Multiscale Distance Matrix for Fast Plant Leaf Recognition

Rongxiang Hu, Wei Jia, Haibin Ling and Deshuang Huang

***Abstract*—In this paper, we propose a novel contour-based shape descriptor, named Multiscale Distance Matrix (MDM), to capture shape geometry while being invariant to translation, rotation, scaling, and bilateral symmetry. The descriptor is further combined with dimensionality reduction to improve its discriminative power. The proposed method avoids the time-consuming point-wise matching used in most of the previous shape recognition algorithms. It is therefore fast and suitable for real-time applications. We applied the proposed method to the task of plan leaf recognition with experiments on two datasets: the Swedish Leaf dataset and the ICL Leaf dataset. The experimental results demonstrate clearly the effectiveness and efficiency of the proposed descriptor.**

***Index Terms*—Shape recognition, plant leaf, multiscale distance matrix, inner distance, cost matrix**

I. Introduction

Shape is one of the most important features of an object. It plays a key role in many object recognition tasks, in which objects are easily distinguished by shape rather than other features such as edge, corner, color, and texture. There are usually two critical parts in a shape recognition approach, shape representation and shape matching. According to choices of shape representation, shape recognition approaches can be generally divided into two classes, i.e., contour-based and region-based, respectively [1].

In the past decade, research on contour-based shape recognition [2-17] is more active than that on region-based due to the following reasons [1]: Firstly, human beings are thought to discriminate shapes mainly by contour features. Secondly, in many shape applications only the shape contour is of interest, while the interior content is less important. Similarly, in this paper, we focus on contour-based shape recognition. Several important contour-based approaches have recently been proposed. Petrakis *et al*. [3] presented an effective contour-based approach using Dynamic Programming (DP), which is invariant to translation, scaling and rotation. Belongie *et al*. [2] proposed a shape feature called Shape Context (SC), which describes a shape by a set of 2-D histograms capturing landmark distributions. Ling *et al*. [10] extended SC to the Inner-Distance SC (IDSC) by replacing the Euclidean distance with the articulation insensitive inner-distance. McNeill *et al*. [6] introduced a multiscale shape matching algorithm named Hierarchical Procrustes Matching (HPM), which investigates shape matching at a variety of different positions. Felzenszwalb *et al*. [9] described a hierarchical shape representation called Shape Tree (ST) to capture shape geometry at different levels of resolution. Xu *et al*. [13] proposed a shape descriptor called Contour Flexibility (CF), which represents the deformable potential at each point on the contour. From these approaches, we conclude that the relative positions between the contour points contain rich information about the structure of objects, and a multiscale representation can better capture the geometric propensities of a shape.

Although the aforementioned contour-based approaches have reported promising recognition performances, they have to face a *crucial* problem, i.e., how to solve the correspondence between two shapes in the matching stage. The solution often requires computing the distance between the two shapes as a sum of matching errors between corresponding points or segments. Many existing contour-based approaches have applied DP procedures to address this problem, which is very time consuming [2-4]. As a result, alternative solutions that are computationally more efficient are desired for real-time applications [1].

In this paper, we propose a novel contour-based shape descriptor named Multiscale Distance Matrix (MDM) to capture the geometric structure of a shape while being invariant to translation, rotation, scaling, and bilateral symmetry. When applying MDM to shape recognition, there is no need to use DP to build point wise correspondence, which makes MDM an efficient shape descriptor. In addition, MDM is flexible in the underlying building distances: either the Euclidean distance or other metrics can be utilized in MDM to compute the dissimilarity of two shapes. Furthermore, we apply dimensionality reduction methods to MDM to extract discriminant information, which further improves the efficiency and accuracy of the proposed method. Compared with other contour-based approaches such as SC and IDSC, MDM can achieve comparable recognition performance while runs much faster.

