Fractional Knapsack using SELECT

$T = \{1 \dots N\}$ type of items

$M = \{m_1 \dots m_N\}$ max supplies of units per item.

$C = \{c_1, \dots c_N\}$ cost per unit of each items

$V = \{v_1, \dots v_N\}$ values "

B = budget & $0 \le f_i \le m_i$

$R = \{\frac{v_1}{c_1}, \dots \frac{v_N}{c_N}\}$

① cal efficiency $R \in r_i = \frac{v_i}{c_i}$

② search in the item set S and budget B to find the smallest r_i s.t. all items. j where grater r_j can be paid for with B.

3. determine quantity of each item i we will buy.

Search (S, B)

If $S \neq \phi$ or $B \le 0$ then return error

If $|S| = 1$ then return (v_1/c_1)

else $(|S| > 1 \quad B > 0)$

Let R be the set $\{r_i\}$ for set S.

~~m ← SELECT (R, ⌈|R|/2⌉)~~ ←

Partition S into

$S_1 = \{i \mid i \in S \text{ and } r_i > m\}$

$S_2 = \{i \mid i \in S \text{ and } r_i = m\}$

$S_3 = \{i \mid i \in S \text{ and } r_i < m\}$

Compute the cost of each partition set

$C_1 = \sum_{i \in S_1} c_i \times m_i$

$C_2 = \sum_{i \in S_2} c_i \times m_i$

$C_3 = \sum_{i \in S_3} c_i \times m_i$

if $B \le C_1$ then return (Search (S_1, B))

else-if $C_1 \le B \le C_1 + C_2$ return m

else return search $(S_3, B - C_1 - C_2)$

endif

endif

endSearch

Given ratio r^* from search,

Partition $(r_i, *)$

	R	B	G
C	1	5	10
V	2	50	10
m	10	10	10
r_i	2/1=2	50/5=10	10/10=1

B = 55 50 5

50 + 5

S_3

	A	B	C	D	E	F	G
R_i =	2	1	4	5	10	9	6

S_1 S_2 m_1 m_2

E F

$S_1 = \{ i \in S \mid v_i/c_i > r^* \}$

$S_2 = \{ i \in S \mid v_i/c_i = r^* \}$

$S_3 = \{ i \in S \mid v_i/c_i < r^* \}$

Set $B' = B - \sum_{i \in S_1} m_i \times c_i$

Set Choices $\in \{(i, m_i) \mid i \in S_1\}$

while $B' > 0$ do

Remove item j from S_2.

if $m_j \times c_j > B'$

then insert $(c_j, B'/c_j)$ into "Choice"

exit the while loop.

else insert (c_j, m_j) into "choice."

Reset $B' \leftarrow B' - m_j \times c_j$

end if

end while

$O(n)$