Uneven Split - Randomized SELECT

2017년 9월 18일 월요일
오후 5:10

Recurrence relation of SELECT with PARTITION

$T(n) \le c_1$ when $n<50$
$T(n) \le T(n/5) + T(3n/4) + cn$ when $n>=50$

Proof by induction on n that $T(n) \le an$:

* Randomized Algorithm (RA)

⇒ Algorithm is randomized if it's behavior
is determine ① input
② values produced by
random-number generator into
Algo.

* Analysis of RA takes expected
run-time over the distribution of
values returned by the random number generator

* probabilistic Analysis

* ② Choosing P.E. m at random.
(assume equal prob of choosing $= \frac{1}{N}$)
: prob(m) = $\frac{1}{n}$

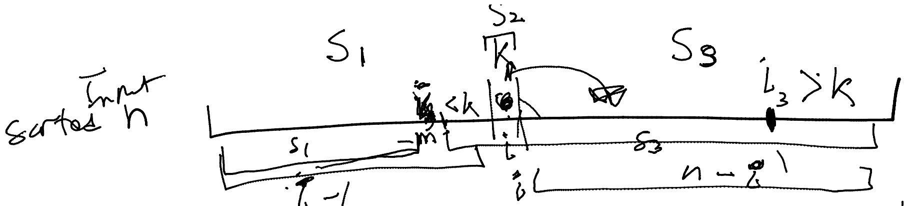
Base : if $|S| = 1$
then return that element
(assume elements are distinct)

Choose index i from any of $\{1, 2, \ldots n\}$ index

Let the rank of P.E. m is i

S_1 $\boxed{k}$ S_2 S_3

Input ... $i_2 > k$



S_2 ① if $i_1 = k$ $(|S_1| < k \leq |S_1| + |S_2|)$ ← return m takes const time

S_3 ② if $i_2 < k$ $(|S_1| + |S_2| < k)$ $|S_3| = n - i$

S_1 ③ if $i_3 > k$ need to recurse on S_1 $(|S_1| = i_3 - 1)$

∴ for give k, expected time of recurrence step:

$$\frac{1}{n}\Big[\underbrace{\sum_{i=1}^{k-1} T(n-i)}_{i<k} + \underbrace{\sum_{i=k+1}^{n} T(i-1)}_{i>k}\Big]$$

pick i

$n-1 \quad n-2 \quad \cdots \quad n-k+1 \qquad k, k+1, \cdots \quad n-1$

$$\approx \frac{1}{n}\Big[\sum_{i=n-k+1}^{n-1} T(i) + \sum_{i=k}^{n-1} T(i)\Big]$$

$n-k+1 \cdots \cdots n-1 \qquad k, k+1 \cdots\cdots n-1$

$$T(n) \leq \max_{1\leq k\leq n}\Big\{\frac{1}{n}\Big[\sum_{i=n-k+1}^{n-1} T(i) + \sum_{i=k}^{n-1} T(i)\Big]$$

* recurrence relationship of Randomized SELECT

$$\begin{cases} T(n) \leq C & n=1 \\ T(n) \leq Cn + \max\limits_{1\leq k\leq n}\Big\{\frac{1}{n}\Big[\sum\limits_{i=n-k+1}^{n-1} T(i) + \sum\limits_{i=k}^{n-1} T(i)\Big]\Big\} \end{cases}$$

claim : $T(n) \leq an$ where $\boxed{a = 4C}$

proof : by induction n.

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Basis (n=1) check latter ✓

$$T(1) \le \boxed{c \le a \cdot 1}$$

4c

Induction step. $(n > 1)$

Assume as IH that claim holds for all $n' < n$

$k \sim n-1$

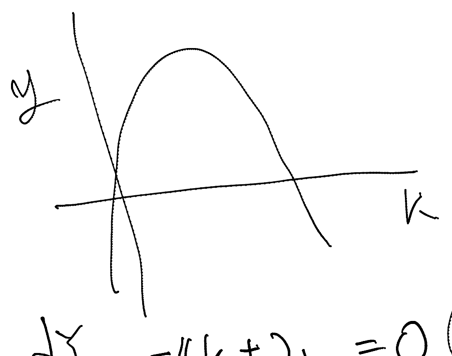
$$T(n) \le cn + \max_{1 \le k \le n} \left\{ \frac{a}{n} \left[\sum_{i=n-k+1}^{n-1} i + \underbrace{\sum_{i=k}^{n-1} i}_{\sum_{i=k+1}^{n} i} \right] \right\}$$

$k+1 \sim n$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\le cn + \max_{1 \le k \le n} \left\{ \frac{a}{n} \left[\frac{(n-1)(n)}{2} - \frac{(n-k)(n-k+1)}{2} + \frac{n(n+1)}{2} - \frac{k(k+1)}{2} \right] \right\}$$

$$= cn + \max_{1 \le k \le n} \left\{ \frac{a}{2n} \underbrace{\left(n^2 - n - n^2 + 2nk - n - k^2 + k + n^2 + n - k^2 - k \right)}_{\frac{a}{2n}\left[-2k^2 + 2nk + n^2 - n\right]} \right\}$$



$$\frac{dy}{dk} = -4k + 2n = 0$$

$$k = n/2$$

$$\le cn + \frac{a}{4}\left[3n - 2\right]$$

$$= \left(c + \frac{3}{4}a\right)n - \frac{a}{2} \le \left(c + \frac{3}{4}a\right)n = an$$

$$c + \frac{3}{4}a = a$$

$$c = \frac{1}{4}a$$

$$\boxed{a = 4c}$$