We applied the proposed method to plant leaf recognition tasks, which has been attracting research attention recently [4]. Automatic plant leaf recognition is very important for phytotaxonomy [18] and real time performance is often desired in electronic field guide or online retrieval systems. Our experiment is conducted on two plant leaf datasets: the Swedish Leaf [19] and the ICL Leaf [22]. The Swedish Leaf, a

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well-known public dataset established by Soderkvist, has been tested by many shape recognition approaches [4, 9, 10, 12, 16, 17, 20, 21]. It however involves only 15 species, which makes the analysis on it less effective to generalize. Recently, in order to provide a larger dataset for more thorough evaluation, we collected the ICL Leaf dataset containing 6000 plant leaf images from 200 species, which can be freely downloaded from [22].

The rest of this paper is organized as follows: Section II gives the definition of MDM. Then Section III reports the experimental results. Finally, Section IV concludes the whole paper.

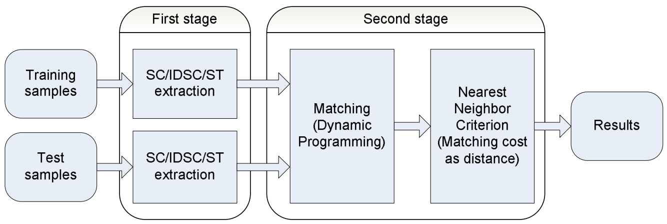

Fig. 1. Two-stage scheme for local-based approaches.

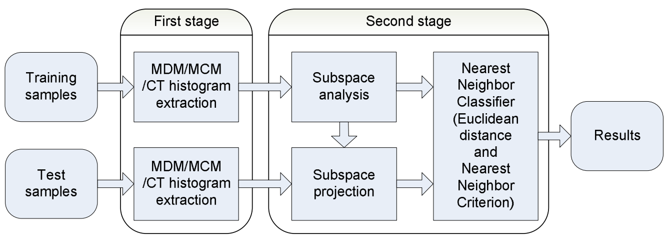

Fig. 2. Two-stage scheme for global-based approaches.

II. Multiscale Distance Matrix

A. Two-stage scheme for shape recognition

A typical contour-based shape recognition approach contains two stages: shape representation and shape matching. We can divide contour-based approaches into two subcategories, i.e., local-based and global-based, as shown in Fig.1 and Fig.2, respectively. The local-based approaches extract local features of a shape and then find the correspondence of contour points in the matching stage, while the global-based approaches extract global features of a shape, and perform the matching without finding the correspondence of contour points. Obviously, all previous approaches we have mentioned in Section I are local-based approaches.

On the other hand, global approaches have also been applied for shape matching. One example is the Principal Component Analysis of Census Transform histograms (PACT) [21]. In PACT, Census Transform (CT) histograms are used to summarize local shape information into global features, and Principal Component Analysis (PCA) [23] is then applied to the CT histograms to extract the most important components among the distribution of CT histograms. PACT, however, is not invariant to rotation and scaling. Our proposed MDM, in comparison, is a global-based feature and invariant to rotation and scaling. Therefore, it has wider range of application for shape recognition problems, at the same time it is also as efficient as PACT.

B. MDM

We define a shape O as a connected and closed subset of $\boldsymbol{R}^2$. Given n sample points $p_1, p_2, \ldots, p_n$ on the contour of shape O with certain order that each point has a coordinate (x_i, y_i), an $n \times n$ distance matrix D can be constructed, where D_{ij} denotes the Euclidean distance between point p_i and point p_j. Obviously, this distance matrix is symmetric, with all diagonal entries being zeros.

Based on D, the MDM can be computed by the following steps:

1) For each column of matrix D, it is shifted up circularly so that the first element becomes zeros. This way, a new matrix D_m is constructed in which the first row has straight zeros.

2) For each row of D_m, its elements are sorted ascendently. This generates a matrix D_{ms}.

3) For D_{ms}, we remove its first and the last $\left\lfloor \frac{n-1}{2} \right\rfloor$ rows to construct a new matrix, which is the basic MDM.

An example for how to construct MDM is illustrated in Fig.3, where n is 4.

