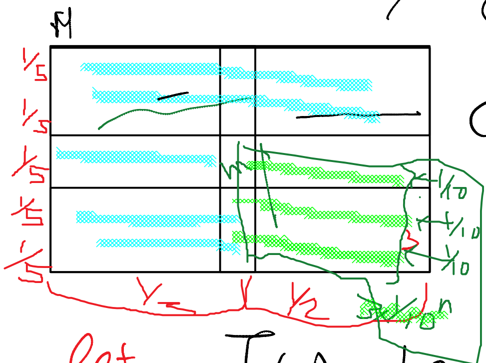
Uneven Split - Selection Problem cont.

2017년 9월 18일 월요일

오후 3:06

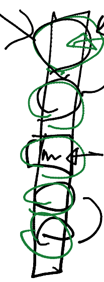
2. Choosing Partition element "near" the middle.

Rough analysis:



① at least $n/4$ elements are $\le m$

② at least $n/4$ elements are $\ge m$



let $T(n)$ be the time req. to select kth smallest element from S

- Set M such that $|M|$ at most $n/5$ $(r=5)$

$\Rightarrow$ recursively call req. at most $T(n/5)$ time

- Subset $\overset{<m}{S_1}$ and $\overset{>m}{S_3}$ are each of size at most $3/4\,n$.

$\Rightarrow S_1$: at least $\lfloor n/10 \rfloor$ elements of $M \ge m$

and there two distinct ($r=5 \times 2$) elements in S

that are at least as large

$\therefore |S_1| \le n - 3\lfloor n/10 \rfloor \le 3/4\, n$

n_0

~~An~~ Sub Analysis

$$1/4\, n \le 3\lfloor n/10 \rfloor$$

$$n/12 \le \lfloor n/10 \rfloor \quad \text{true for } n \ge 50$$

proof

proof

$$\frac{n-9}{10} \le \lfloor n/10 \rfloor \qquad \frac{n}{12} \le \frac{n-9}{10}$$

$$10n \le 12n - 9 \times 12$$

$$54 \le n.$$

$n = 53$

$n = 52$

$n = 51$

$\boxed{n = 50}$

$n = 49$

$n \cancel{\ge} 50$ $\boxed{n \ge 50}$

Recurrence relation of SELECT with PARTITION

T(n) < c when n<50

T(n) < T(n/5) + T(3n/4) + cn when n>=50

Proof by induction on n that T(n) < an :