | 32,576.6 | 1,649.3 |
| 8 | ```SuperMUC-NG - ThinkSystem SD530, Xeon Platinum 8174 24C 3.1GHz, Intel Omni-Path, Lenovo Leibniz Rechenzentrum Germany``` | 305,856 | 19,476.6 | 26,873.9 |  |
| 9 | Titan - Cray XK7, Opteron 6274 16C 2.200 GHz , Cray Gemini interconnect, NVIDIA K20x , Cray Inc. <br> DOE/SC/Oak Ridge National Laboratory <br> United States | 560,640 | 17,590.0 | 27,112.5 | 8,209 |
| 10 | Sequoia - BlueGene/Q, Power BQC 16C 1.60 GHz, Custom , IBM DOE/NNSA/LLNL <br> United States | 1,572,864 | 17,173.2 | 20,132.7 | 7,890 |

## Programming Environment

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| :---: | :---: | :---: | :---: | :---: | :---: |
| 11 | Lassen - IBM Power System S922LC, IBM POWER9 22C 3.1GHz, Dual-rail Mellanox EDR Infiniband, NVIDIA Tesla V100, IBM / NVIDIA / Mellanox DOE/NNSA/LLNL <br> United States | 248,976 | 15,430.0 | 19,904.4 |  |
| 12 | Cori - Cray XC40, Intel Xeon Phi 7250 68C 1.4GHz, Aries interconnect , Cray Inc. <br> DOE/SC/LBNL/NERSC <br> United States | 622,336 | 14,014.7 | 27,880.7 | 3,939 |
| 13 | Nurion - Cray CS500, Intel Xeon Phi 7250 68C 1.4GHz, Intel Omni-Path , Cray Inc. <br> Korea Institute of Science and Technology Information Korea, South | 570,020 | 13,929.3 | 25,705.9 |  |
| 14 | Oakforest-PACS - PRIMERGY CX1640 M1, Intel Xeon Phi 7250 68C 1.4 4 Hz , Intel Omni-Path , Fujitsu Joint Center for Advanced High Performance Computing Japan | 556,104 | 13,554.6 | 24,913.5 | 2,718.7 |
| 15 | HPC4 - Proliant DL380 Gen10, Xeon Platinum 8160 24C 2.1GHz, Mellanox InfiniBand EDR, NVIDIA Tesla P100 , HPE <br> Eni S.p.A. <br> Italy | 253,600 | 12,210.0 | 18,621.1 | 1,320 |
| 16 | Tera-1000-2 - Bull Sequana X1000, Intel Xeon Phi 7250 68C 1.4GHz, Bull BXI 1.2, Bull, Atos Group <br> Commissariat a l'Energie Atomique (CEA) France | 561,408 | 11,965.5 | 23,396.4 | 3,178 |
| 17 | Stampede2 - PowerEdge C6320P/C6420, Intel Xeon Phi 7250 68C $1.4 \mathrm{GHz} /$ Platinum 8160, Intel Omni-Path , Dell EMC Texas Advanced Computing Center/Univ. of Texas United States | 367,024 | 10,680.7 | 18,309.2 |  |

18 K computer, SPARC64 VIIIfx 2.0GHz, Tofu interconnect , Fujitsu $\quad 705,024 \quad 10,510.0 \quad 11,280.4 \quad 12,659.9$ RIKEN Advanced Institute for Computational Science (AICS) Japan
19 Marconi Intel Xeon Phi - CINECA Cluster, Lenovo SD530/S720AP, Intel Xeon $348,000 \quad 10,384.9 \quad 18,816.0$ Phi 7250 68C $1.4 \mathrm{GHz} /$ Platinum 8160, Intel Omni-Path , Lenovo CINECA Italy

20 Taiwania 2 - QCT QuantaGrid D52G-4U/LC, Xeon Gold 6154 18C 3GHz, $\quad 170,3529,000.0 \quad 15,208.2 \quad 797.5$ Mellanox InfiniBand EDR, NVIDIA Tesla V100 SXM2 , Quanta Computer/
Taiwan Fixed Network / ASUS Cloud
National Center for High Performance Computing
Taiwan

