Slotted Aloha Multiaccess Protocol

CSE 390 Class Notes

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1 Slotted Aloha Protocol Model

In the idealized slotted aloha model that we are considering here, time is slotted and all packets are of equal length. Packet transmission time is one full slot. Packets are transmitted in the next slot after they arrive. We also assume that there is no buffering, i.e., it is never the case that a station has more than one packet to transmit in a single slot. If that is the case, the station has to buffer one or more packets for later transmittal. To accommodate the "no buffering" assumption, we assume that there are infinite number of stations and that each new arrival happens at a new station. Note that in the absence of the "infinite stations" assumption, we have to model packet buffering at each node. This is what will happen in a real implementation; but this will complicate the analysis.

If more than one station transmits packets in the same slot, there is a *collision*, and the receivers cannot receive the packets correctly. Successful transmission happens only when there is exactly one packet transmitted in a slot. If no packet is transmitted in a slot, the slot is called *idle*.

Assume that there is an immediate feedback from the receiver about the status of each transmitted packet (i.e., correctly received or not). This feedback is not modeled within our description of the slotted aloha protocol. But in case of a collision, the colliding packets are retransmitted at a later slot after a randomly chosen backoff period. Such packets are also called *backlogged* packets.

2 Analysis

Assume that new packets are arriving at the network at the aggregate rate of λ packets/slot. The actual load on the network is G packets/slot. $G \geq \lambda$ because G includes packets that are retransmitted after collision. The throughput (i.e., rate of successful transmission) of the network is S packets/slot ($S \leq G$).

Assume that packet transmissions in the network is modeled by the Poisson distribution. Poisson is a good distribution to use when there are a very large number of stations using a network but each contributing to a very small fraction of the total load. The probability $P[k]$ that k packets are actually transmitted in a given slot is given by the probability mass function of the Poisson distribution, which is

$$P[k] = \frac{G^k e^{-G}}{k!}. \tag{1}$$

G is the mean of this distribution, meaning that on average G packets are transmitted in each slot. Mathematically, the mean of a discrete distribution such as this, is given by,

$$\text{Mean} = \sum_{k=0}^{\infty} k.P[k], \tag{2}$$

which evaluates to G in this case.

The probability of zero transmissions ($k = 0$) in a slot (i.e, probablity of an idle slot) is e^{-G}. The probability of exactly one transmission ($k = 1$) (i.e., a successful slot) is Ge^{-G}. The probability of a colliding slot thus is given by,

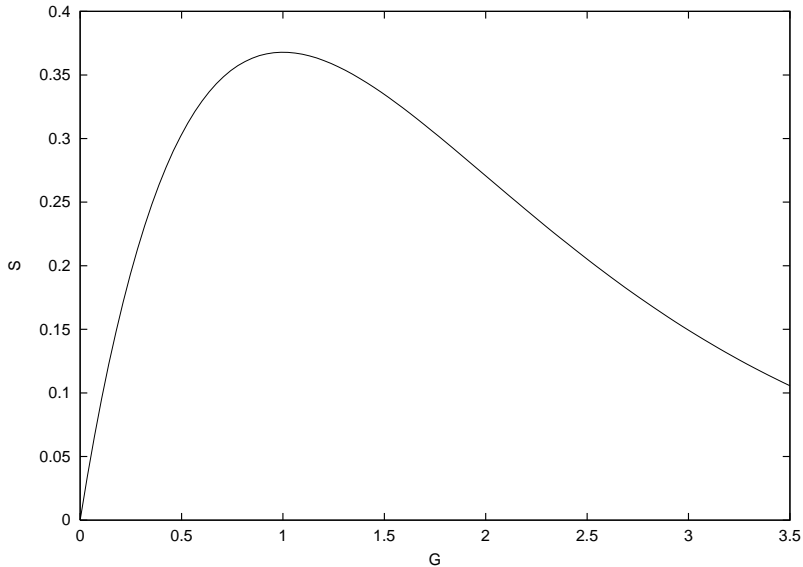$$1 - P[0] - P[1] = 1 - e^{-G}(1 - G). \tag{3}$$

Figure 1: Plot of S (throughput) versus G (network load) in slotted aloha. Note that the maxima occurs at $G = 1$.

Throughput S is the rate of successful transmissions in packets/slot. If each slot has a Ge^{-G} probability of carrying a successful transmission, S is also Ge^{-G}. Average number of attempts made by a packet for transmission is given by $G/S = e^{-G}$.

It is instructive to understand the behavior of S versus G. See Figure 1 for the plot. The maximum of this plot occurs at $G = 1$, where $S = 1/e \approx 0.368$, indicating that the maximum efficiency of slotted aloha is just 36.8%. Note that for $G < 1$, the throughput is less than the maximum possible, because of too many idle slots. For $G > 1$, more than 1 packets are transmitted per slot on an average. This generates too many collisions.

Note that the analysis approach presented here is elementary. This does not provide much feedback into the dynamics of the system. As the number of backlogged packets changes, the load G also changes, leading to a feedback effect, as an increase in G is likely to encounter more collisions and a further increase in G. However, the analysis above correctly identifies the fact that increase in G beyond 1 is not good for performance.