Maximum flow problem

2017년 11월 20일 월요일
오후 3:39

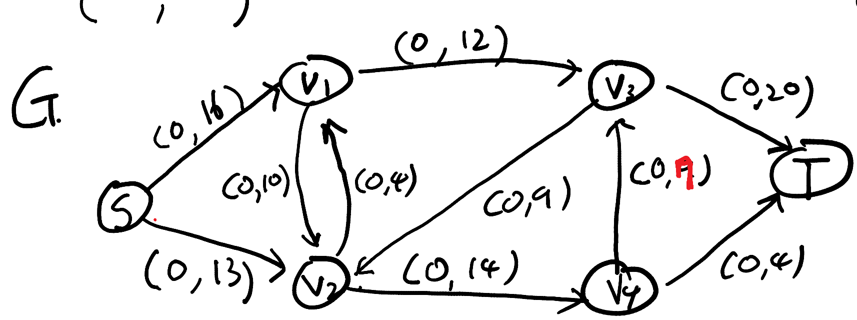
1. Ford - Fulkerson Method

Start with flow $f(u,v)=0$ for all (u,v)
while there $\exists$ an "augmenting path" p do
augment the flow f along the edges of p
endwhile
return f

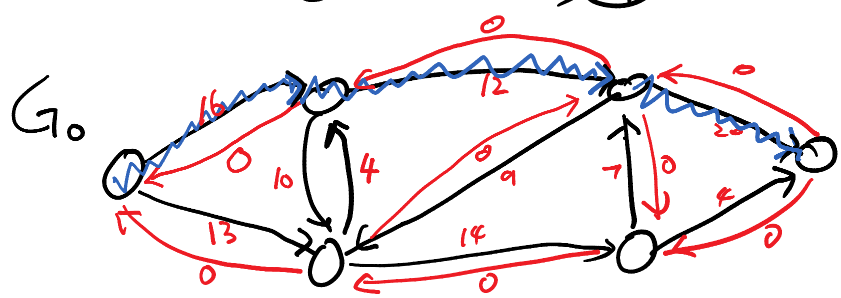
* Residual Network G_f
(flow net. G and flow f)

$$* \; c_f(u,v) = \begin{cases} c(u,v) - \underbrace{f(u,v)}_{\geq 0} & \text{if } (u,v) \in E \\ f(v,u) & \text{if } (v,u) \in E \\ 0 & \text{otherwise} \end{cases}$$

flow ↓ capacity ↓
(,)



max flow 23



Blue path.
12

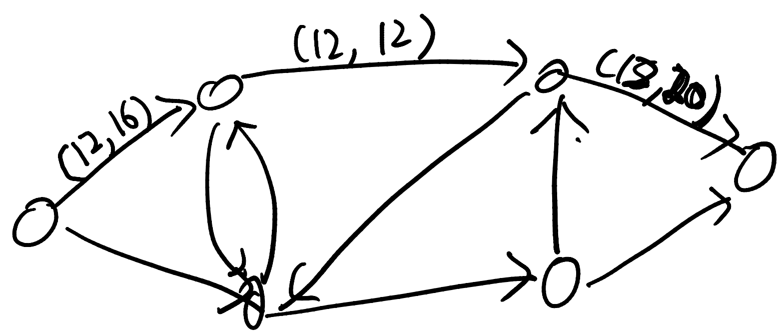
Augmenting Path is path from

s → T in residual Graph G_f with

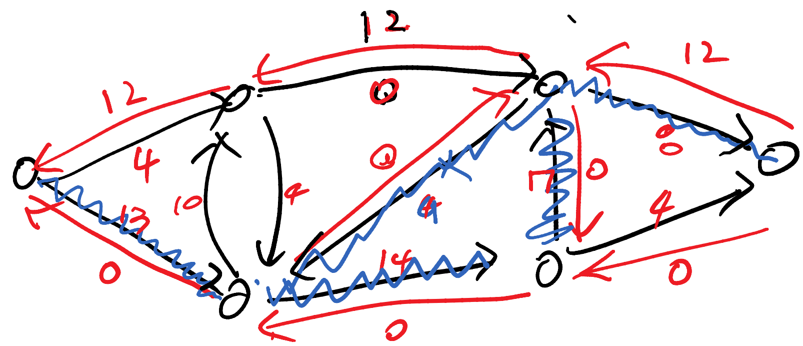
residual capacity of p

$$c_f(p) = \min\{c_f(u,v) \mid (u,v) \text{ in the Path}\}$$

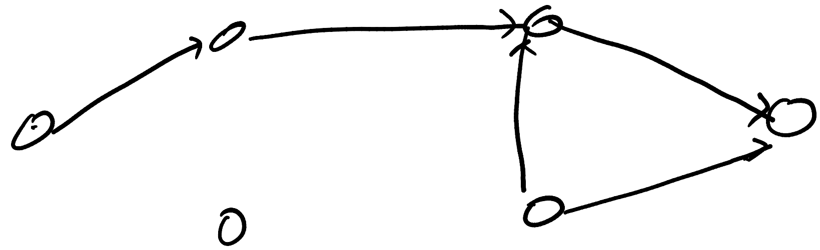
G.