## Programming Environment

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| 1 | Sunway TaihuLight - Sunway MPP, Sunway SW26010 260C 1.45 GHz , <br> Sunway, NRCPC <br> National Supercomputing Center in Wuxi <br> China | 10,649,600 | 93,014.6 | 125,435.9 | 15,371 |
| 2 | Tianhe-2A - TH-IVB-FEP Cluster, Intel Xeon E5-2692 12C 2.200 GHz , TH Express-2, Intel Xeon Phi 31S1P , NUDT National Super Computer Center in Guangzhou China | 3,120,000 | 33,862.7 | 54,902.4 | 17,808 |
| 3 | Piz Daint - Cray XC50, Xeon E5-2690v3 12C 2.6 GHz , Aries interconnect, NVIDIA Tesla P100, Cray Inc. <br> Swiss National Supercomputing Centre [CSCS] Switzerland | 361,760 | 19,590.0 | 25,326.3 | 2,272.0 |
| 4 | Gyoukou - ZettaScaler-2.2 HPC system, Xeon D-1571 16C 1.3GHz, Infiniband EDR, PEZY-SC2 700Mhz ExaScaler Japan Agency for Marine-Earth Science and Technology Japan | 19,860,000 | 19,135.8 | 28,192.0 | 1,350.2 |
| 5 | Titan - Cray XK7, Opteron 6274 16C 2.200 GHz , Cray Gemini interconnect, NVIDIA K20x , Cray Inc. <br> DOE/SC/Oak Ridge National Laboratory <br> United States | 560,640 | 17,590.0 | 27,112.5 | 8,209 |
| 6 | Sequoia - BlueGene/Q, Power BQC 16C 1.60 GHz , Custom , IBM DOE/NNSA/LLNL <br> United States | 1,572,864 | 17,173.2 | 20,132.7 | 7,890 |
| 7 | Trinity - Cray XC40, Intel Xeon Phi 7250 68C 1.4GHz, Aries interconnect, Cray Inc. DOE/NNSA/LANL/SNL <br> United States | 979,968 | 14,137.3 | 43,902.6 | 3,843.6 |
| 8 | Cori - Cray XC40, Intel Xeon Phi 7250 68C 1.4 GHz , Aries interconnect, Cray Inc. <br> DOE/SC/LBNL/NERSC <br> United States | 622,336 | 14,014.7 | 27,880.7 | 3,939 |
| 9 | Oakforest-PACS - PRIMERGY CX1640 M1, Intel Xeon Phi 7250 68C 1.4 GHz , Intel Omni-Path, Fujitsu Joint Center for Advanced High Performance Computing Japan | 556,104 | 13,554.6 | 24,913.5 | 2,718.7 |
| 10 | K computer, SPARC64 VIIIfx 2.0 GHz , Tofu interconnect , Fujitsu RIKEN Advanced Institute for Computational Science (AICS) Japan | 705,024 | 10,510.0 | 11,280.4 | 12,659.9 |
| 11 | Mira - BlueGene/Q, Power BQC 16C 1.60 GHz , Custom , IBM DOE/SC/Argonne National Laboratory United States | 786,432 | 8,586.6 | 10,066.3 | 3,945 |
| 12 | Stampede2 - PowerEdge C6320P/C6420, Intel Xeon Phi 7250 68C 1.4GHz/Platinum 8160, Intel Omni-Path , Dell EMC Texas Advanced Computing Center/Univ. of Texas United States | 368,928 | 8,317.7 | 18,215.8 |  |

## Recommended Textbooks

- A. Grama, G. Karypis, V. Kumar, and A. Gupta. Introduction to Parallel Computing (2nd Edition), Addison Wesley, 2003.
- J. Jádá. An Introduction to Parallel Algorithms (1st Edition), Addison Wesley, 1992.
- T. Cormen, C. Leiserson, R. Rivest, and C. Stein. Introduction to Algorithms (3rd Edition), MIT Press, 2009.
- M. Herlihy and N. Shavit. The Art of Multiprocessor Programming (1st Edition), Morgan Kaufmann, 2008.
- P. Pacheco. Parallel Programming with MPI (1st Edition), Morgan Kaufmann, 1996.


