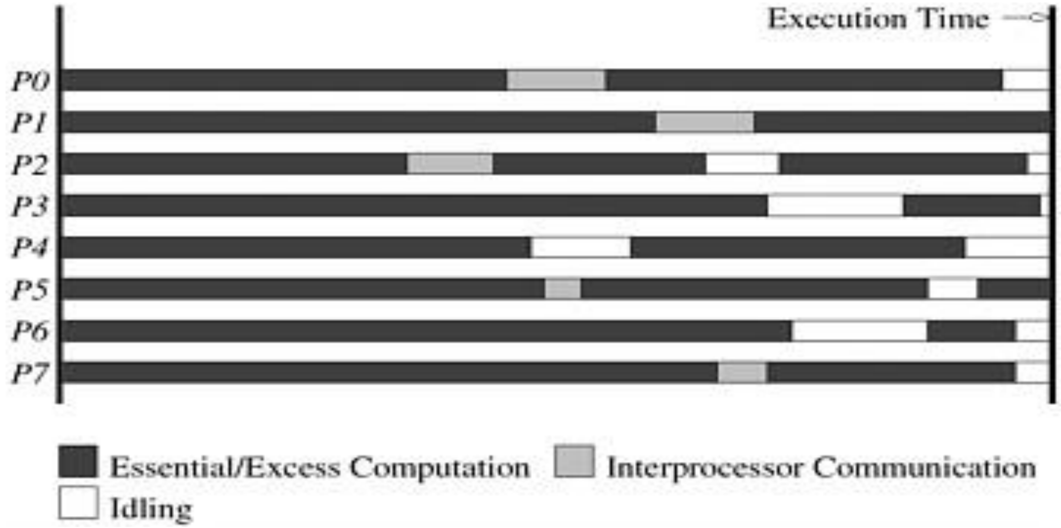
CSE 613: Parallel Programming

Lecture 2
(Analytical Modeling of Parallel Algorithms)

Rezaul A. Chowdhury
Department of Computer Science
SUNY Stony Brook
Spring 2019

Parallel Execution Time & Overhead



Source: Grama et al., "Introduction to Parallel Computing", 2nd Edition

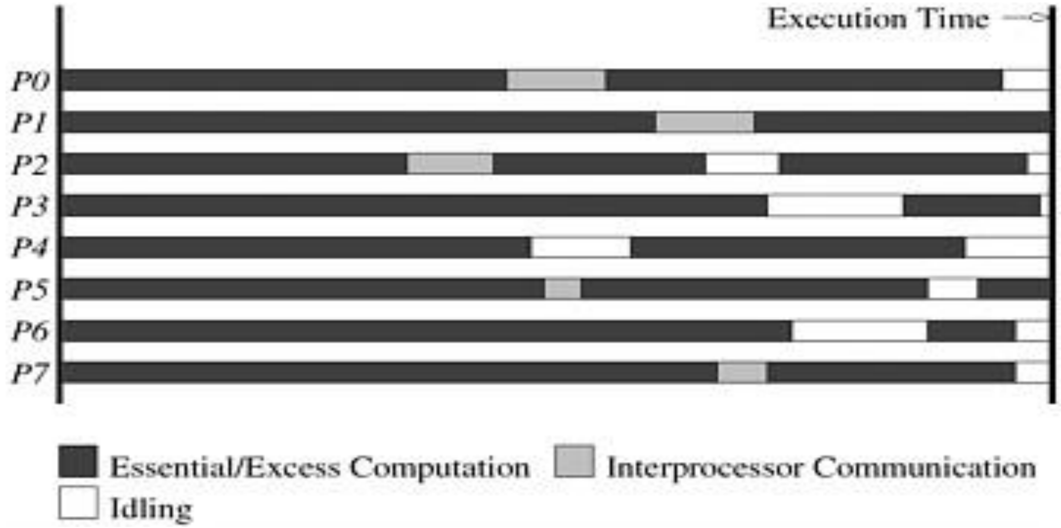
Parallel running time on p processing elements,