For the matrix D_{ms}, each row captures certain range of geometric properties of a shape. For example, the entries of the first row are all zeros, which are the distances between each point and itself. The entries of the second row are all the distances between points and the points next to them, which captures the finest level geometric properties of the shape. For the rest rows of D_{ms}, the entries capture coarser level geometric properties. As the row moves down, the entries can capture even coarser level geometric properties until the row number reaches $\left\lfloor \frac{n}{2} \right\rfloor$, where the coarsest level geometric properties present. It is easy to see that, for a closed contour, the second row of D_{ms} is identical with the last row, both of which depict the distances between the points with interval of 1. For similar reason, about half of rows of D_{ms} are redundant and should be removed.

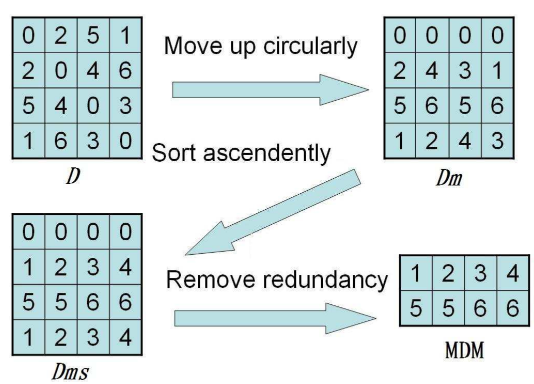

Fig. 3. An example illustration of constructing MDM

In Fig.4, an example of MDM extracted from a leaf shape is given. For this leaf shape, 64 points are sampled on the contour, thus the size of its MDM is 32×64. In the feature matrix (Fig.

4(e)), blue entries represent small values while red entries represent large ones. We select four rows of the matrix to illustrate the features, where the distances between points are in green lines. Fig. 4(a), (c) (d) and (f) depict row 1, 8, 16, 32 of the MDM, respectively. It can be noticed that the first row depicts the finest level of the shape and the last row depicts the coarsest level of the shape.

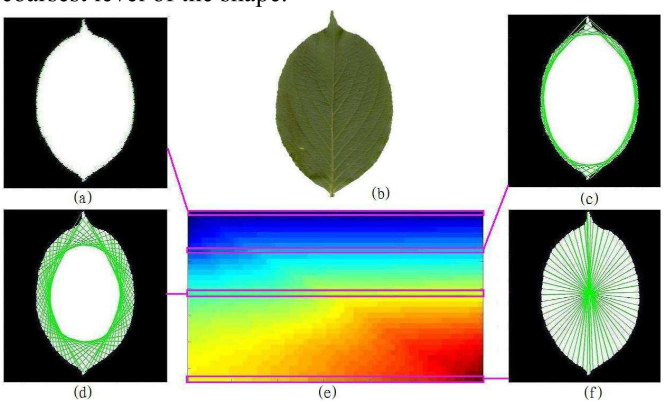

Fig. 4. Different rows of MDM of a leaf. (b) The leaf image, (e) The extracted MDM of the leaf. The green connections in (a) (c) (d) (f) correspond to the first, eighth, sixteenth and last row of the MDM, respectively.

Given the contour of a shape in an image, the translation of the image has nothing to do with the distances between contour points, so the MDM is invariant to translation.

Generally, the rotation of image will change the order of the contour points. When computing MDM, the sorting step naturally neglects the original order of the points set and brings the invariability to rotation. But this step also results in the missing of some important discriminant information of the original distance matrix. In order to keep as much information as possible in D_m, we compute the differences between neighbor elements along each row of D_m circularly and sort them ascendently too. In particular, for a row of D_m with entries $d_1,d_2,\ldots,d_n$, the circular differences of this row are $d_2-d_1,d_3-d_2,\ldots,d_1-d_n$. Then we can add such difference matrix to the bottom of the MDM to construct a new MDM-CD matrix, which doubles the size of MDM.