## Why Parallelism?

## Moore's Law



## Unicore Performance

## Single-Threaded Floating-Point Performance



## Unicore Performance Has Hit a Wall!

Some Reasons

- Lack of additional ILP
( Instruction Level Hidden Parallelism )
- High power density
- Manufacturing issues
- Physical limits
- Memory speed


## Unicore Performance: No Additional ILP

"Everything that can be invented has been invented."

- Charles H. Duell

Commissioner, U.S. patent office, 1899
Exhausted all ideas to exploit hidden parallelism?

- Multiple simultaneous instructions
- Instruction Pipelining
- Out-of-order instructions
- Speculative execution
- Branch prediction
- Register renaming, etc.


## ILP: Instruction Pipelining



Source: Wikipedia

## Unicore Performance: High Power Density

- Dynamic power, $P_{d} \propto V^{2} f C$
- $V=$ supply voltage
- $f=$ clock frequency
- C = capacitance
- But $V \propto f$
- Thus $P_{d} \propto f^{3}$


Source: Patrick Gelsinger, Intel Developer Forum, Spring 2004 ( Simon Floyd )

## Unicore Performance: Manufacturing_Issues

- Frequency, $f \propto 1 / s$
- $s=$ feature size (transistor dimension )
- Transistors / unit area $\propto 1 / s^{2}$
- Typically, die size $\propto 1 / s$
- So, what happens if feature size goes down by a factor of $x$ ?
- Raw computing power goes up by a factor of $x^{4}$ !
- Typically most programs run faster by a factor of $x^{3}$ without any change!


## Unicore Performance: Manufacturing Issues

- Manufacturing cost goes up as feature size decreases
- Cost of a semiconductor fabrication plant doubles every 4 years (Rock's Law )
- CMOS feature size is limited to 5 nm ( at least 10 atoms )


Source: Kathy Yelick and Jim Demmel, UC Berkeley

## Unicore Performance: Physical Limits

Execute the following loop on a serial machine in 1 second:

$$
\begin{aligned}
& \text { for }\left(i=0 ; i<10^{12} ;++i\right) \\
& \quad z[i]=x[i]+y[i] ;
\end{aligned}
$$

- We will have to access $3 \times 10^{12}$ data items in one second
- Speed of light is, $c \approx 3 \times 10^{8} \mathrm{~m} / \mathrm{s}$
- So each data item must be within $c / 3 \times 10^{12} \approx 0.1 \mathrm{~mm}$ from the CPU on the average
- All data must be put inside a $0.2 \mathrm{~mm} \times 0.2 \mathrm{~mm}$ square
- Each data item ( $\geq 8$ bytes ) can occupy only $1 \AA^{2}$ space!
( size of a small atom! )


## Unicore Performance: Memory Wall

Relative
Performance


Source: Rick Hetherington, Chief Technology Officer, Microelectronics, Sun Microsystems

## Unicore Performance Has Hit a Wall!

## Some Reasons

- Lack of additional ILP
( Instruction Level Hidden Parallelism )
- High power density
- Manufacturing issues
- Physical limits
- Memory speed
"Oh Sinnerman, where you gonna run to?"
- Sinnerman (recorded by $\mathcal{N}$ ina Simone )


## Where You Gonna Run To?

- Changing $f$ by $20 \%$ changes performance by $13 \%$
- So what happens if we overclock by $20 \%$ ?



## Where You Gonna Run To?

- Changing $f$ by 20\% changes performance by 13\%
- So what happens if we overclock by $20 \%$ ?
- And underclock by $20 \%$ ?



## Where You Gonna Run To?

- Changing $f$ by 20\% changes performance by 13\%
- So what happens if we overclock by $20 \%$ ?
- And underclock by $20 \%$ ?



