Max-Flow EK

2017년 11월 22일 수요일
오후 4:12

Lemma #2

Total # of flow augmentation by EK algo is $O(mn)$

nodes

edges

proof idea)

m (① at least one edge becomes saturated

② the distance from saturated edge to source s along the aug path must be longer than the last time it was saturated. and that length is at most n.

n

Theorem

The total time for EK algo is

$O(m^2 n)$ $\leftarrow O(mn) \times O(m)$

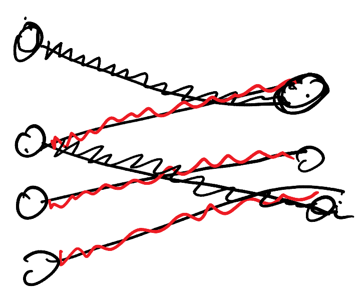
augmenting path

* Application of Max Flow

Maximum Bipartite Matching problem.

Given a undirected graph $G(V,E)$
a match is a subset of $M \subseteq E$ edges
s.t. for any vertex $v \in V$, there is
at most one edge of M incident to it.

ex) L R



← Bipartite graph

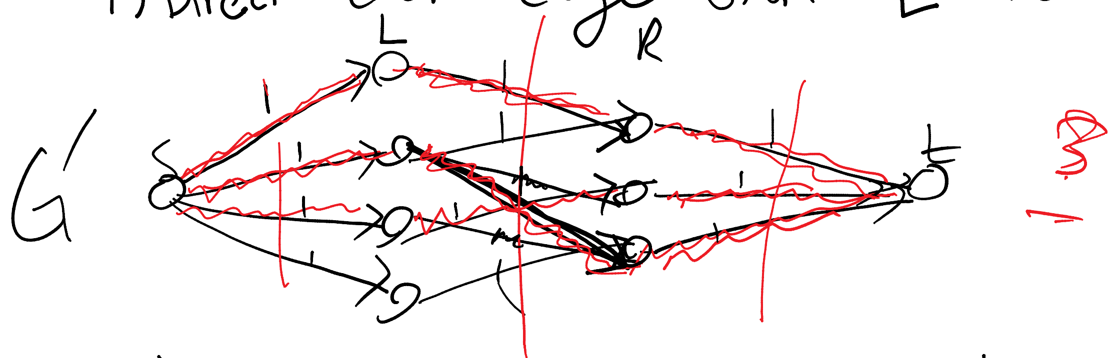
max match : no other matching for that
graph has more edges.

Problem Reduction

Reduce bipartite matching to integer-value max flow

Given undirected graph G, generate G′

1) Direct each edge from L to R



2) introduce two nodes s, t

③ Put directed edge from s to each vertice in L

④ Put " from node in R to t

⑤ put capacity 1 to all edge

solve for max flow

Theorem

If capacity are integers,

the FF finds an integer value flow

corollary

The cardinality of max match in G equals the value of max flow in G' as found in FF method

time * $O(m|F^*|)$

$$|F^*| \le n/2$$

$$\therefore O(mn)$$