$$T_P = t_{end} - t_{start},$$

where, t_{start} = starting time of the processing element that starts first

t_{end} = termination time of the processing element that finishes last

Parallel Execution Time & Overhead

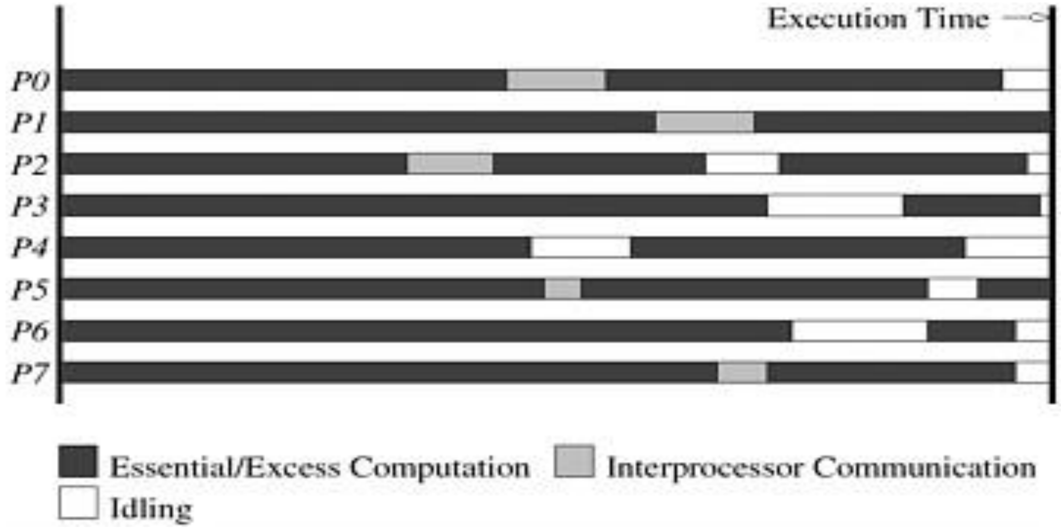


Source: Grama et al., "Introduction to Parallel Computing", 2nd Edition

Sources of overhead (w.r.t. serial execution)

- Interprocess interaction
  - Interact and communicate data (e.g., intermediate results)
- Idling
  - Due to load imbalance, synchronization, presence of serial computation, etc.
- Excess computation
  - Fastest serial algorithm may be difficult/impossible to parallelize

Parallel Execution Time & Overhead



Source: Grama et al., "Introduction to Parallel Computing", 2nd Edition

Overhead function or total parallel overhead,

$$T_O = pT_p - T,$$

where, p = number of processing elements

T = time spent doing useful work

(often execution time of the fastest serial algorithm)

Speedup

Let T_p = running time using p identical processing elements

Speedup, $S_p = \frac{T_1}{T_p}$

Theoretically, $S_p \leq p$ (why?)

Perfect or *linear* or *ideal* speedup if $S_p = p$

Speedup

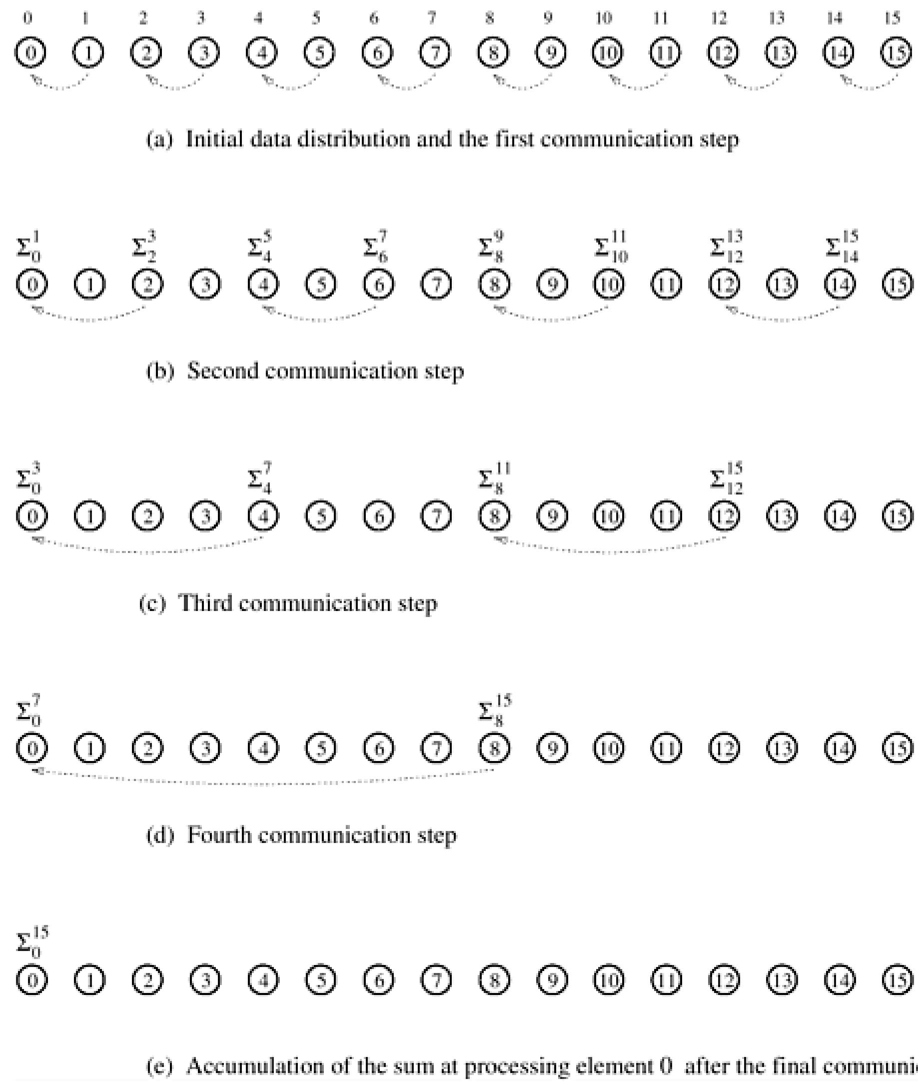
Consider adding n numbers using n identical processing elements.