## Moore's Law Reinterpreted



Source: Report of the 2011 Workshop on Exascale Programming Challenges

## Top 500 Supercomputing Sites ( Cores / Socket)



## Top 500 Supercomputing Sites (Cores / Socket)



## No Free Lunch for Traditional Software



Source: Simon Floyd, Workstation Performance: Tomorrow's Possibilities (Viewpoint Column)

## Insatiable Demand for Performance



Weather Prediction


Genomics Research


Oil Exploration


Financial Analysis


Design Simulation


Source: Patrick Gelsinger, Intel Developer Forum, 2008

## Some Useful Classifications of Parallel Computers

## Parallel Computer Memory Architecture (Distributed Memory)

- Each processor has its own local memory - no global address space
- Changes in local memory by
 one processor have no effect on memory of other processors

Source: Blaise Barney, LLNL

- Communication network to connect inter-processor memory
- Programming
- Message Passing Interface ( MPI)
- Many once available: PVM, Chameleon, MPL, NX, etc.


## Parallel Computer Memory Architecture (Shared Memory)

- All processors access all memory as global address space
- Changes in memory by one processor are visible to all others
- Two types
- Uniform Memory Access ( UMA )
- Non-Uniform Memory Access ( NUMA )
- Programming
- Open Multi-Processing (OpenMP )

- Cilk/Cilk++ and Intel Cilk Plus
- Intel Thread Building Block ( TBB ), etc.


## Parallel Computer Memory Architecture (Hybrid Distributed-Shared Memory).

- The share-memory component can be a cache-coherent SMP or a Graphics Processing Unit (GPU)
- The distributed-memory
 component is the networking of multiple SMP/GPU machines
- Most common architecture for the largest and fastest computers in the world today


| CPU | CPU |
| :--- | :--- |
| CPU | CPU |$\quad$ Memory

## Flynn's Taxonomy of Parallel Computers

Flynn's classical taxonomy ( 1966 ):
Classification of multi-processor computer architectures along two independent dimensions of instruction and data.


## Flynn's Taxonomy of Parallel Computers

SISD

- A serial ( non-parallel) computer
- The oldest and the most common type of computers
- Example: Uniprocessor unicore machines


Source: Blaise Barney, LLNL

Flynn's Taxonomy of Parallel Computers

| prev instruct |
| :---: |
| load $A(1)$ |
| load $B(1)$ |
| $C(1)=A(1)^{*} B(1)$ |
| store $C(1)$ |
| next instruct |
| P1 |


| prev instruct |
| :---: |
| load $A(2)$ |
| load $B(2)$ |
| $C(2)=A(2)^{*} B(2)$ |
| store $C(2)$ |
| next instruct |
| $\mathbf{P} 2$ |


| prev instruct |
| ---: |
| load $A(n)$ |
| load $B(n)$ |
| $C(n)=A(n)^{\star} B(n)$ |
| store $C(n)$ |
| next instruct |
| Pn |



Source: Blaise Barney, LLNL

SIMD

- A type of parallel computer
- All PU's run the same instruction at any given clock cycle
- Each PU can act on a different data item
- Synchronous (lockstep) execution
- Two types: processor arrays and vector pipelines
- Example: GPUs (Graphics Processing Units )


## Flynn's Taxonomy of Parallel Computers

## MISD

- A type of parallel computer
- Very few ever existed

| prev instruct |
| :---: |
| load $\mathrm{A}(1)$ |
| $\mathrm{C}(1)=\mathrm{A}(1)^{* 1}$ |
| store $\mathrm{C}(1)$ |
| next instruct |

P1

| prev instruct |
| :---: |
| load $\mathbf{A}(1)$ |
| $\mathbf{C}(2)=A(1)^{*} 2$ |
| store $\mathbf{C}(2)$ |
| next instruct |
| $\mathbf{P 2}$ |


| prev instruct |
| :---: |
| load $\mathbf{A}(1)$ |
| $\mathbf{C}(\mathrm{n})=\mathrm{A}(1)^{*} \mathrm{n}$ |
| store $\mathrm{C}(\mathrm{n})$ |
| next instruct |
| $\mathbf{P n}$ |