Note that clockwise and counter-clockwise contours of a shape will result in different order of columns of the matrix D_m, but will not affect the matrix D_{ms}. Consequently, both orders generate exactly the same MDM descriptors. In other words, the MDM is insensitive to bilateral symmetry of the shape. In many cases, shapes with bilateral symmetry should be classified to one class, but this invariability may not be a suitable property in some applications.

To make the MDM invariant to scaling, we introduce five normalized versions of MDM, i.e., MDM-M, MDM-A, MDM-C, MDM-RM and MDM-RA, using different normalization schemes. MDM-M, MDM-A and MDM-C normalize the MDM with, respectively, the maximum value of the whole matrix, the average value of the whole matrix and largest distance of the contour points to the centroid of the shape. MDM-RM and MDM-RA normalize each row of the MDM with the maximum value of each row and the average value of each row, respectively.

We now apply dimensionality reduction to the proposed descriptors for improving their efficiency and effectiveness. Generally speaking, dimensionality reduction methods seek to find a low-dimensional subspace in a high-dimensional input space by linear transformation. This low-dimensional subspace can provide a compact representation or extract the most discriminant information of the high-dimensional input data. It is well known that PCA [23] and linear discriminant analysis (LDA) [24] are two typical dimensionality reduction methods, which could be regarded as the simplest unsupervised and supervised dimensionality reduction methods, respectively. Usually, in most cases LDA could achieve better recognition performances than PCA. However, traditional LDA suffers from the Small Sample Size (SSS) problem, and the solution is approximate and sometimes unstable. To address this problem, Jia *et al*. [25] proposed the Decomposed Newton's Method (DNM) to solve LDA in an iterative way. Li *et al*. [26] proposed the Maximum Margin Criterion (MMC) to change the objective function of LDA from the form of ratio to the form of difference. In fact, DNM and MMC are as effective as LDA, but do not suffer from the SSS problem. In our scheme, for the sake of simplicity and robustness, we apply DNM and MMC for dimensionality reduction after extracting MDM features from shape contours, and then use Euclidean distance and the nearest neighbor rule (1NN) for classification.

C. Extension of MDM

The MDM presented above uses the Euclidean distance when building the distance matrix D. We can extend MDM by using different distance measures. One such measure is the inner-distance (ID) used in IDSC [10] to achieve the articulation insensitivity by replacing Euclidean distance in SC [2]. The inner-distance between two points is the length of the within-shape shortest path between these two points, and the computing of the inner-distance matrix could refer to [10]. To further explore the discriminability of MDM, we extend the matrix D by using the inner-distance, and keep unchanged other components. We name such descriptor as Multiscale Distance Matrix with Inner-Distance (MDM-ID).

Besides the Euclidean distance and the inner-distance, other dissimilarity measures between a pair of points can be used to extend the MDM descriptors. These metrics may not have clear geometric explanations, but they may be effective and useful to certain applications. For example, in MDM, we can also use the histogram comparison-based cost distance matrix, like the one used in [2,10]. In this case, each entry in the distance matrix is the matching cost of two points on different shapes, where each point is represented by its SC or IDSC histogram. To compute the dissimilarity between points on the same shape we only need to apply the histogram comparison method on one shape and its own. We call it Multiscale Cost Matrix (MCM). Generally speaking, each row of MCM can capture the statistical property of dissimilarity between contour points in a certain scale. Note that MCM is irrelevant to the real geometric distance and is intrinsically invariant to scaling, so it does not have the versions with different normalizations.

III. Experiments and Discussions

To validate the proposed methods, we apply them to leaf shape classification tasks using two leaf datasets: the Swedish Leaf dataset and the ICL Leaf dataset. The latter is collected by ourselves to compensate to the limitation of the former. Note that there is another public dataset, the Smithsonian Leaf dataset [10] which contains 343 leaves from 93 species. It, however, has in average less than four images per species and therefore not suitable for an extensive experiment.