Serial runtime, $T_1 = \Theta(n)$

Parallel runtime, $T_n = \Theta(\log n)$

Speedup, $S_n = \frac{T_1}{T_n} = \Theta\left(\frac{n}{\log n}\right)$

Speedup not ideal.



Source: Grama et al., "Introduction to Parallel Computing", 2nd Edition

Superlinear Speedup

Theoretically, $S_p \leq p$

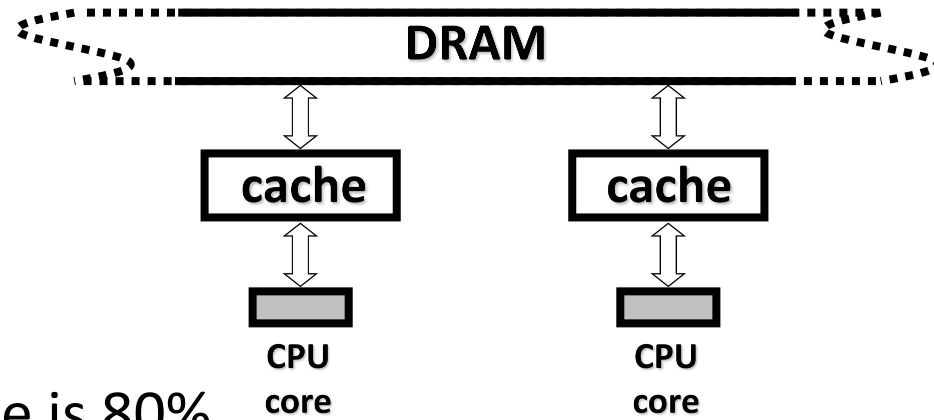
But in practice *superlinear speedup* is sometimes observed,

that is, $S_p > p$ (why?)

Reasons for superlinear speedup

- Cache effects
- Exploratory decomposition

Superlinear Speedup (Cache Effects)



Let cache access latency = 2 ns

DRAM access latency = 100 ns

Suppose we want solve a problem instance that executes *k* FLOPs.

With 1 Core: Suppose cache hit rate is 80%.

If the computation performs 1 FLOP/memory access, then each FLOP will take 2 × 0.8 + 100 × 0.2 = 21.6 ns to execute.

With 2 Cores: Cache hit rate will improve. (why?)

Suppose cache hit rate is now 90%.

Then each FLOP will take 2 × 0.9 + 100 × 0.1 = 11.8 ns to execute.

Since now each core will execute only *k* / 2 FLOPs,

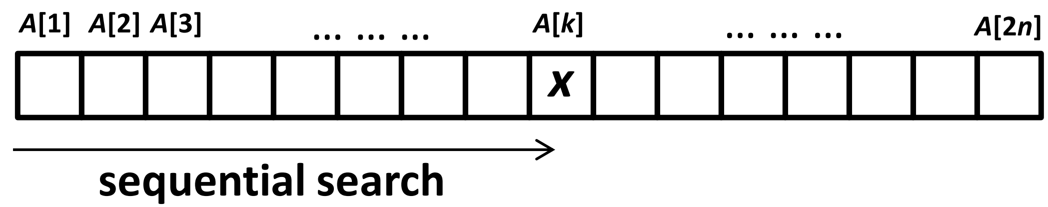
Speedup, $S_2 = \frac{k \times 21.6}{(k/2) \times 11.8} \approx 3.66 > 2$

Superlinear Speedup
(Due to Exploratory Decomposition)

Consider searching an array of $2n$ unordered elements for a specific element x.

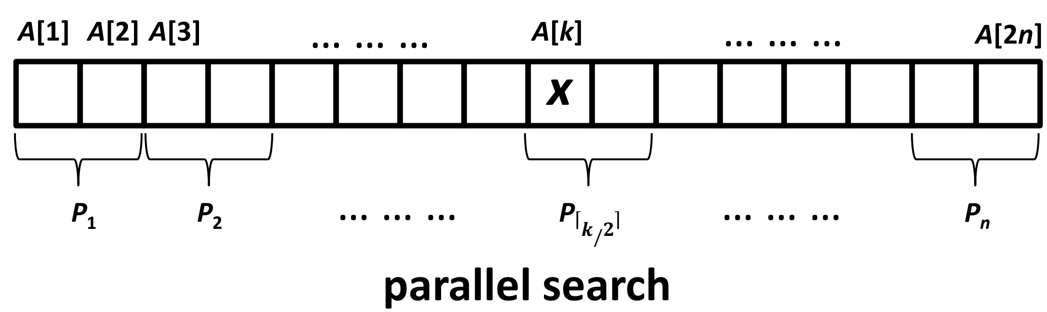
Suppose x is located at array location $k > n$ and k is odd.

Serial runtime, $T_1 = k$



Parallel running time with n processing elements, $T_n = 1$

Speedup, $S_n = \frac{T_1}{T_n} = k > n$



Speedup is superlinear!

