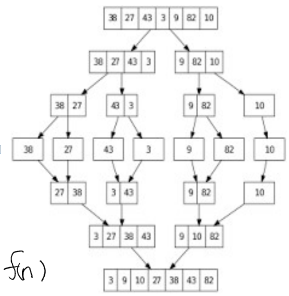
① Divide : "instance" (smaller)

② Conquer: [recurrence case / Base case

③ combine

Recurrence



- substitution method
- recurrence three
- Master theorem

$\Theta \to$ $T(n) = aT(n/b) + f(n)$

$O \to$ $\leq$

$\Omega \to$ $\geq$

Even split

ex) mergesort (A, i, j) // Sort A[i:j]

```
if i=j then return A
else m ← ⌊(i+j)/2⌋
    merge sort ( A , i , m )          ← T[n/2]
    merge sort ( A , m+1 , j )        ← T[n/2]
    for k=i to m B[k-i] = A[k] endfor          ] Θ(n)
    for k = m+1 to j C[k-m-1] = A[k] endfor    ]
    B[m-i+1] = C[j-m] = INF
    b=0 , c=0                                          Cn
    for k = i to j
        if C[c] < B[b]                          ] Θ(n)
        then A[k] = C[c++]
        else A[k] = B[b++]
        endif
    end for
endif
```

running time

$T(1) = C$

$T(n) = T(\lfloor n/2 \rfloor) + T(\lceil n/2 \rceil) + Cn$

Let m be a smallest power of 2 s.t $m \geq n$

$T(m) = 2T(m/2) + cm$

Claim : $T(m) = cm\lg m + cm$ $\boxed{O(m\lg m)}$

proof by induction on m

Basis : $m=1$

$T(1) = c \times \lg 1 + c \times 1 = c$ ✓

Induction step : $m > 1$

Assume as the induction hypothesis (IH) that claim hold for $m' < m$

$$T(m) = 2T(\underbrace{m/2}_{m'}) + cm$$

$$= \cancel{2}\left(c\left(m/\cancel{2}\right)\lg\left(\frac{m}{2}\right) + c\left(\frac{m}{\cancel{2}}\right)\right) + cm$$

$$= cm\lg\left(\frac{m}{2}\right) + 2cm$$

$$= cm(\lg m - \underbrace{\lg 2}_{=1}) + 2cm$$

$$= \underline{cm\lg m + cm}$$

ex 2) Multiplying two n-bit numbers.

mult(n, x, y) // assum $n = 2^l$, $l \ge 0$

if $n=1$ then return xy

else // $x = a2^{n/2} + b$

// $y = c2^{n/2} + d$

// $xy = ac2^n + (ad+bc)2^{n/2} + bd$

$5 = 101$, $m=3$

$2 \times 2 + 1$, 10, $\lceil n/2 \rceil + 1$

$u \leftarrow (a+b)(c+d)$ // mult(n/2, a+c, c+d)

$ac + ad + bc + bd$

$v \leftarrow ac$ // mult(n/2, a, c)

$w \leftarrow bd$ // mult(n/2, b, c)

$z \leftarrow v2^n + (u - v - w)2^{n/2} + w$

return z

$n = 8$

$x = 1011\ 0010$ $y = 0110\ 0011$