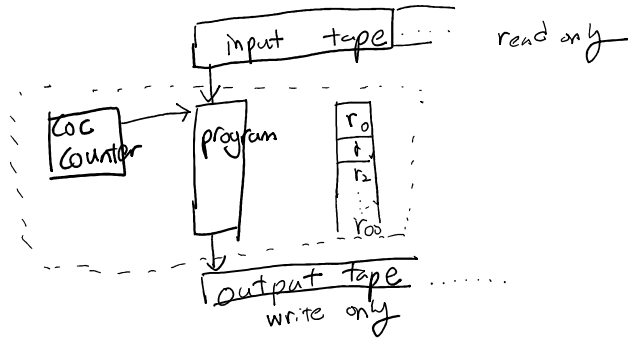


Random access machine (RAM)



* assumptions about memory

- ① reg. can hold integers of arbitrary size
- ② # reg. infinite

o * assumption about program.

- ① program does not modify itself
- ② program is seq of operations (instructions)

o two complexities in RAM

• uniform cost criterion for time:

→ instructions req. one unit of time

o logarithmic cost criterion for time

a instruction^l req. $l \log |i|$ time on integer i

$$l(i) = \begin{cases} 1 & i = 0 \\ \lfloor \log |i| \rfloor + 1 & i \neq 0 \end{cases}$$

o uniform cost criterion for space

: each reg. used one unit of space

o log. cost criterion for space

$$\sum_{\text{location } i} l(x_i) \text{ where } x_i \text{ is the largest value ever stored in } i$$

(ex) Factoring n by "trial division"
 $m \leq n$

$F \leftarrow \phi$
 $i \leftarrow 2$

While $i \leq \lfloor \sqrt{n} \rfloor$ do

if i divides evenly into m
 then insert i into F
 $m \leftarrow m/i$
 else $i \leftarrow i+1$
 ENIF
 endwhile

if $m > 1$, then insert m into F

* ab

• # times through the while loop

$$\leq \lfloor \sqrt{n} \rfloor + \lg n$$

ex) $2^4 = \cancel{16 \times 16} = 2 \times 2 \times 2 \times 2$

$\begin{matrix} i=2 & m=2 \times 2 \times 2 \times 2 \\ i=2 & m=2 \times 2 \times 2 \\ i=2 & m=2 \times 2 \\ i=2 & m=2 \\ i=2 & m=1 \\ i=3 \\ i=4 \end{matrix}$

$$\sqrt{2^4} = 4$$

• max # of factor ? $\lg n$

• uniform cost

time $O(\lfloor \sqrt{n} \rfloor + \lg n) = O(\sqrt{n})$

space $O(\lg n)$

• logarithmic cost

trial division

$n = 4^8$ $F = 2, 2, 2, 2, 2, 2, 2, 2$
 $i = 2 \times \frac{24}{m}$

$$i = 2 \times 12$$

$$i = 2 \times 6$$

$$i = 2 \times 3$$

$$m = 3 > \#$$

• logarithmic cost

time: worst loop time for each ^{trial division} ~~loop~~

is when m never gets updated

$\rightarrow \lg(n)$ for each trial division (\sqrt{n})

$$\therefore O(\sqrt{n} \cdot \lg(n))$$

Space :

$$F = (n = f_1 \times f_2 \times \dots \times f_k$$

$$\lg n = \lg f_1 + \lg f_2 + \dots + \lg f_k) \underline{O(\lg n)}$$

$$m \Rightarrow \underline{\lg n}$$

~~*~~

$$i = \lg \sqrt{n} = \frac{\lg n}{2} \therefore \text{space } O(\lg n)$$