

o "size" of the problem of quantity of input in

time complexity :  
o space "

o asymptotic time(space) complex

$$\lim_{n \rightarrow \infty} ( ) = \square$$

o Asymptotic notations

o Big-Oh (asymptotic upper bound)

def >  $f(n) = O(g(n))$

there exists const.  $n_0 + C > 0$  s.t. for all  $n \geq n_0$

$$0 \leq f(n) \leq Cg(n)$$

ex) "running time is  $O(n^2)$ " = worst case running time is  $O(n^2)$

ex)  $f(n) = 2n \lg n + 5n$

$\lg = \log_2$

choose  $n_0 = 2$

$$\frac{2n \lg n + 5n}{C} \leq 2n \lg n + 5n \leq Cn \lg n \quad \therefore f(n) = O(n \lg n)$$

$$2n \lg n + 5n \leq 2n^2 + 5n^2 = 7n^2$$

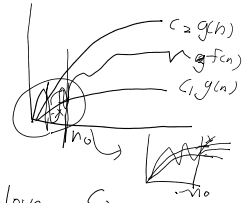
$$C = 7 \quad f(n) = O(n^2)$$

o theta  $\Theta$  (asymptotic tight bound)

def )  $f(n) = \Theta(g(n))$  :

there  $\exists$  three const.  $n_0, C_1, C_2 > 0$  s.t. for all  $n > n_0$

$$C_1 g(n) \leq f(n) \leq C_2 g(n)$$



ex)  $f(n) = 2n \lg n + 5n$

choose  $n_0 = 2$

$g(n) = n \lg n$

$$C_1 n \lg n \leq 2n \lg n + 5n \leq 2n \lg n + 5n \lg n = C_2 n \lg n$$

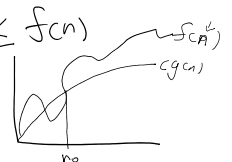
o Big-Omega ( $\Omega$ ) (asymptotic lower-bound)

def >  $f(n) = \Omega(g(n))$

there  $\exists$  two const.  $n_0, C > 0$  s.t.

for all  $n \geq n_0$

$$0 \leq Cg(n) \leq f(n)$$



o little-o

" (  $0 \leq f(n) < \epsilon g(n)$  )  $2n = o(n^2)$   
 $2n^2 \neq o(n^2)$

$(0 \leq f(n) \leq eg(n)) \quad \begin{matrix} 2n = o(n^2) \\ 2n^2 \neq o(n^2) \end{matrix}$

o little-w

$0 \leq cg(n) \leq f(n)$

ex)  $2n^2 + 3n + 1 = 2n^2 + \Theta(n)$   
 $3n^2 + 5n + 1 = 3n^2 + \Theta(n)$

ex)  $T(n) = T(n/2) + \Theta(n)$

ex)  $2n^2 + \Theta(n) = \Theta(n^2) = O(n^2)$

o functions of n

- $O(\lg n)$  : logarithmic
- $O((\lg n)^k)$  : poly logarithmic (const. k)
- $O(\sqrt{n})$  :
- $O(n)$  : linear
- $O(n^2)$  : quadratic
- $O(n^k)$  : polynomial (some const. k)
- $O(c^n)$  : exponential  $c > 1$
- $O(n!)$