In this paper, we compare the recognition performance of the proposed methods with IDSC and the classic Fourier shape descriptor (FD) [1] for several reasons. Firstly, IDSC has been successfully applied for foliage shape analysis and has been used in a real electronic field guide system [4,10]. Secondly, IDSC is a typical local-based shape descriptor and has a computation complexity similar to many others, which makes it a reasonable choice for comparison since one of our major concern is about efficiency. Thirdly, FD is a classic global-based descriptor that runs very fast. It is therefore an excellent choice for evaluating the accuracy-efficiency trade-off of the proposed methods. Other methods mentioned in Section I such as ST [9], HPM [6] and CF [13] are difficult to be implemented fairly due to lacking some technical details in literatures.

For each shape, we uniformly sample 128 points on its contour and build MDM over these points. Sample points are used for IDSC as well. Notice that to apply the dimensionality reduction methods, the MDM is transformed into a vector and the final feature dimension is experimentally determined, ranging from 20 to 50. Notice that DNM and MMC have quite similar objective function and similar performances on most datasets, but none of them is always superior to the other. So in our experiments, we report the best results among them on the Swedish Leaf dataset and ICL Leaf dataset, respectively.

A. *Experimental results on Swedish Leaf dataset*

The Swedish Leaf dataset [19] contains isolated leaves from 15 different Swedish tree species, with 75 leaves per species. Fig.5 shows some representative examples. Following the protocol of previous works [9, 12, 19, 21], the first 25 images from each class are used for training and the rest 50 images are exploited for test.

In Table I, we report the recognition rates of different versions of MDM, in which the method of DNM is used for dimensionality reduction. In this table, we also list the recognition rates of IDSC and FD reported in [10]. From the results we have several observations: First, the recognition performances of MDM versions with '-CD-' are better than that of the basic versions, which suggests the usefulness of circular difference in MDM-CD. Second, the recognition performances of MDM versions with '-RA' or '-RM' are a little worse than

Fig. 5. Eight samples from Swedish Leaf dataset

Fig. 6. Eight samples from Clean Swedish Leaf dataset

TABLE II
Recognition rates (%) on Clean Swedish leaf

IDSC	MDM-CD-C	MDM-CD-M	MDM-CD-A
85.07	91.07	91.20	**91.33**
FD	MDM-ID-CD-C	MDM-ID-CD-M	MDM-ID-CD-A
83.60	89.60	90.80	90.67

the ones of the versions with '-A' or '-M' or '-C'. This indicates that normalizing the matrix along each row is less effectively than normalizing the matrix globally. Third, for MDM the inner-distance based versions with labels '-ID-' achieve better performances than Euclidean distance based versions do, which demonstrates the merits of inner-distance on this dataset.

It should be noted that the original Swedish leaf images contain footstalks. Obviously, the length and orientation of those footstalks heavily depend on the collecting and imaging process. Though these footstalks might provide some discriminant information for recognition, they may be unreliable when extracting from the images. For this reason, we cut them off to construct a clean dataset, named the Clean Swedish Leaf, as shown in Fig.6. Here, we only test the versions of MDM with '-CD-' on the Clean Swedish Leaf dataset, since they have better recognition performances in the experiments on original Swedish Leaf dataset. The experimental results obtained from Clean Swedish Leaf dataset are listed on Table II. We notice that the performance of IDSC decreases dramatically from 93.73% to 85.07% in this experiment, which may indicate that those footstalks indeed provide useful information for recognition and the method of IDSC can not benefit from them in the clean dataset anymore. All performances of MDM versions decline too, but within a much small range. The version of MDM-ID-CD-M decreases only from 93.60% to 90.80%, and the version of MDM-CD-A