Parallelism & Span Law

We defined, T_p = runtime on p identical processing elements

Then span, T_∞ = runtime on an infinite number of identical processing elements

Parallelism, $P = \frac{T_1}{T_\infty}$

Parallelism is an upper bound on speedup, i.e., $S_p \leq P$ (why?)

Span Law

$$\boxed{T_p \geq T_\infty}$$

Work Law

The cost of solving (or work performed for solving) a problem:

On a Serial Computer: is given by T_1

On a Parallel Computer: is given by pT_p

Work Law

$$T_p \geq \frac{T_1}{p}$$

Work Optimality

Let T_s = runtime of the optimal or the fastest known serial algorithm

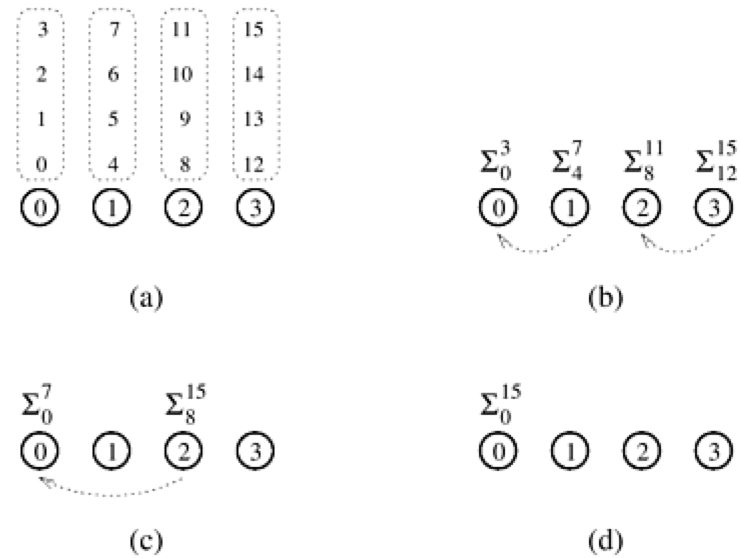
A parallel algorithm is *cost-optimal* or *work-optimal* provided

$$pT_p = \Theta(T_s)$$

Our algorithm for adding n numbers using n identical processing elements is clearly not work optimal.

Adding n Numbers Work-Optimality

We reduce the number of processing elements which in turn increases the granularity of the subproblem assigned to each processing element.



Source: Grama et al., "Introduction to Parallel Computing", 2nd Edition

Suppose we use p processing elements.

First each processing element locally adds its $\frac{n}{p}$ numbers in time $\Theta\left(\frac{n}{p}\right)$.

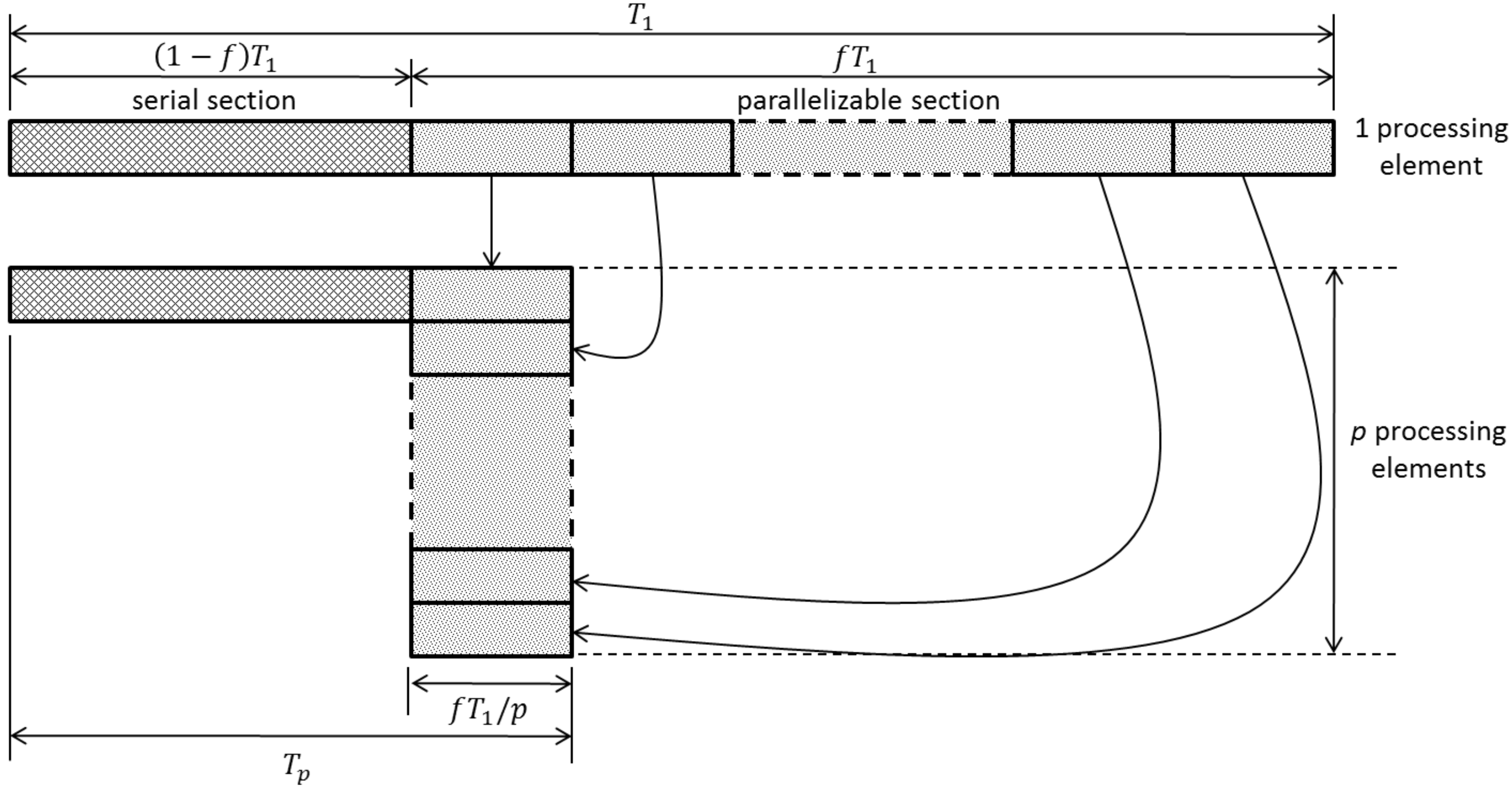
Then p processing elements adds these p partial sums in time $\Theta(\log p)$.

