Divide and Conquer Cont.

2017년 9월 6일 수요일

오전 11:28

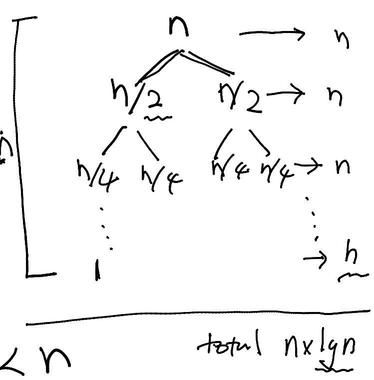
$T(n) = 2T(n/2) + n$

ⓐ $T(n) = O(n \lg n)$

Claim $T(1) \le c$, $T(n) \le cn\lg n$ ($n \ge 3$), $n = 1$ ✗, $c > 0$

* Base case : $T(1) \le c$

Assume by IH that Claim holds for $n' < n$



$T(n) = 2T(\underbrace{n/2}_{<n}) + n$ $\downarrow$ IH $\quad T(1) \le c$

$\le \cancel{2}c\frac{n}{\cancel{2}}\lg\frac{n}{2} + n$

$= cn(\lg n - \lg 2) + n$ ($\lg 2 = 1$)

$= cn\lg n \underbrace{- (c-1)n}_{<0} \Rightarrow \boxed{c - 1 > 0 \quad c > 1}$

$\le cn\lg n$

$n = 8 \quad x = \underbrace{1011}_{a}\underbrace{0010}_{b} \times y = \underbrace{0110}_{c}\underbrace{0011}_{d}$

$a \times c$: $\underbrace{10}_{a}\underbrace{11}_{b} \times \underbrace{01}_{c}\underbrace{10}_{d}$ $\quad b \times d$: 0010×0011 $\quad (a+b)(c+d)$: $\underbrace{11}_{a}\underbrace{01}_{b} \times \underbrace{10}_{c}\underbrace{01}_{d}$

$a \times b$: $\underbrace{1}_{a}\underbrace{0}_{b} \times \underbrace{0}_{c}\underbrace{1}_{d}$ $\quad b \times d$: $\underbrace{1}_{a}\underbrace{1}_{b} \times \underbrace{1}_{c}\underbrace{0}_{d}$ $\quad (a+b)(c+d)$: 0011 $\quad$ $a \times c$ $\quad d b \quad (a+b)(c+d)$] k

$a \times c$

lg n

at level k.

subproblems : 3^k

size of subproblem : $n/2^k$

spent on k? $(3^k \ldots$

$\left(\frac{}{2}\right) n \cdot \approx O(n)$

when $n=1$ $k = \lg n$ $\rightarrow$

$\begin{cases} 3^{\lg n} \\ 1/2^{\lg n} \end{cases} \Rightarrow \left(\frac{3}{2}\right)^{\lg n} O(n) = n\left(\frac{3}{2}\right)^{\lg n}$

$= O(3^{\lg_2 n})$

$= O(n^{\lg 3})$

- ac : $T(n/2)$
- bd : $T(n/2)$
- $(a+b)$: at most $n/2 + 1$ bits
  $(c+d)$: " "
  $a+b = p2^{n/2} + q$
  $c+d = r2^{n/2} + s$
- $(a+b)(c+d) = T(n/2+1) \approx T(n/2)$

$*\ q, s \approx 1$ bit

$K > 0$

time $\begin{cases} T(1) \le k & n=1 \\ T(n) \le 3T(n/2) + kn & \text{if } n>1 \end{cases}$