TABLE I
Recognition rates (%) on The Swedish leaf dataset

IDSC[10]	MDM-M	MDM-A	MDM-RM	MDM-RA	MDM-C
	92.53	92.40	91.20	91.60	92.40
94.13	MDM-CD-M	MDM-CD-A	MDM-CD-RM	MDM-CD-RA	MDM-CD-C
	93.33	92.67	92.13	92.53	93.20
FD[10]	MDM-ID-M	MDM-ID-A	MDM-ID-RM	MDM-ID-RA	MDM-ID-C
	92.67	92.67	91.60	92.13	93.07
89.60	MDM-ID-CD-M	MDM-ID-CD-A	MDM-ID-CD-RM	MDM-ID-CD-RA	MDM-ID-CD-C
	93.60	92.80	92.53	92.67	93.47

TABLE III
RECOGNITION RATES (%) ON THE ICL PLANT LEAF SUBSET

Subset	Situations	IDSC	FD	MDM-CD-C	MDM-ID-CD-C	MCM-CD	MCM-ID-CD
A	30T15	**95.79**	92.90	95.47	94.97	95.73	94.91
	30T29	98.00	96.00	**98.20**	**98.20**	**98.20**	97.76
B	30T15	65.42	60.00	**68.75**	68.13	63.25	55.13
	30T29	70.32	66.56	**74.20**	74.08	70.44	59.64
C	30T15	63.99	59.37	73.88	**73.93**	67.27	63.95
	30T29	66.64	62.96	**80.88**	80.80	73.52	69.72

achieves the best performance of 91.33%. We should notice that the extended versions with inner-distance achieve worse performances than the basic Euclidean distance based MDM on this dataset.

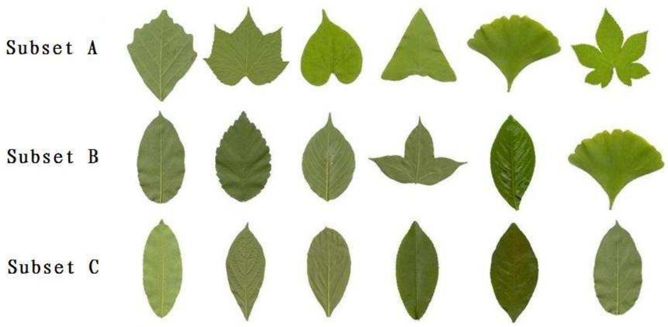

Fig. 7. Samples of different species from three subsets

In IDSC, the distance is in the log space and is insensitive to long footstalk length, while in MDM linear space of distance is used. So IDSC is more robust to different footstalk lengths, and this may explain why IDSC achieves better performance than MDM methods on the original Swedish Leaf dataset. The inner-distance is insensitive to articulation variations, which makes the methods based on this distance metric more robust to footstalk changes than those methods based on Euclidean distance, so the versions of MDM with label '-ID-' yield better performance in the first experiment. While on the Clean Swedish Leaf dataset, the merits of inner-distance are no longer available and the traditional Euclidean distance seems to better represent the geometric statistical properties of the shape. For the same reason, IDSC could no longer benefit from the footstalks and its performance decreases dramatically.

B. Experimental results on ICL Leaf dataset

In the ICL Leaf dataset, all images were collected at the Botanical Garden of Hefei, Anhui Province of China by the members of Intelligent Computing Laboratory (ICL) in Institute of Intelligent Machines, Chinese Academy of Sciences. The ICL Leaf dataset contains 6000 plant leaf images from 200 species, in which each class has 30 samples. All the footstalks have been cut off and this dataset is totally clean. We intended to use the whole dataset to test the algorithms, but the DP process in IDSC need a long time to complete the matching procedure when the number of samples is large. So we construct three subsets, named subsets A, B, and C, from the whole dataset for performance evaluation. Each subset contains 50 species and each class contains 30 samples. In subset A, all classes are carefully selected and most of the shapes could be distinguished easily by human. In subset B, all classes are randomly selected from the whole dataset, which approximates the distribution of the whole dataset while being smaller in size, and there are some classes with similar shapes. In subset C, all classes are also carefully selected and most of the shapes are similar but still distinguishable. Fig.7 shows some examples from these three subsets. Obviously, subset B is more difficult than subset A for recognition, and subset C is the most difficult for recognition among three subsets. We test methods in two situations. In the first situation, half samples of each class are selected for training, and the rest 15 samples are used for test. In the second situation, 29 samples of each class are randomly selected for training and the rest 1 sample of each class is used for test. All these tests are repeated for 50 times, consequently, the average recognition rates are reported.