Thus $T_p = \Theta\left(\frac{n}{p} + \log p\right)$, and $T_s = \Theta(n)$.

So the algorithm is work-optimal provided $n = \Omega(p \log p)$.

Scaling Laws

Scaling of Parallel Algorithms (Amdahl's Law)



Suppose only a fraction f of a computation can be parallelized.

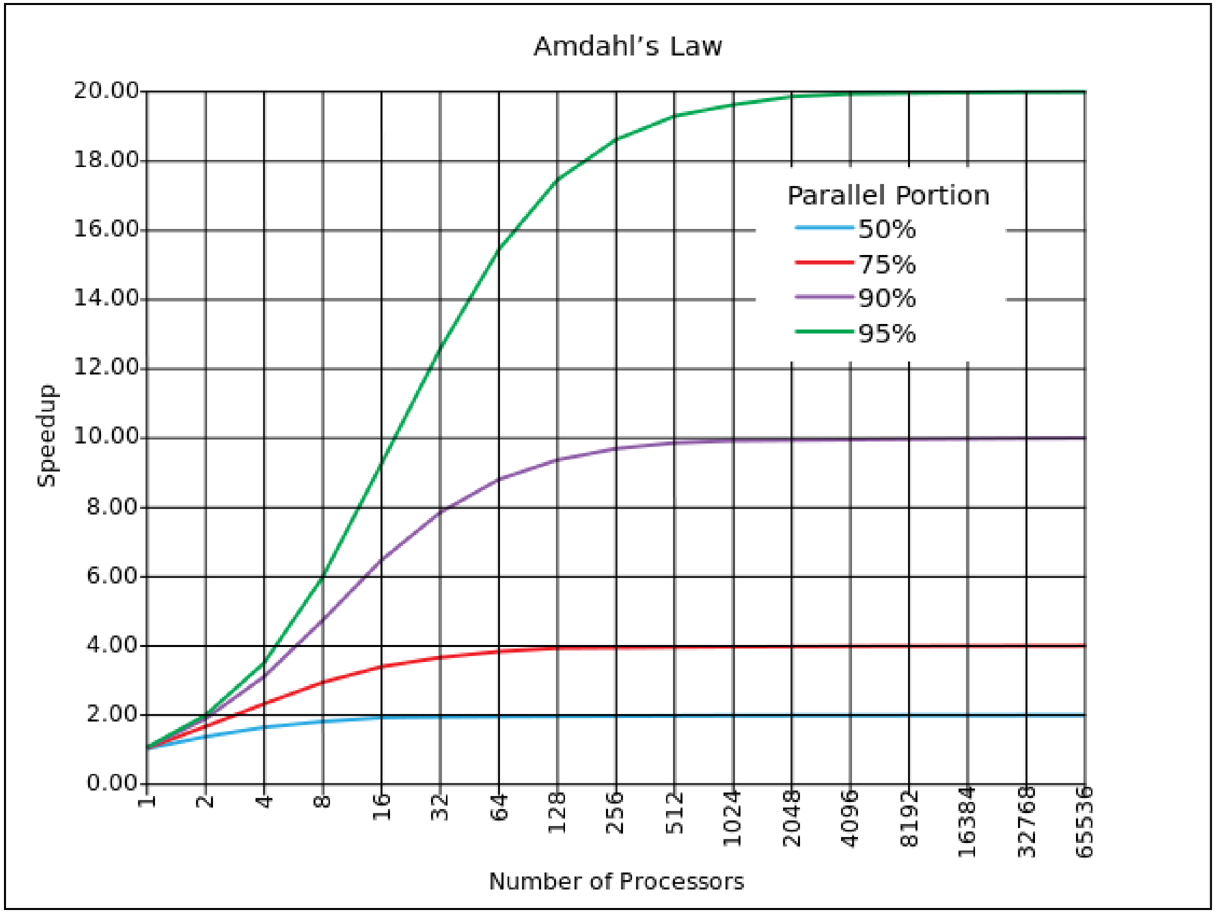
Then parallel running time, $T_p \geq (1-f)T_1 + f\,\frac{T_1}{p}$

