CSE548 Quiz 2017-09-18 Name: ______________________________

1. Assuming the following recurrence.

$$T(n) = c \qquad \text{if } n < 7$$

$$T(n) = 2T(\lfloor n/7 \rfloor) + T(\lceil 2n/5 \rceil) + cn \qquad \text{if } n \geq 7$$

Find and prove a <u>good</u> bound on T(n).

2. Let there be a sequence of numbers with the following properties:

$$L(0) = 0, L(1) = 1$$

$$L(n) = 3L(n-1) + L(n-2) \qquad \text{if } n > 2,$$

where L(n) denotes the nth number. Obvious method for computing the series of numbers with the above properties is by computing each value in the sequence L(0), L(1), …, L(n) in turn, taking constant time per value, by using previously-computed values.

We claim that sequence of numbers has the following property:

$$L(n) = L(a+1)L(n-a) + L(a)L(n-1-a)$$

for a ≥ 0 and $n \geq a + 1$.

Prove that this is true by induction on ***a*.**

Basis (a = 0)

Induction step (a>0)

Assume that the claim holds for a-1

$$L(n) = L((a-1)+1)L(n-(a-1)) + L(a-1)L(n-1-(a-1))$$