Claim: $T(n) \le \overset{c_1}{\boxed{3k}} n^{\lg 3} \overset{c_2}{\boxed{-2k}} n$

proof : by induction on n

Basis $(n=1)$ $T(1) \le 3k - 2k = k$

IS $(n>1)$ assume as IH that claim holds for $n' < n$

$$T(n) \le 3\,T(n/2) + kn$$

IH

$$\le 3\left(\underbrace{3k\left(\frac{n}{2}\right)^{\lg 3} - \cancel{2}k\frac{n}{\cancel{2}}}_{T(n/2)}\right) + kn$$

$$= \frac{\cancel{9}^{\,3}\,k\,n^{\lg 3}}{\underbrace{2^{\lg 3}}_{=\cancel{3}}} \underbrace{- 3kn + kn}_{-2kn}$$

$$= 3kn^{\lg 3} - 2kn$$

$$\therefore T(n) = O(n^{\lg 3})$$

Master Theorem

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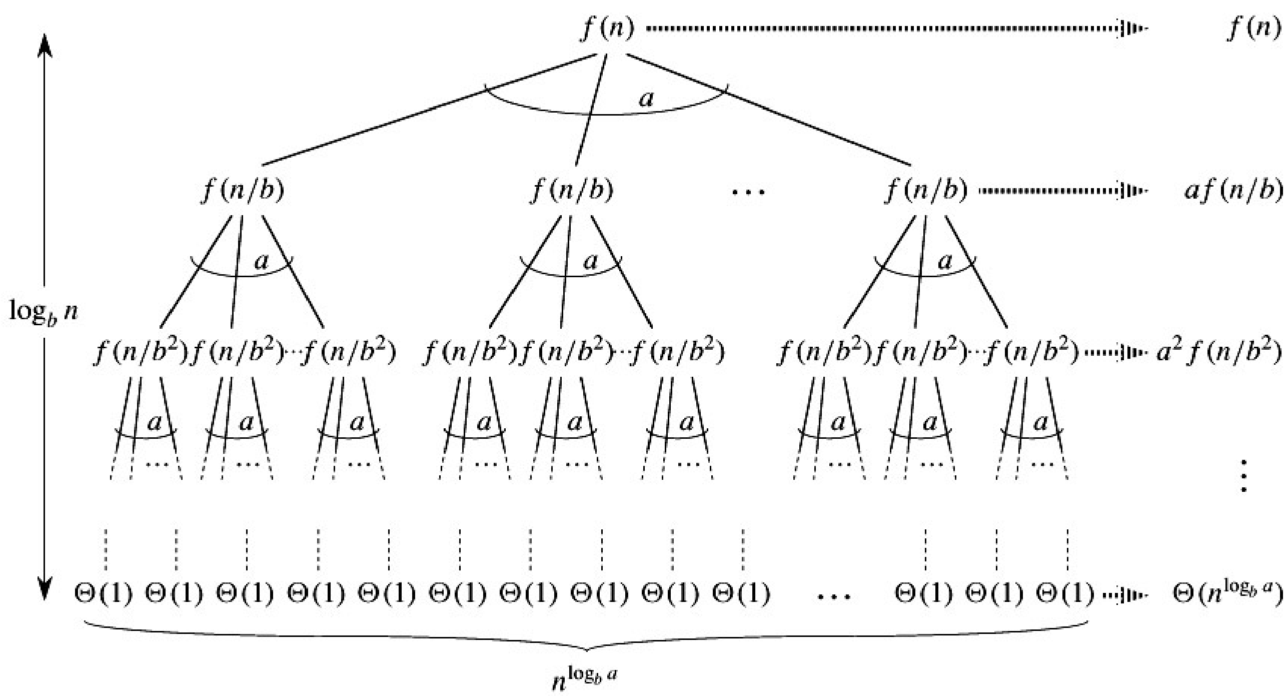
Recursion Tree for Master Theorem :
T(n) = aT(n/b) + f(n)



Let $a \geq 1$ $b > 1$ be const.
$f(n)$ be a function.
$T(n)$ be defined on non-neg. inte by recurrence

$$T(n) = aT(n/b) + f(n)$$

n/b can be either $\lfloor n/b \rfloor$ or $\lceil n/b \rceil$

Then $T(n)$ has the asymptotic bounds:

1. If $f(n) = O(n^{\log_b a - \varepsilon})$ $\varepsilon > 0$

$\rightarrow T(n) = \Theta(n^{\log_b a})$

2. If $f(n) = \Theta(n^{\log_b a})$

$T(n) = \Theta(n^{\log_b a} \lg n)$

3. If $f(n) = \Omega(n^{\lg_b a + \varepsilon})$

$T(n) = \Theta(f(n))$