Speedup, $S_p = \frac{T_1}{T_p} \leq \frac{p}{f+(1-f)p} = \frac{1}{(1-f)+\frac{f}{p}} \leq \frac{1}{1-f}$

Scaling of Parallel Algorithms (Amdahl's Law)

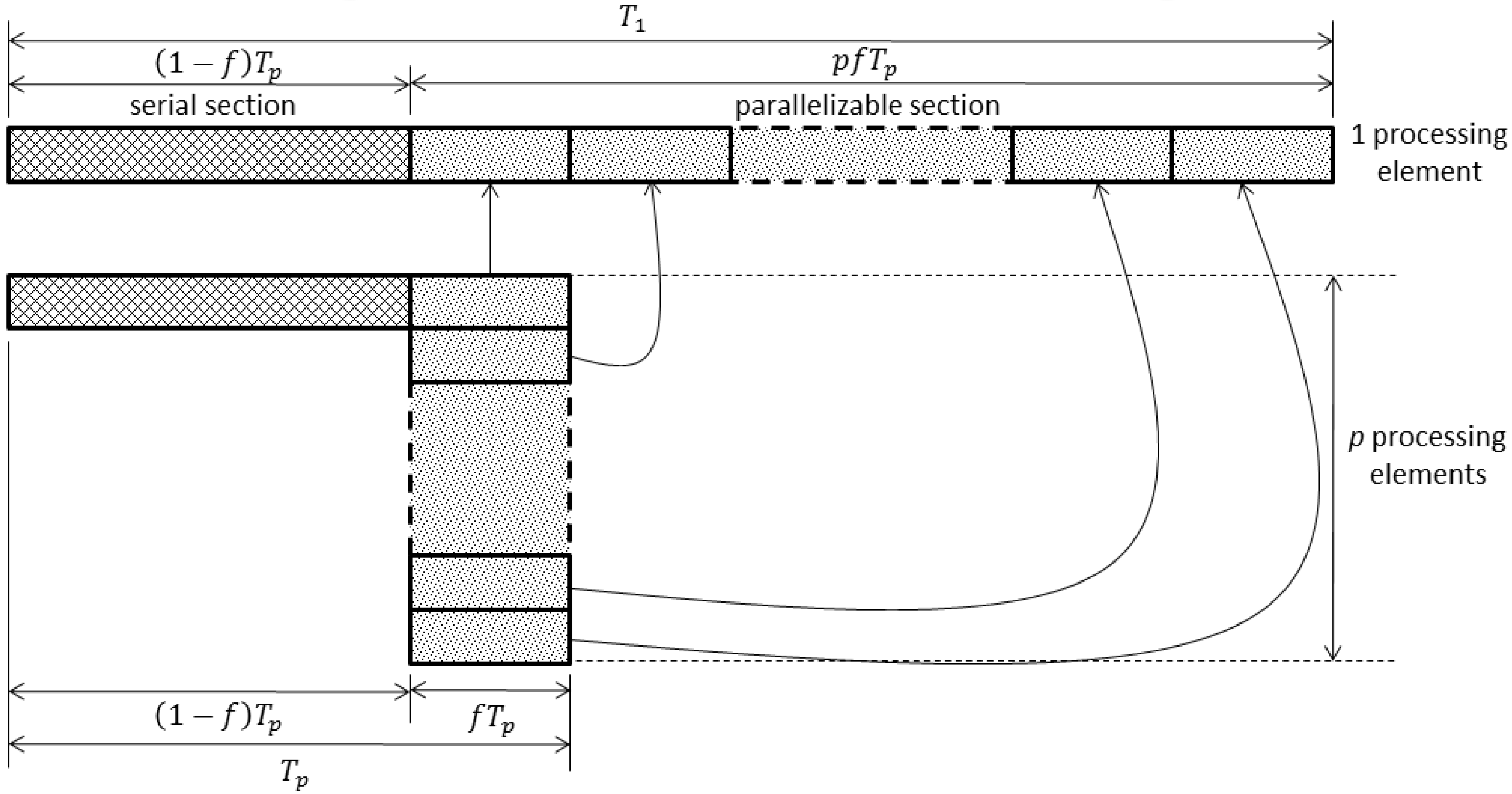
Suppose only a fraction f of a computation can be parallelized.

Speedup, $S_p = \frac{T_1}{T_p} \leq \frac{1}{(1-f)+\frac{f}{p}} \leq \frac{1}{1-f}$



Source: Wikipedia

Scaling of Parallel Algorithms (Gustafson-Barsis' Law)



Suppose only a fraction f of a computation was parallelized.

