Uneven Split - Prune and Search

2017년 9월 13일 수요일

오후 3:11

$\Theta(0)$

$\Theta(n^2)$ $n \times n$ □ # people $= n$.

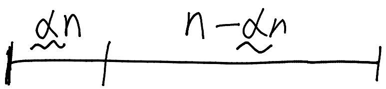
$T(n) = 7T(n/2) + \Theta(n^2)$

MT case 1

$n^{\log_2 7} \approx n^{2.8\ldots} > n^2 = f(n)$

$\quad n^{0.8\sim}$

$\therefore \Theta(n^{2.8})$



$0 < \alpha \le \frac{1}{2}$

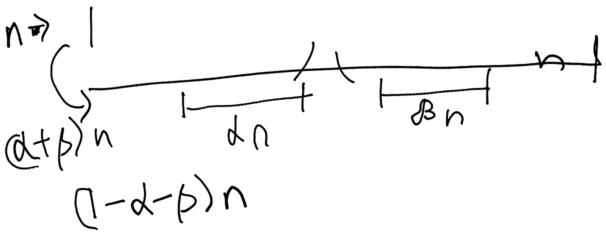
Theorem (Case 2)

let c, α, and β be const.
with $\alpha > 0$, $\beta > 0$, $\alpha + \beta < 1$, $c > 0$

let $T(n) \le c$ if $0 < n < \max\{\frac{1}{\alpha}, \frac{1}{\beta}\}$

$T(n) \le cn + T(\lfloor \alpha n \rfloor) + T(\lfloor \beta n \rfloor)$

then $T(n)$ is $O(n)$



$\alpha n + \beta n < n$

$(1-\alpha-\beta)n$

claim : $T(n) \le an$

proof by induction on n

Basis $T(n) \le C < a \le an$ $a = \frac{c}{1-\alpha-\beta}$

Induction ~~Step~~ :

assume by IH that the claim holds for all $n' < n$

$T(n) \le cn + T(\lfloor \alpha n \rfloor) + T(\lfloor \beta n \rfloor)$

$\le cn + a\lfloor \alpha n \rfloor + a \lfloor \beta n \rfloor$ claim $\le cn + an(\alpha+\beta)$

$a = c + a(\alpha+\beta))$

$= n(\underbrace{c + a(\alpha+\beta)}_{=a})$

$= an$

$a = \frac{c}{1-\alpha-\beta}$

$c = a(1-\alpha-\beta)$

* order statistic :

* selection problem :

Input: A set S of n (distinct) numbers and an int k $1 \le k \le n$

* Selection prob

Input: A set S of n (distinct) numbers and an int k $1 \leq k \leq n$

Output: The element $x \in S$ that is larger than exactly $k-1$ other elements of S

* Approach 0
  sort S and index in k
  $\Rightarrow O(n \lg n)$

* Approach 1:
  Choose appropriate partition element m
  $m \in S$

Partition S into

$S_- = \{x < m \mid x \in S\}$

$S_1 = \{x < m \mid x \in S\}$

$S_2 = \{x = m \mid x \in S\}$ $S_3 = \{x > m \mid x \in S\}$

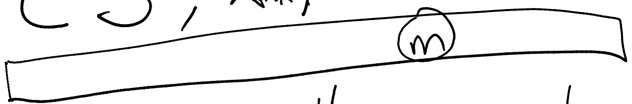
Reduce problem into smaller problem

- S involving k' and one of S_i

$|?|$

Algorithm

SELECT (S, k_{th})



if $|S|$ is small enough

sort S and return k^{th} smallest

else find partition element $\underline{m}$

then Determine S_1 S_2 S_3

$k \le |S_1|$: return SELECT(S_1, k)

$|S_1| < k \le |S_1| + |S_2|$: return m

$|S_1| < k \le |S_1| + |S_2|$: return m

$|S_1| + |S_2| < k$: return SELECT$(S_3, k - |S_1| - |S_2|)$

endcase

endif

* If m could be found s.t. $|S_1|, |S_3| \approx \frac{1}{2}|S|$

then $T(n) = T(n/2) + O(n)$

* Choosing of ~~par~~ partition element m ① choose ele

* Choosing of partition element m ① choose element at random

② choose element "near" the middle (Escatch)

- let r be fixed small odd #
- Divide S into $\lfloor n/r \rfloor$ subset of size $T(n = |S|)$ with at most one subset of size n mod

- find median in each subset and let M be the set of medians.
- recursively find $\lceil M/2 \rceil$ the smallest element in M, let that be $\boxed{m}$

5 5 5 5 5 /5 /5/

38 r_1 = 9 23 28 30 43 42

5 4.5 37 25 44 62 60 34

M $\boxed{32}$ $\boxed{38}$ $\boxed{36}$ $\boxed{56}$ $\boxed{63}$ $\boxed{61}$ $\boxed{43}$

53 7

58 77 59 93 69 99

0 at least 41 75 64 80 68