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오후 3:58

2. Adversary Argument

⇒ devise a strategy to construct a worst case input for an unknown <u>correct</u> algorithm that solve the problem.

⇒ Adversary interacts with some algorithm in such a way that the worst case input for the algo is generated.

Ex > Element Uniqueness

in a comparison based mode.

Adversary' <u>private</u> DB.

- initially require all the values in set be within interval [0, n]

- all the value v_i will have interval $[a_i, b_i]$ associated with
- each time a comparison is done certain interval may be shrunk.
- if intervals of two values overlaps, then the value could be equal $[^{v_i} \; [\;] \; ^{v_j}]$
- if the algorithm would halt before all intervals are pairwise-nonoverlapping, assuming the adversary has not yet said "=",

then the adversary could answer either ">" or "<".
Thus the algorithm is not correct.

Adversary Action for Comp. E.U.P.

// when algo ask : compare v_i vs v_j

// Adversary Action:

<u>If</u> $i = j$ <u>then</u> answer "$v_i = v_j$" <u>endif</u>

[redacted] ① v_i v_j

$m_i \leftarrow (a_i + b_i)/2$

$m_j \leftarrow (a_j + b_j)/2$

① if $m_i \le m_j$ then

$b_i \leftarrow m_i$

$a_j \leftarrow m_j$ " $v_i < v_j$ "

answer

② else /* $m_i > m_j$ */ then

$a_i \leftarrow m_i$

$b_j \leftarrow m_j$ " $v_j < v_i$ "

answer

endif

Analysis of Adversary Action

When the intervals are nonoverlapping.

their total length $\le n$

$\therefore \sum_{i=1}^{n} l_i \le n$ l_i = length $[a_i, b_i]$ at end of Algo

- when $l_i = n/2^{c_i}$ where c_i is # of interesting comparisons for v_i

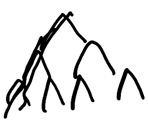
② $m_i \le m_j$ v_i v_j m_j m_i

$$\sum_{i=1}^{n} l_i = \sum_{i=1}^{n} n/2^{c_i} \le n$$

$$\Rightarrow \sum_{i=1}^{n} 1/2^{c_i} \le 1$$

Claim

$\underbrace{\sum_{i=1}^{n} c_i}_{\text{time complexity}}$ is minimized when.

 $c_j = c_k$ for all j, k

assume

$$1/2^{c_i} \le \frac{1}{n} \qquad 2^{c_i} \ge n$$

$$c_i > \lg n$$

$$\sum_{i=1}^{n} c_i \ge n \lg n$$

$\therefore$ Thus ~~dt~~ U.P. is $\Omega(n \lg n)$