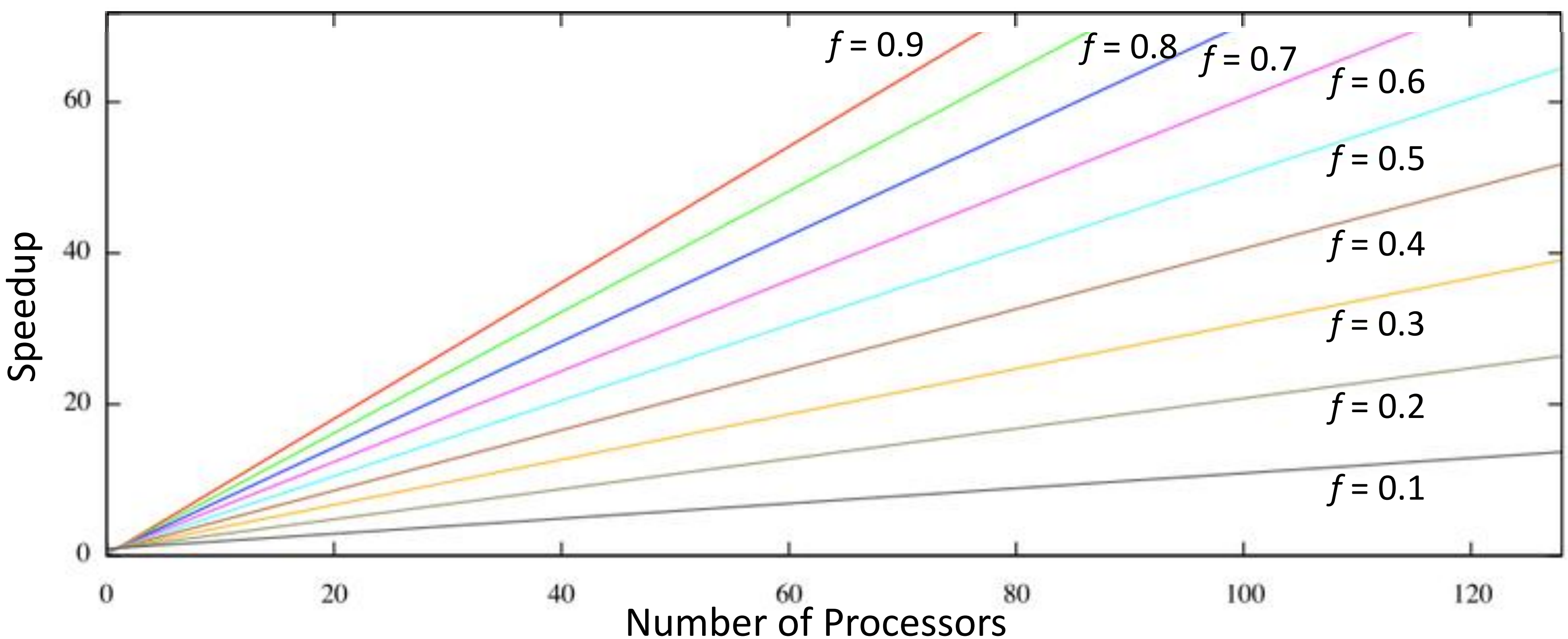
Then serial running time, $T_1 = (1-f)T_p + pfT_p$

Speedup, $S_p = \frac{T_1}{T_p} = \frac{(1-f)T_p + pfT_p}{T_p} = 1 + (p-1)f$

Scaling of Parallel Algorithms (Gustafson-Barsis' Law)

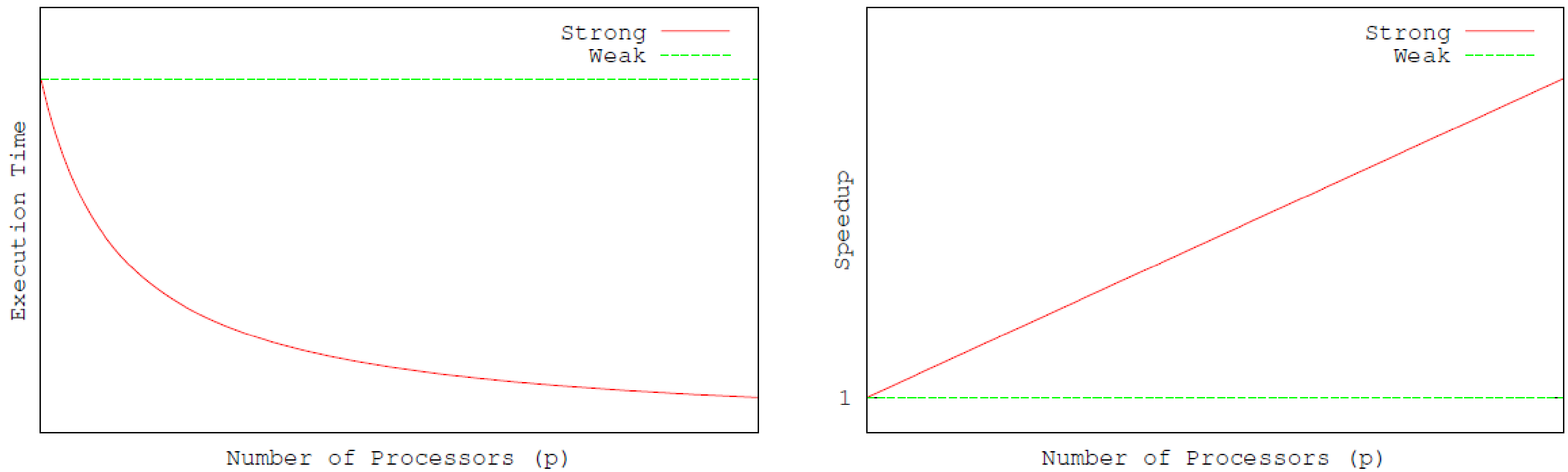
Suppose only a fraction f of a computation was parallelized.