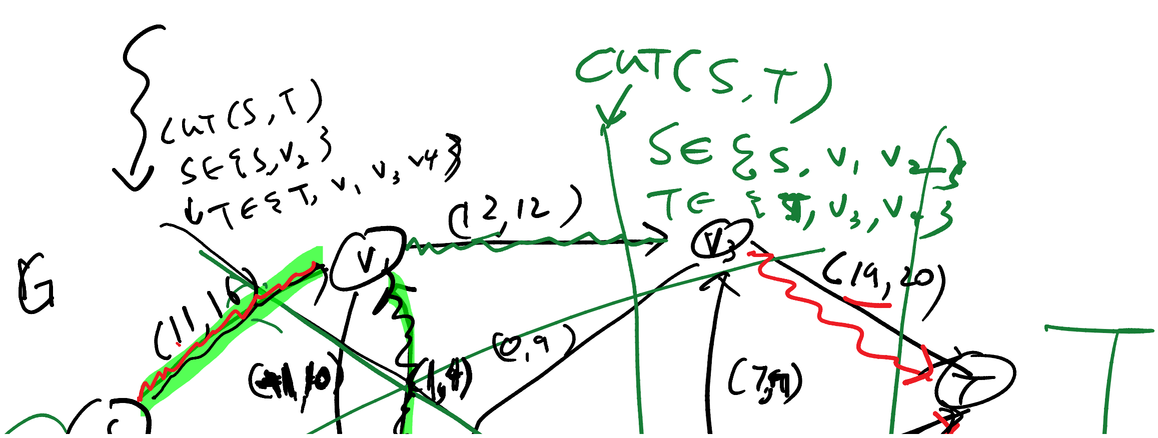
G_0

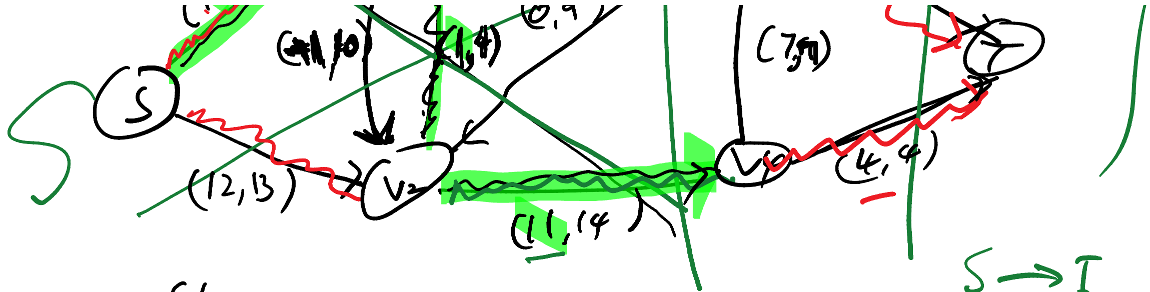


G



net G





max - flow

① $\sum_{(s,v)\in E} f(s,v) = 11 + 12 = 23$

② $\sum_{(v,T)\in E} f(v,T) = 19 + 4$

* CUT(S,t) of flow net :

partitions vertice set V into

S or T = V-S s.t. $s \in S$ and $t \in T$

* flow across a cut (S,T)

$$f(S,T) = \sum_{u\in S}\sum_{v\in T} f(\vec{u,v})$$

* Time of Ford-Fulkerson Method

Assume the cappasity to be integers

for FF. Let Graph be represented by

adj list. Given a flow f, can generate G_f and find an augmenting path in $O(m)$

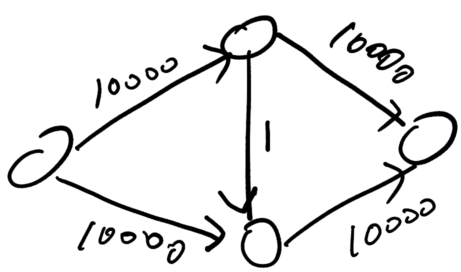
← # of edges

If each capacity is an integer.

$$c_f(P) \geq 1$$

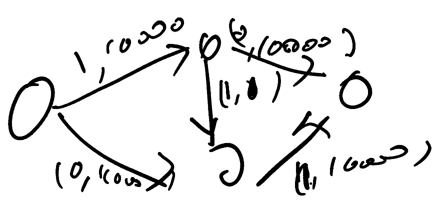
time is $\underline{O(m \times |f^*|)}$ where f^* is flow of maximum value.

ex) G.



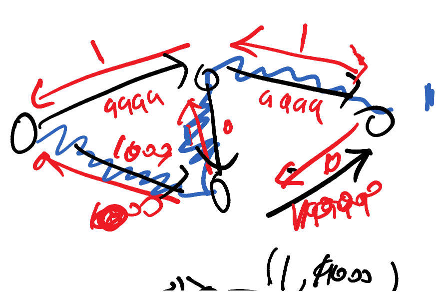
G_f

G



↓1

G_f

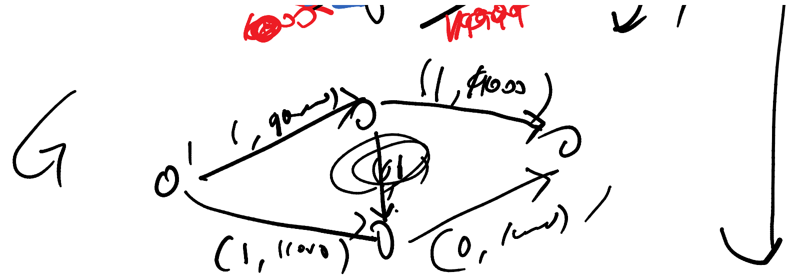


↓1

f^*

↓

20000



Edmonds-Karp Algorithm

* augment along shortest path first
  ≡ fewest edge

Q: Why is shortest path good?

Lemma EK1

For all vertices $v \in V - \{s, t\}$, the shortest distance $d_f(s,v)$ from s to v in residual graph G_f is monotonically non-decreasing as f is augmented in EK algo.