## MIMD

- A type of parallel computer
- Synchronous /asynchronous execution
- Examples: most modern supercomputers, parallel

| prev instruct |
| :---: |
| load $\mathrm{A}(1)$ |
| load $\mathrm{B}(1)$ |
| $\mathrm{C}(1)=\mathrm{A}(1)^{*} \mathrm{~B}(1)$ |
| store $\mathrm{C}(1)$ |
| next instruct |

P1


P2

| prev instruct |
| :---: |
| do $10 \mathrm{i}=1, \mathrm{~N}$ |
| alpha= $\mathrm{w}^{* * 3}$ |
| zeta $=\mathrm{C}(\mathrm{i})$ |
| 10 continue |
| next instruct |
| Pn |

## Parallel Algorithms Warm-up

"The way the processor industry is going, is to add more and more cores, but nobody knows how to program those things. I mean, two, yeah; four, not really; eight, forget it."
—Steve Jobs, $\mathcal{N} \Upsilon$ Times interview, June 102008

## Parallel Algorithms Warm-up (1).

Consider the following loop:

$$
\begin{aligned}
& \text { for } i=1 \text { to } n \text { do } \\
& \qquad C[i] \leftarrow A[i] \times B[i]
\end{aligned}
$$

- Suppose you have an infinite number of processors/cores
- Ignore all overheads due to scheduling, memory accesses, communication, etc.
- Suppose each operation takes a constant amount of time
- How long will this loop take to complete execution?


## Parallel Algorithms Warm-up (1).

Consider the following loop:

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\begin{aligned}
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\end{aligned}
$$

- Suppose you have an infinite number of processors/cores
- Ignore all overheads due to scheduling, memory accesses, communication, etc.
- Suppose each operation takes a constant amount of time
- How long will this loop take to complete execution?
- O( 1 ) time


## Parallel Algorithms Warm-up (2).

Now consider the following loop:

$$
\begin{aligned}
& c \leftarrow 0 \\
& \text { for } i=1 \text { to } n \text { do } \\
& \quad c \leftarrow c+A[i] \times B[i]
\end{aligned}
$$

- How long will this loop take to complete execution?


## Parallel Algorithms Warm-up (2).

Now consider the following loop:

$$
\begin{aligned}
& c \leftarrow 0 \\
& \text { for } i=1 \text { to } n \text { do } \\
& \quad c \leftarrow c+A[i] \times B[i]
\end{aligned}
$$

- How long will this loop take to complete execution?
$-O(\log n)$ time


## Parallel Algorithms Warm-up (3).

Now consider quicksort:

$$
\begin{aligned}
& \operatorname{QSort}(A) \\
& \text { if }|A| \leq 1 \operatorname{return} A \\
& \text { else } p \leftarrow A[\operatorname{rand}(|A|)] \\
& \quad \operatorname{return} \operatorname{QSort}(\{x \in A: x<p\}) \\
& \quad \#\{p\} \# \\
& \quad \operatorname{QSort}(\{x \in A: x>p\})
\end{aligned}
$$

- Assuming that $A$ is split in the middle everytime, and the two recursive calls can be made in parallel, how long will this algorithm take?


## Parallel Algorithms Warm-up (3).

Now consider quicksort:

$$
\begin{aligned}
& \operatorname{QSort}(A) \\
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& \quad \operatorname{return} \operatorname{QSort}(\{x \in A: x<p\}) \\
& \quad \#\{p\} \# \\
& \operatorname{QSort}(\{x \in A: x>p\})
\end{aligned}
$$

- Assuming that $A$ is split in the middle everytime, and the two recursive calls can be made in parallel, how long will this algorithm take?
$-O\left(\log ^{2} n\right) \quad$ (if partitioning takes logarithmic time )
$-O(\log n) \quad($ but can be partitioned in constant time $)$