Speedup, $S_p = \frac{T}{T_p} \leq \frac{T_1}{T_p} = \frac{(1-f)T_p + pfT_p}{T_p} = 1 + (p-1)f$



Source: Wikipedia

Strong Scaling vs. Weak Scaling



Source: Martha Kim, Columbia University

Strong Scaling

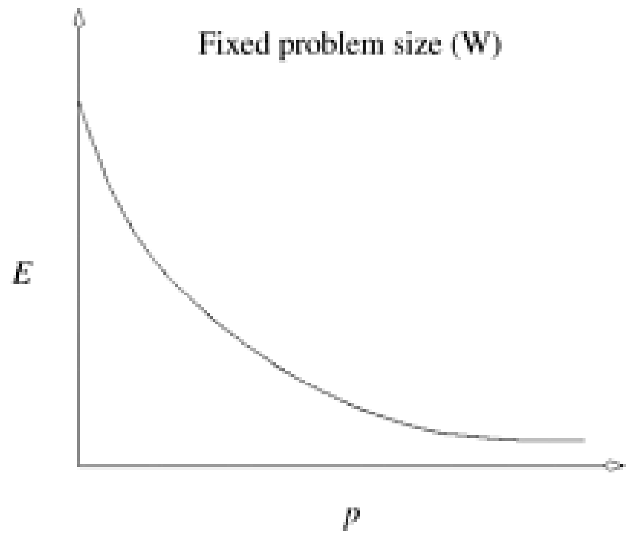
How T_p (or S_p) varies with p when the problem size is fixed.

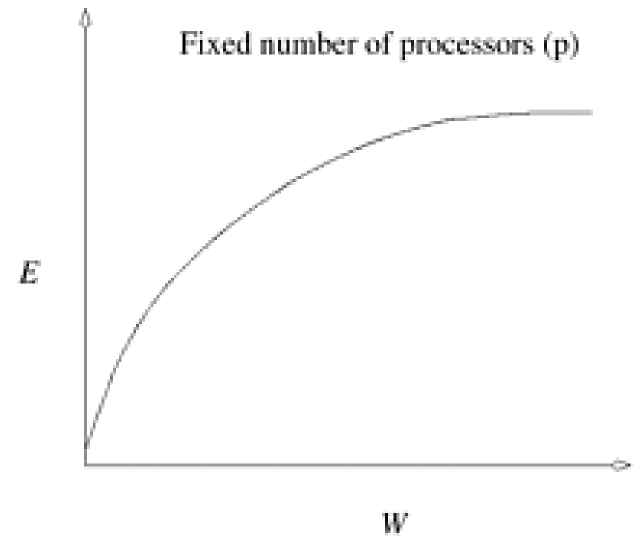
Weak Scaling

How T_p (or S_p) varies with p when the problem size per processing element is fixed.

Scalable Parallel Algorithms

Efficiency, $E_p = \frac{S_p}{p} = \frac{T_1}{pT_p}$





Source: Grama et al., "Introduction to Parallel Computing", 2nd Edition

A parallel algorithm is called *scalable* if its efficiency can be maintained at a fixed value by simultaneously increasing the number of processing elements and the problem size.

Scalability reflects a parallel algorithm's ability to utilize increasing processing elements effectively.

Scalable Parallel Algorithms

In order to keep E_p fixed at a constant *k*, we need

$$E_p = k \Rightarrow \frac{T_1}{pT_p} = k \Rightarrow T_1 = kpT_p$$

For the algorithm that adds *n* numbers using *p* processing elements:

$$T_1 = n \ \text{ and } \ T_p = \frac{n}{p} + 2\log p$$

So in order to keep E_p fixed at *k*, we must have:

$$n = kp\left(\frac{n}{p} + 2\log p\right) \Rightarrow n = \frac{2k}{1-k} p \log p$$

n	*p* = 1	*p* = 4	*p* = 8	*p* = 16	*p* = 32
64	1.0	***0.80***	0.57	0.33	0.17
192	1.0	0.92	***0.80***	0.60	0.38
320	1.0	0.95	0.87	0.71	0.50
512	1.0	0.97	0.91	***0.80***	0.62

Fig: Efficiency for adding *n* numbers using *p* processing elements

Source: Grama et al., "Introduction to Parallel Computing", 2nd Edition