TABLE IV
EXECUTIVE TIME (S) OF IDSC, FD AND MDM ON DIFFERENT SITUATIONS

Situations	IDSC	FD	MDM
Test 30T15	27344	8	37
Test 30T29	3481	7	55
One-to-one Matching	7.295	0.001	0.001

With extensive experiments of different MDM versions, we find that for all three subsets, the versions with label '-C-' achieve the best performances in most cases. So in table III, we list them and results from MCM on all three subsets, and MMC is applied for dimensionality reduction. Meanwhile we report the recognition rates of IDSC and FD for comparison. From the results we can see: First, on subset A the MCM methods achieve the best among all methods, while on subsets B and C the MCM methods perform the worst in all the situations. This indicates that different distance metrics have quite different properties and different suitable situations. Second, from subset A to B to C, the difficulty of recognition is increasing as the performances of IDSC demonstrate. Surprisingly, though the performances of MDM methods decrease from subset A to B, they show much better discriminability on subset C than B. This proves the potential of MDM methods to classify shapes with small variations due to the further discriminant feature extraction of dimensionality reduction methods.

In the first two rows of Table IV, we report the average CPU time of completing one test including feature extraction and matching, which gives us a rough feeling of the advantage of MDM. Since the final feature of MDM is only a vector, Euclidean distance is directly computed for matching, on the contrary, DP is applied to match two SCs. We would only regard the computing of Euclidean distance or DP as matching while other processes are feature extraction. All algorithms are coded in Matlab except that the DP procedure of IDSC is coded in C in the mex form. Then we show the comparison of one-to-one matching time in the third row, where the DP procedure of IDSC is also coded in Matlab rather than in C. In both cases, the MDM approach costs much less time than IDSC. Noted that in the ICL Leaf dataset, all leaf images have be rotated to a canonical direction, which means that no starting point difference needs to be taken into account in these experiments. While in a real application, either direction normalization or a more adaptable matching is necessary. This would affect the matching of IDSC, but would not affect the matching of MDM. So IDSC needs more time to match two common leaves, and the MDM approach is more suitable for a real-time recognition system.

IV. Conclusions

In this paper, we proposed a novel shape descriptor, i.e., MDM, and its extensions to capture multi-scale geometric prosperities of a shape. In leaf recognition experiments on two datasets, our approach achieves comparable or better performances in comparison with IDSC, which shows our approach is much more efficient and effective. Besides good recognition performance and invariability to translation, rotation, and scaling, the proposed method for shape recognition has three additional advantages in comparison with the state-of-the-art approaches:

1) Significantly fewer parameters to tune. Specifically, only one parameter is needed, i.e., the number of points on the shape contour;
2) Extremely fast evaluation speed compared with DP-based procedure;
3) Very easy to implement since it is based only on the distance matrix of the shape.

There are several important issues about the MDM that have to be addressed here. First, to compute the MDM, the distance matrix of the shape boundary points are assumed to be known. This limits the proposed approach to those applications where the segmentation of the shape image is unstable or unavailable. Second, the metric selection is critical, and the discriminability highly depends on the properties of the metric. If the metric is sensitive to shape topology such as inner-distance, the methods may cause some problems. For example, occlusion may cause the topology of shapes to change, and in such cases MDM will surely have a degrading performance due to the disadvantages of the metric. Third, MDM is data-independent while the dimensionality reduction used is data-dependent, so the proposed approach may be limited in shape recognition applications. At last, though we only tested the proposed method on two leaf shape datasets, the MDM approach could be applied to other shape recognition tasks involving single closed contours. How to extend MDM to capture the features of an open curve or a multi-contour shape is our future research focus.

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